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Enhanced Time Demand Analysis

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Abstract: Since its introduction, Rate Monotonic Analysis (RMA) has frequently been promoted as a way of attaining real time system predictability, yet RMA (in its exact form), exhibits pseudo-polynomial time complexity and becomes impractical for larger task set to be analyzed online. To tune-up RMA for online systems, researchers are attempting to lower the complexity of RMA in terms of reducing the number of scheduling points. However, reducing number of scheduling points does not actually lower the number of points, which will be tested in actual, at run time. In this paper, we tackle the high complexity associated with current algorithms by identifying and propagating false points that arise during system feasibility analysis. The contribution of this paper is two-fold; first, it highlights an implicit drawback found in a recently proposed technique and secondly, a novel exact feasibility test called enhanced time demand analysis is presented, which significantly reduces the number of inequalities that would be tested otherwise.

Key words: Real-Time Systems · Fixed-priority Scheduling · Feasibility Analysis · Online Schedulability

INTRODUCTION

A major issue in the design of real-time multitasking systems is that of timing correctness i.e. they must provide predictable behavior under all circumstances. The solution lies in the optimal scheduling algorithm that assigns priorities to task on the basis of some predefined criteria such as activation rate or deadline etc. The most commonly used approach to schedule real time tasks is priority driven which falls into two types: fixed priority and dynamic priority [1]. A fixed-priority algorithm assigns the fixed/same priority to all jobs in each task, which should be different from the priorities assigned to jobs generated by other tasks in the system. In contrast, dynamic-priority scheduling algorithms place no restrictions upon the manner in which priorities are assigned to individual jobs. Although, dynamic algorithms are considered better theoretically [2], they become unpredictable when transient overload occurs [3]. This paper thus considers only fixed-priority scheduling due to its applicability, reliability and simplicity [2, 4-5].

The problem of scheduling periodic task under fixed-priority scheme was first addressed by Liu and Layland [6] in 1973 under simplified assumptions (for details see [6]), they derived the optimal static priority scheduling algorithm for implicit-deadline model (when deadlines coincide with respective periods) called rate monotonic (RM) algorithm. RM assigns static priorities on task activation rates (periods) such that for any two tasks τ_i and τ_j, priority (τ_i)> priority (τ_j) ⇒ period (τ_i) < period τ_j, while ties are broken arbitrarily. For constrained deadline systems, where deadlines are not greater than periods, an optimal priority ordering has been shown in [7] to be Deadline-Monotonic (DM) scheduling, where priority assigned are inversely proportional to relative deadlines. The RM and DM are identical when relative deadline of every task is proportional to its period.

Let $\tau = \{\tau_1,\ldots,\tau_n\}$ denotes a periodic task system. A task τ_i is represented by its parameters, c_i, p_i and d_i, where c_i is the execution time, p_i is the time period and d_i is the deadline of τ_i. For each task τ_i, its utilization is defined as: $u_i = \frac{c_i}{p_i}$. We define a cumulative utilization u_{tot} of periodic task system τ as:

$$u_{tot} = \sum_{i=1}^{n} \frac{c_i}{p_i} \qquad (1)$$

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For validating timing constrains, feasibility tests-given a task set and system model, determining weather it is possible to meet all the deadlines-are performed to achieve system predictability [4-5, 7-19]. The first feasibility test for RM was proposed by the same authors in [6], called LL-bound; a periodic task system is static-priority feasible if

$$u_{tot} \leq n(2^{1/n} - 1) \tag{2}$$

Where n denotes the number of tasks in τ. The term $n(2^{1/n} - 1)$, decreases monotonically from 0.83 when $n = 2$ to $\ln(2)$ as as $n \to \infty$. This shows that any periodic task set of any size is static priority feasible upon a preemptive uniprocessor, if the RM scheduling is used and u_{tot} is not greater than 0.693. This result gives a simple $O(n)$ procedure to test task feasibility online, where tasks can arrive at run time, however, it is a sufficient condition only; it is quite possible that an implicit-deadline synchronous periodic task system which exceeding the LL-bound be static-priority feasible. The classic work of Liu and Layland [6] is extended in [1, 13-15], however all these tests are sufficient conditions (SC) only and trade utilization for performance.

To overcome the 3% theoretical difference in performance proposed by LL-bound, necessary and sufficient condition (NSC) based tests were proposed in literature [8-10, 12, 15-16]. These feasibility tests can be divided into two schemes: straight forward approaches [4-5, 8, 16] and iterative techniques [9, 10]. The former analyzes task feasibility only at times when tasks arrive, called scheduling points, while the later answers the task feasibility by employing iterative technique. Whatever is the implementation mechanism, their time complexity remains pseudo-polynomial [19]. Recently, authors in [4] extended the work in [8] by proposing an exact feasibility test that reduces the number of scheduling points. However, we show in Section 3 that reducing the number of scheduling point does not necessarily mean lowering the number of points which are actually being utilized by the test.

Unlike the previous work [4], this paper also extends the work in [8] and presents a novel technique to handle the complexity of exact test by reducing the number of points in actual that need to be tested otherwise for determining system feasibility. Though the main idea of this paper can be easily integrated into any fixed priority-scheduling algorithm, however in order to align with previous results, we focus on RM here. Simulation results show that our technique shows better performance, when compared to other techniques lying in the same category, without any modification to the underlying scheduling. As a related work, we report that recursive technique such as the one proposed in [4], involves recursive function calling and, in some cases even needs more points for analyzing feasibility of a task (details in Section 3).

