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Lanczos Potential for Arbitrary Spacetimes of Petrov Types O, N and III

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Abstract: For arbitrary geometries of Petrov types III, N and O, we construct the Lanczos potential for the corresponding Weyl tensor.

Key words: Conformal tensor · Lanczos generator · Canonical null tetrad · Petrov classification

INTRODUCTION

The Lanczos potential $K_{\mu\nu\alpha}$ [1-8] satisfies the algebraic symmetries:

$$K_{\mu\nu\alpha} = -K_{\nu\mu\alpha}, \qquad K_{\mu\nu\alpha} + K_{\nu\alpha\mu} + K_{\alpha\mu\nu} = 0 \qquad K^{\mu\nu}{}_{\nu} = 0, \tag{1}$$

and it generates the Weyl tensor [9, 10] via the expression [11]:

$$C_{\mu\nu\alpha\beta} = K_{\mu\nu\alpha;\beta} - K_{\mu\nu\beta;\alpha} + K_{\alpha\beta\mu;\nu} - K_{\alpha\beta\nu;\mu} + \frac{1}{2}[\left(K_{\mu\beta} + K_{\beta\mu}\right)g_{\nu\alpha} + (K_{\nu\alpha} + K_{\alpha\nu})g_{\mu\beta} - \left(K_{\mu\alpha} + K_{\alpha\mu}\right)g_{\nu\beta} - \left(K_{\nu\beta} + K_{\beta\nu}\right)g_{\mu\alpha}], \qquad K_{\mu\nu} \equiv K_{\mu\sigma\nu}{}^{;\sigma}. \tag{2}$$

If we select:

$$K_{\mu\nu\alpha} = \frac{1}{3}(2F_{\mu\nu;\alpha} + F_{\alpha\nu;\mu} - F_{\alpha\mu;\nu} + F_{\nu\lambda}{}^{;\lambda}g_{\alpha\mu} - F_{\mu\lambda}{}^{;\lambda}g_{\alpha\nu}), \tag{3}$$

for arbitrary $F_{\mu\nu} = -F_{\nu\mu}$, then (2) implies the relation:

$$C_{\mu\nu\alpha\beta} = C_{\sigma\nu\alpha\beta}F^{\sigma}{}_{\mu} - C_{\sigma\mu\alpha\beta}F^{\sigma}{}_{\nu} + C_{\sigma\beta\mu\nu}F^{\sigma}{}_{\alpha} - C_{\sigma\alpha\mu\nu}F^{\sigma}{}_{\beta}, \tag{4}$$

which gives 0 = 0 for any conformally flat space, that is, (3) is a Lanczos potential for arbitrary Petrov type O spacetime.

Now we consider two Petrov types in the canonical null tetrad [9, 12, 13], for a certain $F_{\mu\nu}$:

a). Type N:

$$C_{\mu\nu\alpha\beta} = \psi_4 V_{\mu\nu}V_{\alpha\beta} + \overline{\psi}_4 \overline{V}_{\mu\nu}\overline{V}_{\alpha\beta}, \qquad V_{\mu\nu} = l_\mu m_\nu - l_\nu m_\mu, \tag{5}$$

$$F_{\alpha\beta} = q\left(n_\alpha l_\beta - n_\beta l_\alpha\right), \tag{6}$$

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thus (4) takes the form $C_{\mu\nu\alpha\beta} = 2q\, C_{\mu\nu\alpha\beta}$, therefore (3) is a Lanczos potential with (6) for $q = \frac{1}{2}$.

b). Type III:

$$C_{\mu\nu\alpha\beta} = \psi_3\left(V_{\mu\nu}M_{\alpha\beta} + M_{\mu\nu}V_{\alpha\beta}\right) + \bar{\psi}_3\left(\bar{V}_{\mu\nu}\bar{M}_{\alpha\beta} + \bar{M}_{\mu\nu}\bar{V}_{\alpha\beta}\right),$$
$$M_{\mu\nu} = m_\mu \bar{m}_\nu - m_\nu \bar{m}_\mu + n_\mu l_\nu - n_\nu l_\mu, \qquad (7)$$

then (4), (6) and (7) give the relation $C_{\mu\nu\alpha\beta} = q\, C_{\mu\nu\alpha\beta}$, that is, (3) is a Lanczos generator with (6) for $q = 1$.

Hence the Lanczos potential for arbitrary Petrov types N and III spacetimes has the structure (3) if we employ the corresponding canonical null tetrad and $F_{\mu\nu}$ is given by (6) with $q = \frac{1}{2}$ and $q = 1$ respectively; in Petrov type O geometries, we can use (3) with any $F_{\alpha\beta}$. The construction of $K_{\mu\nu\alpha}$ for arbitrary 4-spaces of types I, II and D, is an open problem.

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