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Andersson-Edgar's Potential

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Abstract: We indicate particular cases of the Andersson-Edgar's potential which generates the Lanczos spintensor for the Weyl tensor.

Key words: Lanczos potential · Conformal tensor · Gödel spacetime · Petrov types

INTRODUCTION

Andersson-Edgar [1-4] proved that any Lanczos spinor can be generated via the relation:

$$L_{ABCD} = \nabla^{E}{}_{D} T_{ABCE}, \qquad \mathrm{T}_{ABCE} = T_{(ABC)E}, \tag{1}$$

such that:

$$T_{ABCE} = l_A l_B l_C(\Lambda_0 l_E - \Lambda_4 O_E) + (l_A l_B O_C + (O_A * l_B) l_C)(-\Lambda_1 l_E + \Lambda_5 O_E) + \\ (O_A O_B l_C + (O_A * l_B) O_C)(\Lambda_2 l_E - \Lambda_6 O_E) + O_A O_B O_C(-\Lambda_3 l_E + \Lambda_7 O_E), \tag{2}$$

and [5-9]:

$$T_{ABCD} = l_A l_B l_C(\Omega_0 l_D - \Omega_4 O_D) + (l_A l_B O_C + (O_A * l_B) l_C)(-\Omega_1 l_D + \Omega_5 O_D) + \\ (O_A O_B l_C + (O_A * l_B) O_C)(\Omega_2 l_D - \Omega_6 O_D) + O_A O_B O_C(-\Omega_3 l_D + \Omega_7 O_D), \tag{3}$$

in terms of the spinors associated to the null tetrad of Newman-Penrose (NP) [10-14]:

$$l^{\mu} \leftrightarrow O^A O^B, \qquad n^{\mu} \leftrightarrow l^A l^B, \qquad m^{\mu} \leftrightarrow O^A l^B, \qquad \overline{m}^{\mu} \leftrightarrow l^A O^B. \tag{4}$$

The tensorial version of (1) means the existence of the tensors $F_{\mu\nu} = -F_{\nu\mu}$ and $U_{\mu\nu\alpha\beta}$ with the algebraic symmetries of the Weyl tensor, such that:

$$K_{\mu\nu\alpha} = U_{\mu\nu\alpha\beta}{}^{;\beta} + \frac{1}{3}\left(2F_{\mu\nu;\alpha} + F_{\alpha\nu;\mu} - F_{\alpha\mu;\nu} + F_{\nu\lambda}{}^{;\lambda} g_{\alpha\mu} - F_{\nu\lambda}{}^{;\lambda} g_{\alpha\nu}\right), \tag{5}$$

thus, the Lanczos potential has the properties [15-17]:

$$K_{abc} = -K_{bac}, \qquad K_{abc} + K_{bca} + K_{cab} = 0, \qquad K_a{}^b{}_b = 0, \tag{6}$$

and it generates the conformal tensor via the relation:

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$$C_{\mu\nu\alpha\beta} = K_{\mu\nu\alpha;\beta} - K_{\mu\nu\beta;\alpha} + K_{\alpha\beta\mu;\nu} - K_{\alpha\beta\nu;\mu} + \frac{1}{2}[\left(K_{\mu\beta} + K_{\beta\mu}\right)g_{\nu\alpha} + \left(K_{\nu\alpha} + K_{\alpha\nu}\right)g_{\mu\beta} - \left(K_{\mu\alpha} + K_{\alpha\mu}\right)g_{\nu\beta} - \left(K_{\nu\beta} + K_{\beta\nu}\right)g_{\mu\alpha}], \qquad K_{\mu\nu} \equiv K_{\mu\sigma\nu}{}^{;\sigma}. \tag{7}$$

We shall consider two applications of (5):

a). $U_{\mu\nu\alpha\beta} = 0$. Then the Lanczos potential acquires the structure:

$$K_{\mu\nu\alpha} = \frac{1}{3}\left(2F_{\mu\nu;\alpha} + F_{\alpha\nu;\mu} - F_{\alpha\mu;\nu} + F_{\nu\lambda}{}^{;\lambda}g_{\alpha\mu} - F_{\nu\lambda}{}^{;\lambda}g_{\alpha\nu}\right), \tag{8}$$

whose application in the Weyl-Lanczos equations [7, 18, 19], for arbitrary $F_{\mu\nu}$, gives 0 = 0 for any conformally flat space, that is, (8) is a Lanczos generator for arbitrary spacetimes of Petrov type O. If now we select the expression:

$$F_{\mu\nu} = q(n_\mu l_\nu - n_\nu l_\mu), \tag{9}$$

then (8) is a Lanczos potential for arbitrary geometries types N and III, in the canonical null tetrad [12, 20], for $q = \frac{1}{2}$ and $q = 1$, respectively [4, 21-23], reproducing the results obtained in [24].

b). $F_{\mu\nu} = 0$. Hence from (5):

$$K_{\mu\nu\alpha} = U_{\mu\nu\alpha\beta}{}^{;\beta}, \tag{10}$$

which is important in the Gödel cosmological model [12, 25-30]:

$$ds^2 = (dx^0)^2 + 2e^{x^3}dx^0dx^1 + \frac{1}{2}e^{2x^3}(dx^1)^2 - (dx^2)^2 - (dx^3)^2, \tag{11}$$

because in [31] was constructed the Lanczos generator $K_{\mu\nu\alpha} = -\frac{2}{9}C_{\mu\nu\alpha\beta}{}^{;\beta}$ with the structure (10).

We think that (5) with $U_{\mu\nu\alpha\beta} \neq 0$ and $F_{\mu\nu} \neq 0$ will be useful in the search of Lanczos potentials for arbitrary spacetimes of Petrov types I, II and D.

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