The rest of paper is organized as follows. In Section 2, we introduce some basic notations to formulate our problem. The exact test [8], which provides the basis of our work, is presented in Section 3, in addition to the critical evaluation of hyperplane exact test [4]. The main result of this paper is presented in Section 4, where we derive a novel algorithm to determine the feasibility of task set at run time. Experimental results are discussed in Section 5 and finally, conclusion is drawn in Section 6.

Problem Formulation and Notation: Our work is targeting independent, preemptable and hard periodic task in which the response of the tasks may be earlier than the ends of their corresponding periods. In our model of a hard real-time task set, each task τ_i generates a job at each integer multiple of *pi* and each such job has an execution requirement of c_i time units that must be completed by the next integer multiple of p_i. Moreover, all tasks immediately get ready for execution on uniprocessor as soon as they are released, deadlines are equal to periods, all tasks overheads such as task swapping times are ignored and the number of priority levels is unlimited. Initially all tasks arrives simultaneously at $t = 0$. To begin with, we describe a pseudo-polynomial time exact schedulability test in Section 3 for the feasibility of fixed priority systems, where the response time of the jobs are smaller than or equal to their respective periods. To determine whether a task can meet all its deadlines, we compute the total demand for processor time by a task τ_i and check whether this demand can be met by its deadline.

Before we describe exact test in Section 3, we introduce some notations here, which will be needed latter in this paper. The workload constituted by τ_i at time t consists of its execution demand c_i as well as the interference it encounters due to higher priority tasks from τ_{i-1} to τ_1 and can be expressed mathematically as

$$W_i(t) = c_i + \sum_{j=1}^{i-1} \left\lceil t/p_j \right\rceil c_j \tag{3}$$

A periodic task τ_i is feasible if we find some $t \in [0, t]$ satisfying

Table 1: RM scheduling points for 3-tasks set

τ_i	c_i	p_i	S_i
1	c_1	3	3
2	c_2	7	3,6,7
3	c_3	20	3,6,7,9,12,14,15,18,20

$$L_i = \min_{0<t\le p_i}(W_i(t)\le t) \qquad (4)$$

In other words, task τ_i completes its computation requirements at time $t \in [0, t]$, if and only if the entire request from the $i-1$ higher priority tasks and computation time of τ_i, is completed at or before time t. As t is a continuous variable, there are infinite numbers of points to be tested. The entire task set τ is feasible iff

$$L = \max_{1\le i\le n}\{\min_{0<t\le p_i}\frac{W_i(t)}{t}\}\le 1 \qquad (5)$$

Privious Results on Exact Tests

Time Demand Analysis: The first attempt to limit the infinite number of points in interval $t \in [0, t]$ is made in [8]. The authors' show that $W_i(t)$ is constant, except at finite number of points, where tasks are released, called RM scheduling points. Consequently, to determine whether τ_i is schedulable, $W_i(t)$ is computed only at multiples of $\tau_i \le \tau_j$, $1 \le j \le i$. Specifically, let

$$S_i = \{ap_b \mid b = 1,\dots,i; a = 1,\dots,\lfloor p_i/p_b \rfloor\} \qquad (6)$$

We have the following fundamental theorem to determine whether an individual task is feasible or not.

Theorem 1: Given a set of n periodic tasks $\tau_1,\dots,\tau_n$, τ_i can be feasibly scheduled for all tasks phasings using RM iff

$$L_i = \min_{t\in S_i}\frac{W_i(t)}{t}\le 1 \qquad (7)$$

With above theorem, L_i is needed to be analyzed only at a finite number of points. The time complexity of Theorem 1 depends on both the number of tasks and maximum task period i.e. $O(n\frac{p_n}{p_1})$ [1]. In rest of the paper, we represent this technique by Time Demand Analysis (TDA). We now apply results parented in Theorem 1 to a set of three tasks with parameters given in Table 1.

In Algebraic Terms, We Have:

List. 1

1. Task τ_1 is RM-schedulable iff

$$c_1 \le 3 \qquad (8)$$

II. Task τ_2 is RM-schedulable iff

$$\begin{aligned} &c_i + c_2 \le 3 \textit{ OR}\\ &2\,c_i + c_2 \le 6 \textit{ OR}\\ &3\,c_i + c_2 \le 7 \end{aligned} \qquad (9)$$

III. Task τ_3 is RM-schedulable iff

$$\begin{aligned} &c_i + c_2 + c_3 \le 3 \textit{ OR}\\ &2c_i + c_2 + c_3 \le 6 \textit{ OR}\\ &3\,c_i + c_2 + c_3 \le 7\\ &3c_i + 2c_2 + c_3 \le 9 \textit{ OR}\\ &4\,c_i + 2c_2 + c_3 \le 12 \textit{ OR}\\ &4\,c_i + 2c_2 + c_3 \le 14 \textit{ OR}\\ &5\,c_i + 3c_2 + c_3 \le 15 \textit{ OR}\\ &6\,c_i + 3c_2 + c_3 \le 18 \textit{ OR}\\ &7\,c_i + 4c_2 + c_3 \le 20 \textit{ OR} \end{aligned} \qquad (10)$$

For any task τ_i, the number of elements in set S_i is of particular interest. Every element of S_i contributes a RM scheduling point and constitutes one inequality for τ_i, which can be seen in List 1. The number of elements in S_i becomes huge especially when ratio p_n/p_1 is large [4]. To avoid confusion, by scheduling point we mean every $t \in S_i$, while the point which is tested in actual during analysis is denoted by actual-point, in rest of the paper. Clearly, an efficient technique would be the one which is capable of restricting the number of scheduling points as well as the number of actual points during online feasibility analysis.

