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Stirling Functions

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Abstract: We apply the operators of central differences to the Stirling functions to obtain identities involving the ascending factorial fucntion.

Key words: Interpolation by central differences · Stirling functions · Ascending factorial function

INTRODUCTION

In the interpolation by central differences [1-4] are important the following operators:

$$\delta f(x) = f(x+1) - 2\,f(x) + f(x-1), \qquad \gamma f(x) = \frac{1}{2}\,[f(x+1) - f(x-1)], \tag{1}$$

such that:

$$[\gamma \delta^k f(x)](0) = \frac{1}{2(k+1)}\,\left[\delta^{k+1}\left(x f(x)\right)\right](0), \qquad [\delta^j g(x)](0) = \sum_{r=0}^{2j} (-1)^r \binom{2j}{r} g(j-r), \tag{2}$$

in particular, for arbitrary functions $h(x)$ [even] and $p(x)$ [odd]:

$$[\gamma \delta^q\, h(x)](0) = 0, \qquad [\delta^q p(x)](0) = 0, \quad q \geq 0. \tag{3}$$

In Sec. 2 the relations (3) are applied to the Stirling functions to obtain identities for the ascending factorial function [5].

Stirling Functions: The Stirling expansion to interpolate by central differences involves the following odd and even functions [1-4]:

$$\Phi_{2k-1}(x) = \frac{1}{(2k-1)!}\,(x-k+1)_{2k-1}, \qquad \Phi_{2k}(x) = \frac{x}{2k}\,\Phi_{2k-1}(x) = \frac{x}{(2k)!}\,(x-k+1)_{2k-1}, \tag{4}$$

with $\Phi_0(x) = 1$ and $\Phi_{-r}(x) = 0$, $r \geq 1$, which have interesting properties, for example:

$$x\,\Phi_{2k}(x) = (2k+1)\Phi_{2k+1}(x) + \frac{k}{2}\,\Phi_{2k-1}(x), \tag{5}$$

that is:

$$(x^2 - k^2)\binom{x+k-1}{2k-1} = 2k\,(2k+1)\binom{x+k}{2k+1}, \tag{6}$$

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and also:

$$\Phi_k(x+1) + \Phi_k(x-1) - 2\Phi_k(x) = \Phi_{k-2}(x), \tag{7}$$

or equivalently:

$$(m-1)\,[(x+1)\binom{x+m}{2m-1} + (x-1)\binom{x+m-2}{2m-1} - 2x\binom{x+m-1}{2m-1}] = m\,x\,\binom{x+m-2}{2m-3}. \tag{8}$$

The application of (2) and (3) to the Stirling functions (4) gives the following identities involving the ascending factorial function [5]:

$$\sum_{j=-(q+1)}^{q+1} (-1)^j \binom{2q+2}{q+1+j} j^2 (j+1-k)_{2k-1} = 0, \quad \sum_{j=-q}^{q} (-1)^j \binom{2q}{q+j} (j+1-k)_{2k-1} = 0, \tag{9}$$

which could be written in terms of Stirling numbers of the first kind because [5-7]:

$$(x)_k = \sum_{m=0}^{k} (-1)^{k-m}\; S_k^{(m)}\; x^m\,. \tag{10}$$

It is interesting indicate the transformation of the Stirling functions under the action of the operators (1):

$$\delta\Phi_j(x) = \Phi_{j-2}(x), \quad \forall j, \qquad \gamma\Phi_{2k-1}(x) = \Phi_{2k-2}(x), \qquad \gamma\Phi_{2r}(x) = \Phi_{2r-1} + \tfrac{1}{4}\,\Phi_{2r-3}\,. \tag{11}$$

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