Hyper-Planes Exact Test: To reduce scheduling points, E. Bini and G.C. Buttaazo provided a formulation, called Hyper-planes Exact Test (HET) recently in [4] that reduces scheduling point for τ_i from set S_i to a reduced set $H_i(t)$. For any task τi, their test begins with p_i and expands its search space by

$$H_i(t) = H_{i-1}\left(\left\lfloor\frac{t}{p_i}\right\rfloor p_i\right)\cup H_{i-1}(t) \qquad (11)$$

Where H_0 (t)={t}. Two interesting observations can be made here:

Observation 1: To offer a tradeoff between complexity versus performance, the test can be tuned accordingly through a parameter $\boldsymbol{\delta}$ in the range of 0.1 – 1.0. Giving a lower value to $\boldsymbol{\delta}$ (up to 0.5), HET exhibits lower complexity (comparable to LL-bound), however it becomes sufficient condition only, in this case. Putting $\boldsymbol{\delta} = 1$ makes HET both necessary and sufficient at the expense of high complexity [4]. Though efficient when δ has lower values, however, care is required to use HET in exact form ($\delta = 1$). In such case, the cordiality of $H_i(t)$ can reach 2^i in worst case, thus, useless in exact form, when applied to a larger task set i.e., $n \to \infty$.

Observation 2: A convincing argument for the superiority of any algorithm over exact test would be that $\exists S_{i(reduced)}$: $S_{i(reduced)} \subseteq S_{,i}$ On one hand, HET efficiently provides a reduced set $H_{i-1}(p_i) \subseteq S_i$, while on the other, this result has a potential drawback of testing the same point repeatedly due to its recursive nature i.e. because of calling *WorkLoad*() function recursively.

Since this paper is focused on exact tests, therefore, by HET we mean HET in its exact form ($\delta = 1$). Below we re-write the same pseudo code given in [4] for reference.

```
procedure Boolean RMTest ($\tau_n$)
int i;
for all ($i$ = 0; $i$<$n$;$i$ + +) do
if($c_i$ + WorkLoad ($i$ −1, $p_i$) then
return fasle;
else
return true;
end if
end for
end procedure

/* Recursive function */
double last $\psi$ [BIG_ENOUGH];
double lastWorkLoad [BIG_ENOUGH];
function double WorkLoad(int i, double b)
{
int f,g;
double branch0,branch1;
if (i ≤ 0) then
return 0;
end if
if (p ≤ last $\psi$ [$i$]) then
return lastWorkLoad [i];
end if
f= $\left\lfloor \frac{b}{p_i} \right\rfloor$; g = $\left\lceil \frac{b}{p_i} \right\rceil$;
branch0 = b-$f(p_i - c_i)$+WorkLoad(i-1,$f \times p_{i-1}$);
branch1 =g × $c_i$+WorkLoad( i-1, b);
last $\psi$ [$I$] = $b$;
lastWorkLoad(i]=min(branch0, branch1);
return lastWorkLoad[i];}
end function
```

Table 2: Construction of set S_i and H_i

τ_i	c_i	p_i	S_i	H_i
1	1	3	3	3
2	2	8	3, 6, 8	8, 6
3	3	20	3, 6, 8, 12, 158, 16, 18, 20	15, 16, 18, 20

Table 3: Actual points needed

TDA	HET
Actual points	Actual points
3	3
3	8, 6, 8
3, 6, 8	20, 16, 15, 16, 20, 18, 20

The above recursive function calling has implicit tendency for manipulating a large sequence of points (up to 2^i for τ_i). To make the point clear, the feasibility of a task set given in Table 2 is answered in the light of TDA and HET respectively, where more actual-points are needed to be evaluated by HET than TDA. The advantage of TDA over HET is shown in Table 3.

Illustrative Example: The RM scheduling points constructed by both TDA and HET are given in Table 2, where task feasibility has to be tested. At first glance, the advantage of HET is clear, as $H_i \subseteq S_i$ and $\Sigma|H_k| \leq \Sigma|S_k|, 1 \leq k \leq n$. However, the important results are given in Table 3. Results can be compared on relaxed task sets, where case $\left\lfloor \frac{t}{p_j} \right\rfloor p_j = t$ may happen, such formulation does not contribute any point to the intended search space [4]. In order to provide more appropriate task set for HET, we deliberately use the task set in Table 1 to avoid the case when $\left\lfloor \frac{t}{p_j} \right\rfloor p_j = t$.

In our example this case never arises, as p_3 is not integer multiple of p_1 or p_2. The difference between the number of scheduling points constructed and the actual-points for respective tasks, can be observed in Table 2 and 3 respectively (more actual-points (multi-set) for HET in Table 3 than set H_i in Table 2). With TDA, for task τ_3, S_3 consist of a large number of scheduling points, however only 3 actual-points tested during analysis, as $W_3(t) \leq t$ is satisfied at $t = 8$ (third point in set S_3). In contrast, for the same task (τ_3), though H_i consists of fewer points; it starts testing feasibility at multiple points at run-time (Table 3). We graphically show this behavior in Fig. 1.

From Eq.(11), we can see that for task τ_i, H_i is expanding in descending order; it commences feasibility test with largest t among all points in set H_i.

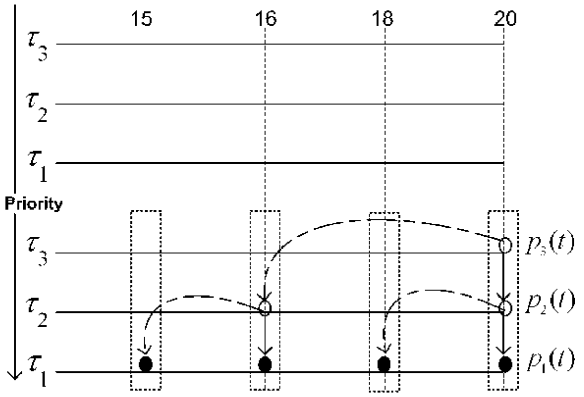


Fig. 1: Inconsistency between scheduling points and actual-points

The small circles in Fig. 1 reflects points which are repeated due to recursive function calling (*WorkLoad*), while filled circles shows $t \in H_3$. As long as number of scheduling points and actual-points are concerned, for the referred τ, TDA offers 54% improvement over HET. Eventually, for larger task set ($n \to \infty$), we can conclude that TDA suppress HET in terms of time complexity as $O(n\frac{p_n}{p_1}) < O(2^n)$.

Similarly, when $p_i = p_j \forall\ i \neq j\ |1 \le i, j \le n$ multiple points are contributed to search space produced by HET. Another interesting case would be the feasibility analysis of a low computation task set where $\sum_{i=1}^{n} c_i \le p_{min}$.

In this case, applying HET, results in a large array of scheduling points due to union operator involved, which is shown in Eq. (11). For such task sets, it is sufficient to test only n number of actual-points by TDA, as feasibility is positively answered at $\tau_i, \sum_{j=1}^{i} c_j \left\lceil \frac{p_1}{p_j} \right\rceil \le p_1$

holds true (p_1, being the smallest $t \in S_i$, is the first actual-point to be tested for any task τ_i).

Enhanced Time Demand Analysis: Using Theorem 1, it is equally important for a low priority task to be tested in interval $[0, p_1]$ despite the fact that there is a fair chance of not fulfilling the cumulative workload constituted by current task because interference from the higher priority tasks (τ_{i+1} to τ_n) is also added to computation demands of current task τ_i e.g. any false inequality for τ_2 through Eq.(9) also remains false with Eq.(10) for τ_3. We avoid this redundancy of points by proposing that when the time demand for task τ_1 is not fulfilled at any point t, then it becomes unnecessary to test feasibility of the lower priority tasks (τ_{i+1} to τ_n) at the same point t, as demand must remains unsatisfied at point t.

Mathematically:

$$\sum_{j=1}^{i} \left\lceil \frac{t}{p_j} \right\rceil c_j + c_{lower} > t \tag{12}$$

is always true, where c_{lower} is the execution demand of tasks whose priority is lower than τ_i, since $c_{lower} > 0\ \forall\ c_{lower} \in [c_{i+1}, ..., c_n]$.

To reduce the intended search space, we present some definitions and theorems, which eventually lead us to the main contribution of this section.

Theorem 2: For any given periodic tasks set τ, scheduled by RM, task τ_i has a set of RM scheduling points S_i:$S_i \subseteq S_{i+1}$.

Proof: To prove this theorem, we must prove that if there exist a scheduling point $t \in S_i$, then $t \in S_{i+1}$ also holds good. For an arbitrary element of S_i, it follows that $t = ap_b$, where $a \in \{1, ..., \lfloor p_i / p_j \rfloor$, $j \in 1, ..., i$and $b \in 1,...i$. According to RM, tasks priorities are inversely proportional to their periods, tasks τ_i has higher priority than τ_{i+1} as $p_{i+1} \ge p_i$, so it can be seen that $a \in \{1, ..., \lfloor p_{i+1}/p_j \rfloor\}$ and $j \in 1, ..., i+1$, hence $t \in S_i$ and therefore $S_i \in S_{i+1}$ holds true.

Definition 1: False Point (FP): A point $t \in S_i$, is said to be false for any task τ_i if it satisfies the inequality constraint: $W_i(t) > t$.

Theorem 3: Given a set of n periodic tasks $\{\tau_1, ..., \tau_n\}$ scheduled by RM, every false point for τ_i must also be a false point for τ_{i+1}.

Proof: According to the definition of false point for task τ_i;$W_i(t) > t$, thus for task τ_{i+1}

$$W_{i+1}(t) = \sum_{j=1}^{i+1} \left\lceil \frac{t}{p_j} \right\rceil c_j = \sum_{j=1}^{i} \left\lceil \frac{t}{p_j} \right\rceil c_j + \left\lceil \frac{t}{p_{i+1}} \right\rceil c_{i+1}$$
$$= W_i(t) + c_{i+1} > W_i(t) \tag{13}$$

As $c_{i+1} > 0$, therefore $W_{i+1}(t) > t$ and hence t is also a false point for τ_{i+1}.

Theorem 4: Given a set of n periodic tasks $\{\tau_1, ..., \tau_n\}$ scheduled by RM, every false point for τ_i is also a false point for task having priority lower than τ_i.

Proof: This theorem can be directly deduced from Theorem 3.

Definition 2: Chained Point: A false point t for task τ_i becomes chained point for the task with priority lower than τ_i. We represent all the false points contributed by τ_i with a set X_i such that X_i becomes a set of chained points for $\{\tau_{i+1},...,\tau_n\}$.

Theorem 5: Given a set of n periodic tasks $\{\{\tau_1,...,\tau_n\}$, τ_i can be feasibly scheduled for all tasks phasings using RM *iff*.

$$L_i = \min_{t\in Z_i} \frac{W_i(t)}{t} \leq 1 \qquad (14)$$

Where $Z_i = S_i - X_{i+1}$.
By Extension $X_0 = \varnothing$

Proof: The proof is straightforward, from Theorem 4, we known that, if t is a false point for τ_{i+1} then it must also be a false point for τ_i.

Consequently to calculate L_i, we confined our search to a reduced set $Z_i \subseteq S_i$, by excluding all chained points.

Theorem 6: The entire task set τ is schedulable for all task phasings using RM if

$$L = \max_{1\leq i\leq n} L_i \leq 1 \qquad (15)$$

We have replaced S_i given in Theorem 1 by Z_i, which immediately reflects the reduced complexity of our technique $Z_i \subseteq S_i$; allows us to search reduced number of inequalities for determining task feasibility. Task feasibility is drawn immediately when $W_i(t) \leq t$ is satisfied at the smallest $t \in Z_i$. We use the term Enhanced Time-Demand Analysis (ETDA) to refer our scheme hereafter in this paper. The effectiveness of ETDA becomes more prominent when it is applied to a large task set.

Pseudo Code for ETDA Can Be Written as Follows:

```
Procedure ETDA ($\tau$)
{$X_0 = \varnothing$
For all $\tau_i \in \tau$ do
    Compute $S_i = ap_b \mid b = 1,...,i; a = 1,...,\lceil \frac{p_i}{p_b} \rceil$
    $Z_i = S_i - X_{i+1}$
    For all $t \in Z_i$
        If ($L_i \leq 1$) then
            $\tau$ is schedulable; break;
        else update $X_i$
        end if
    end for
    if ($L_i \leq 1$) then $\tau$ is feasible
    Else is infeasible;
    end if
end for
end procedure.
```

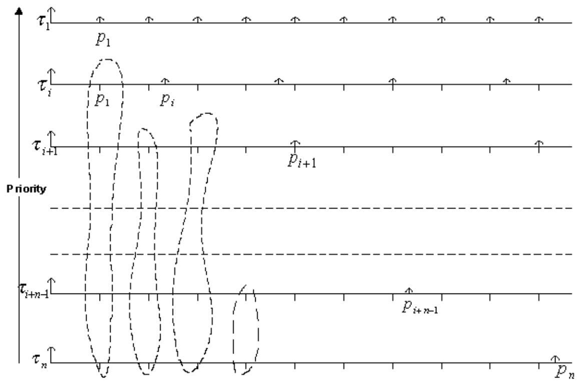


Fig. 2: Identification of false points

A graphical explanation for the identification of FP is given in Fig. 2. The symbol ↑ shows the time period of the current task τ_i, while | denotes the testing points contributed from higher priority tasks. Lets assume task τ_i is infeasible at point $t = p_1$ (the deadline for the highest priority task τ_1) or algebraically: $\sum_{j=1}^{i} c_j \lceil \frac{t}{p_j} \rceil > t$

This fact makes it unnecessary to analyze feasibility of lower priority tasks (from τ_i to τ_n) at $t_i = p_1$ execution demand of lower priority task is also added to the left side of already false inequality constraint i.e. $\sum_{j=1}^{i} c_j \lceil \frac{t}{p_j} \rceil + c_{lower} > t$

never holds true at $t = p_1$, since the workload cannot be reduced while considering additional lower priority tasks. We mark p_1 as FP and avoid this inequality constraint to be tested by lower priority tasks. The same treatment is applied to all FP that arise as ETDA proceeds.

In Fig. 3, we visualize Theorem 5. The feasible interval $[0, p_i]$ for any task τ_i is divided into three sub portions. The filled rectangle represents all candidate scheduling points Z_i. At each point $t \in Z_i$, L_i is evaluated, which is either true or false. A true answer declares feasibility of τ_i, otherwise t is marked as FP. The thin horizontal rectangle with crossed lines depicts the false region, which consists of all FP, where task feasibility of current task τ_i is definitely false. False region becomes larger and larger as the feasibility test proceeds, due to emergence of more FP. Once task feasibility is confirmed, our algorithm skips further search space, which is denoted by blank rectangle in Fig. 3.

Experimental Results and Anlysis: In this section, we evaluate the performance of ETDA. In order to make a comparison, both TDA and HET are also implemented.

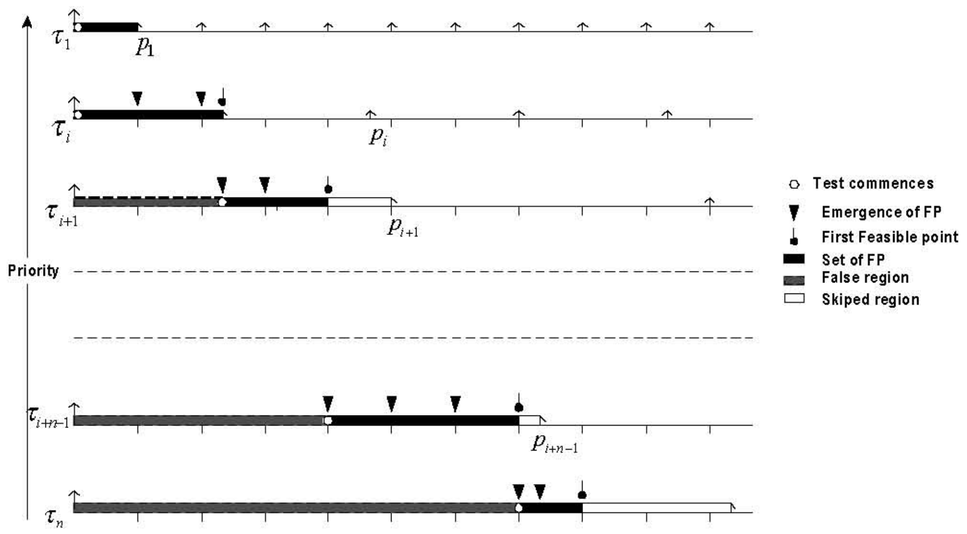


Fig. 3: Illustration of Theorem 5

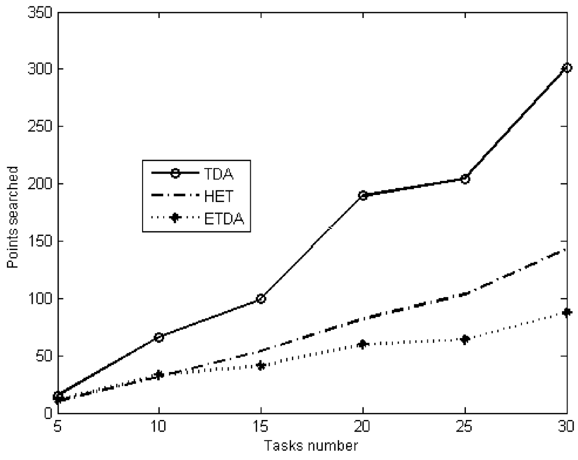
For our experiments, we generate random task periods in the range of [10,1000] with uniform distribution. Similarly, for corresponding task execution demands, random values are taken in the range of [1, p_i], also with uniform distribution. Tasks priorities are assigned according to RM assignment algorithm. The work is compared, in light of three performance evaluation criterions i.e. i) the number of RM scheduling points used, ii) the number of actual-points tested and iii) the execution time, which is the intended contribution of this paper.

Number of Sheduling Points: In this section, we demonstrate the effectiveness of ETDA in terms of reducing the number of scheduling points. Fig. 4 provides a comparison among the tests by counting the number of scheduling points i.e.

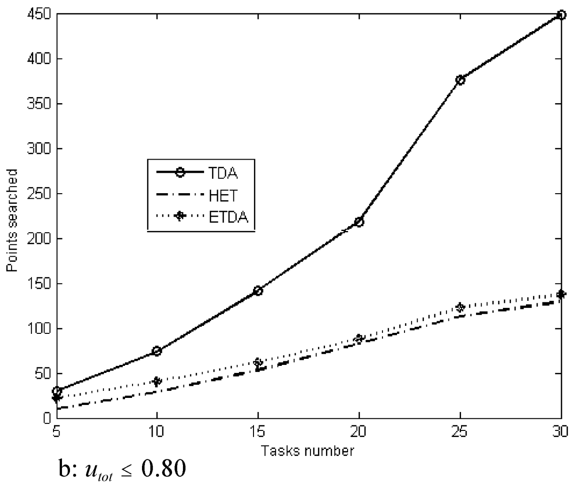
$$\sum_{i=1}^{n} S_i, \sum_{i=1}^{n} H_i, \sum_{i=1}^{n} Z_i$$

for TDA, HET and ETDA respectively. Together, a series of periodic tasks is generated by varying the range from 5 to 50 with the increase of 5 tasks and average is taken after 200 iterations. We analyze all tests by varying system utilization form In(2) to 1.0. When tasks set size increases ($n \to \infty$), more inequalities are needed to be tested as p_n/p_1 is becoming larger. Since τ is always feasible when $u_{tot} \leq ln(2)$, LL-bound would be sufficient to determine feasibility, however, we are intended to find feasibility through exact tests and hence the numbers of scheduling points are discussed. At higher system utilization, c_i becomes larger and more points are need to reach the conclusion (more inequalities to test). The graph of TDA and ETDA is increasing with increased utilization, since both commence testing t in ascending order. HET enjoys the advantage here of being non-increasing here, for any task τ_i it starts testing feasibility with largest element from H_i and therefore task feasibility is answered quickly. It can be seen that both HET and ETDA have effectively reduced the number of scheduling points as compared to TDA. This improvement is due to so-called reduced sets ($H_{i-1} \subseteq S_i$ and ($Z_i \subseteq S_i$) required for any task τ_i by HET and TDA respectively.

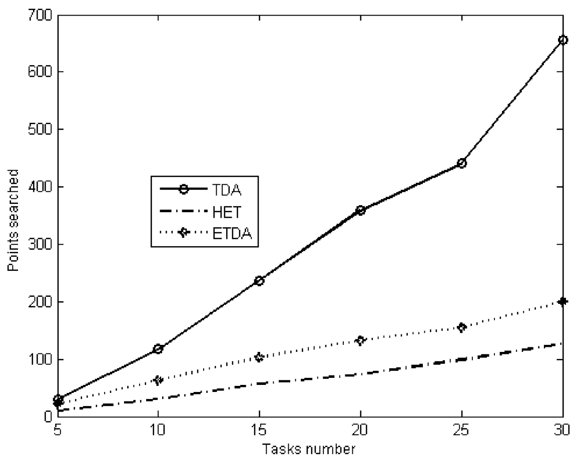
Number of Actual Points: In this important experiment, we extract the number of actual-points, which are being tested by all necessary and sufficient tests discussed earlier in Section 3.1, 3.2 and 4 respectively. Results in Fig. 5 are obtained by performing computation over 200 task sets, each consist of 5 to 30 tasks. Here we also varied the total utilization from $n(2^{1/n} - 1)$ to 1.0. We observed that the number of actual-points tested is directly influenced by variation in system utilization. The reason is that for higher utilization the execution demand of each task is high, since uniform distribution is applied. This fact makes the cumulative workload of individual task τ_i higher and its schedulability becomes possible at some later stage in the interval [0, p_i]. At lower system utilization $u_{tot} \leq ln(2)$, all tasks $\tau_1, \tau_2, ..., \tau_n$ are feasible, so feasibility of all n tasks is tested in sequence and thus, more actual-points are utilized. While for higher u_{tot}, there exist some unschedulable task $\tau_i, 1 < i \leq n$, where feasibility of τ is determined early and therefore the feasibility tests quits immediately (skipping lower priority tasks). It is found that HET uses union operator and has implicit tendency for testing repeated pints from intended search space obtained with Eq. (8).



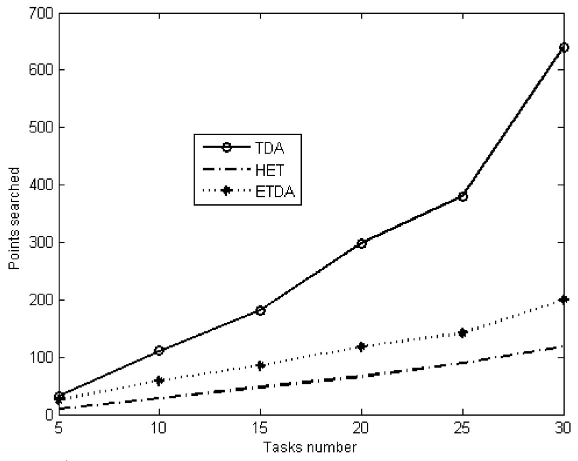
a: $u_{tot} \leq$ In 2



b: $u_{tot} \leq 0.80$



c: $u_{tot} \leq 0.90$



d: $u_{tot} \leq 1.00$

Fig. 4: Required number of scheduling points

In contrast, On one hand, for feasible task set having $u_{tot} \leq n(2^{1/n} - 1)$ both ETDA and TDA converges early (fewer inequalities required), while on the other, more actual-points are needed to be analyzed for higher utilization when u_{tot} (again in ascending order). Clearly, the improved performance of ETDA is due to ($Z_i \subseteq Si$), which is a direct conclusion of Theorem 5, that suppresses others in terms of testing reduced number of actual-points as shown in Fig. 4. For TDA and ETDA the number of scheduling points are always more than or equal to actual-points, since both are implemented non-recursively. In our experiment it is revealed that HET uses repeated points. This fact is witness by the difference in Fig. 4 and Fig. 5, where no agreement is followed between number of scheduling points and actual-points tested by HET. As mentioned before, though HET reduces the number of scheduling points in theory, it looses its effectiveness at run time by scanning redundant points from the search space due to its recursive implementation. In our proposed test, there is a direct mapping between Fig. 4 and Fig. 5.

Execution Times: The time a test takes to solve feasibility problem is an obvious criteria for measuring the performance of a given algorithm, especially when efficiency is the primary concern. This experiment supports our claim that only reducing number of scheduling points is useless unless actual number of points utilized is also handled carefully. The task set is constructed with the same criteria, as discussed earlier. The effect of changing system utilization is also observed. It is note that both TDA and ETDA increases with higher utilization, because more points are needed to be tested in this case, which eventually takes longer to

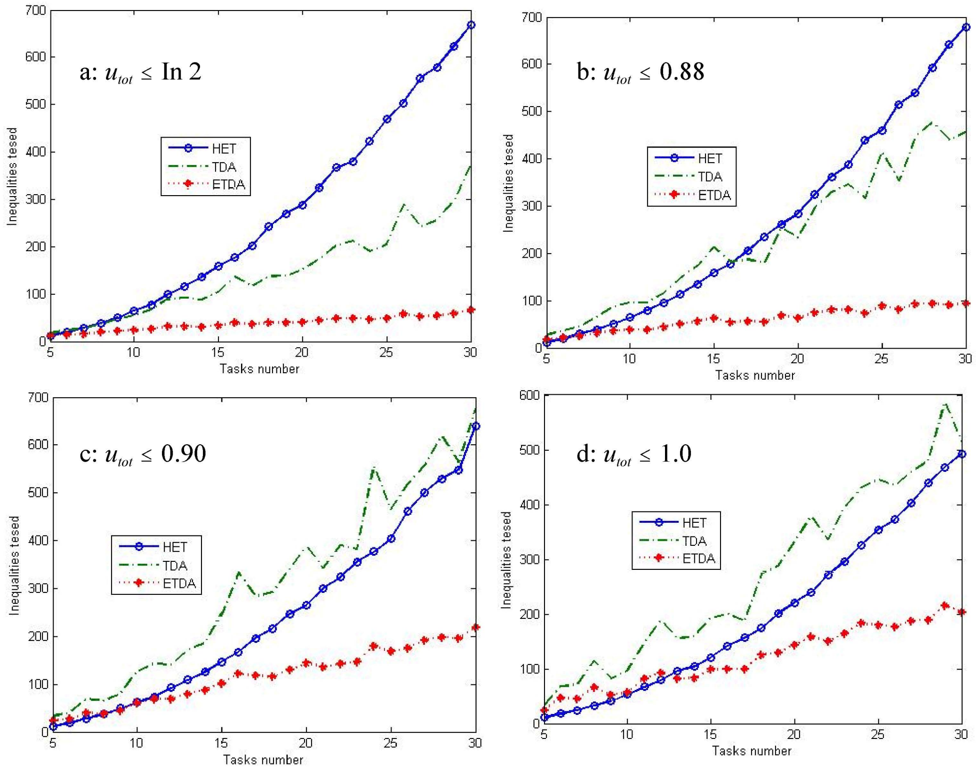


Fig. 5: Actual number of scheduling points tested

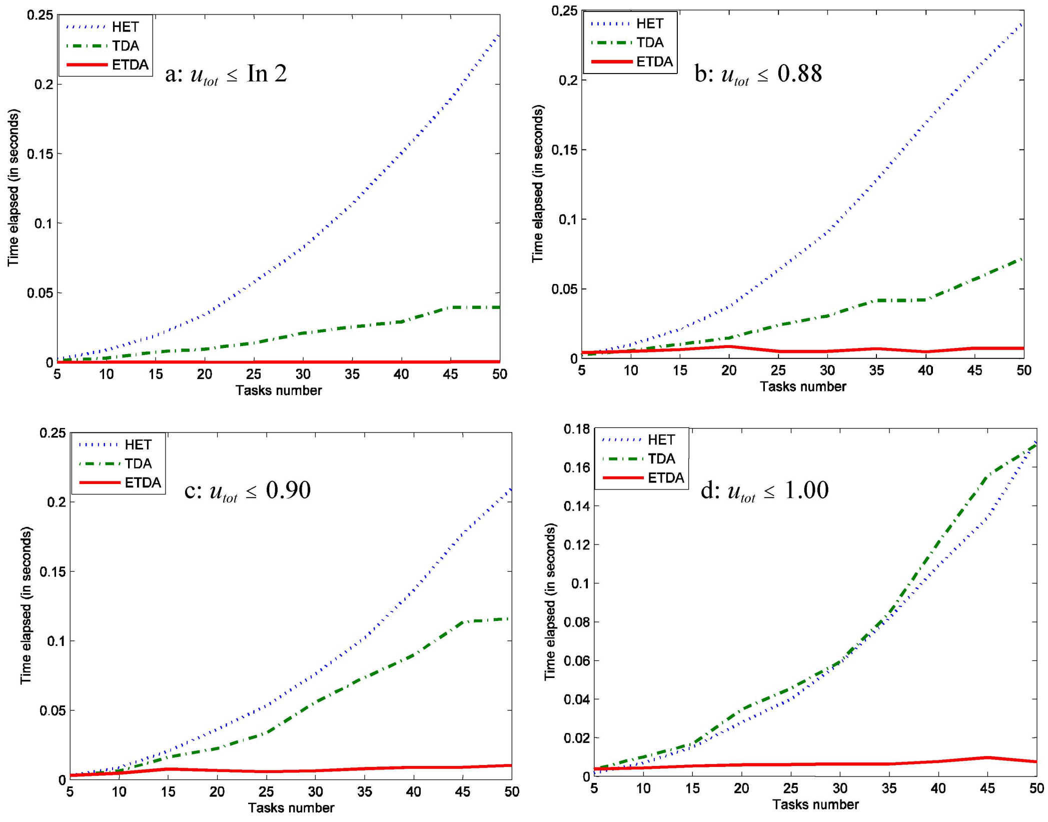


Fig. 6: Execution times of exact tests

conclude feasibility of the task set as given in Fig. 6. TDA's graph is increasing according to utilization, since it start testing feasibility with smallest scheduling point in ascending order, that's why at higher utilization the commutative workload can not be satisfied and more actual-points are needed to be evaluated. From Fig. 5 and 6, with increased number of actual-points, the taken by TDA also increases in Fig. 6. As expected ETDA takes negligible amount of time, because of skipping huge number false points. The non-increasing behavior of HET with increasing utilization is for certain, since number of actual-points are also reduced (Fig. 5). In the light of Fig. 4, 5 and 6, it can be seen that, though the number scheduling point for HET is lowered (Fig. 4), the time taken by HET for the same task set is still high (Fig. 6). For HET Fig. 5 and 6 are in agreement and not Fig. 4 and 6. Thus, it is concluded from the experimental analysis that restricting actual-points is more important for online analysis of periodic task set than just reducing scheduling points.

CONCLUSION

We addressed the basic question of testing feasibility of periodic task under fixed priority assignment algorithms by eliminating redundant points in feasibility analysis. To reach the conclusion, merits and demerits of both SC and NSC are discussed. As a background work, we have shown the impracticality of exact tests for large tasks set to be performed online for real time systems. Similarly, a recently proposed efficient feasibility test is critically evaluated and its shortcomings are explored both theoretically and experimentally. Lastly, we have proposed ETDA, a state of the art solution for testing online feasibility of real time systems employing RM scheduling.

We evaluated the correctness and goodness of ETDA by means of mathematical proofs and simulation results. The experiment aimed at testing the algorithm under different performance conditions and comparing its results with previous solutions. The experimental results obtained, show the effectiveness of ETDA in providing an efficient online analysis for periodic tasks and validated our theoretical results.

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