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Study of Mixed Three-Flavor Neutrino Oscillation with Berry Phase

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Abstract: Three-flavors form a useful basis for describing the state of a neutrino. In this investigation we study the time evaluation of a mixed state for a composition of neutrino with different flavors in presence of Berry (topological) phase. The berry phase obtained in this study is a function of the mixing angle only.

Key words: Neutrino oscillations · Berry phase · Neutrino mixing · PACS: 03.65.Bz · 11.10.-z · 14.60.Pq

INTRODUCTION

The discovery of neutrino oscillation in the past decade [1-8], has turned neutrino physics into one of the most exciting and active fields of physics. Different types of neutrino have been discovered by now namely solar, atmospheric and reactor neutrino. In spite of different masses identified for different types of neutrino, they alternatively change their identities in time as they travel in vacuum [9] and also especially in matter [10]. This process called neutrino oscillation. The physics of neutrino oscillation is currently under very active investigation, since it leads to physics beyond the standard model. The oscillation effects are closely related to the phases of neutrino mixture. Neutrino oscillation in a mixed system is an interesting subject in theoretical research about neutrino. Particularly, in recent years strong activities have been done to perform experiments for better understanding of neutrino.

Two-flavor neutrino oscillation, however, is adopted in most analyzes of the data, although everyone knows that there are three active neutrino flavors. Two-flavor oscillation is easy to investigate in comparison with three-flavor oscillation because there are only two parameters: a mass-squared difference and a mixing angle. In addition, some exact solutions of the oscillation probability are known in the two-flavor oscillation scheme.

Quantum mechanically, if interactions of particle with background matter are known, its wave function and energy spectrum immediately be the solutions of the associated time independent Schrödinger equation. Once the wave function of the particle known in an specific time, its evolution can be determined. This approach is useful when Hamiltonian of the system do not depend on time.

Berry has shown that if the Hamiltonian depends on time via a set of adiabatic parameters, besides the usual dynamic phase, a geometrical phase (the Berry phase) can be used [11]. Several authors have considered the phase properties in neutrino oscillations [12-14]. When the neutrinos move through a medium, a Berry phase is then expected to be generated. It has been shown that the mass squared differences and the mixing angles have strong constraints from various experiments [15-21]. This paper organized as follow. In next section, we describe the notion of Berry phase completely and apply it to explain oscillation of two-flavor neutrino system. In third section we develop this approach to study the oscillation of three-flavor neutrino system. Finally we summarize our results and compare them with the results obtained from other theoretical methods for two-flavor neutrino system.

Berry Phase for Two-Flavor Oscillating Neutrinos: Two-flavor neutrino states in t = 0 can be show by the following relation,

$$\begin{aligned} \left|v_e(0)\right\rangle &= \cos\theta\left|v_1\right\rangle + \sin\theta\left|v_2\right\rangle, \\ \left|v_\mu(0)\right\rangle &= -\sin\theta\left|v_1\right\rangle + \cos\theta\left|v_2\right\rangle, \end{aligned} \tag{1-2}$$

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At time t, the states $|\nu_e\rangle$ and $|\nu_\mu\rangle$ evolve to states,

$$|\nu_e(t)\rangle = \exp(-i\omega_1 t)\cos\theta|\nu_1\rangle + \exp(-i\omega_2 t)\sin\theta|\nu_2\rangle, \quad (2\text{-}2)$$
$$|\nu_\mu(t)\rangle = -\exp(-i\omega_1 t)\sin\theta|\nu_1\rangle + \exp(-i\omega_2 t)\cos\theta|\nu_2\rangle.$$

Where $H|\nu_i\rangle = \omega_i|\nu_i\rangle$ and ω_i are energies associated to the mass Eigen states $|\nu_i\rangle$ with $I = 1,2$. Since at a time $T = \frac{2\pi}{\omega_2 - \omega_1}$ the states are same as the initial ones a part from a phase factor,

$$|\nu_e(t)\rangle = \exp(i\varphi)|\nu_e(0)\rangle, \quad (3\text{-}2)$$
$$|\nu_\mu(t)\rangle = \exp(i\varphi)|\nu_\mu(0)\rangle.$$

Where $\varphi = -\frac{2\pi\omega_1}{\omega_2 - \omega_1}$. Now we show how such a time evolution does contain a purely geometric part, i.e. the Berry phase. It is a straight forward calculation to separate the geometric and dynamical phases which is summarized by the following standard procedure,

$$\beta_e = \varphi + \int_0^T \langle\nu_e(t)|H|\nu_e(t)\rangle dt, \quad (4\text{-}2)$$
$$= -\frac{2\pi\omega_1}{\omega_2 - \omega_1} + \frac{2\pi}{\omega_2 - \omega_1}\left(\omega_1\cos^2\theta + \omega_2\sin^2\theta\right),$$
$$= 2\pi\sin^2\theta.$$

Here, appeared a non-zero geometrical phase β which is related to mixing angle θ and independent of neutrino masses m_1, m_2 and energies $\omega_1\ \omega_2$ In an similar way, we obtain the Berry phase for the muon neutrino states,

$$\beta_\mu = \varphi + \int_0^T \langle\nu_\mu(t)|H|\nu_\mu(t)\rangle dt = 2\pi\cos^2\theta. \quad (5\text{-}2)$$

Since $\beta_\mu + \beta_e = 2\pi$, we can rewrite equation (3-1) as,

$$|\nu_e(t)\rangle = \exp(i2\varphi\sin^2\theta)\exp(-\omega_{ee}T)\ |\nu_e(0)\rangle \quad (6\text{-}2)$$
$$|\nu_\mu(t)\rangle = \exp(i2\varphi\cos^2\theta)\exp(-\omega_{\mu\mu}T)\ |\nu_\mu(0)\rangle.$$

Where we have used the notations

$$\langle\nu_e(t)|H|\nu_e(t)\rangle = (\omega_1\cos^2\theta + \omega_2\sin^2\theta) = \omega_{ee} \quad (7\text{-}2)$$
$$\langle\nu_\mu(t)|H|\nu_\mu(t)\rangle = (\omega_1\sin^2\theta + \omega_2\cos^2\theta) = \omega_{\mu\mu}.$$

In analogy with above relation, it is convenient to obtain

$$\langle\nu_e(t)|H|\nu_\mu(t)\rangle = \langle\nu_e(t)|H|\nu_\mu(t)\rangle, \quad (8\text{-}2)$$
$$= \frac{1}{2}(\omega_2 - \omega_1)\sin 2\theta,$$
$$= \omega_{e\mu} = \omega_{\mu e}.$$

It is convenient to show that time evaluated states are not eigen states of Hamiltonian,

$$\langle\nu_e(0)|H|\nu_e(t)\rangle = \exp(-i\omega_1 t)\cos^2\theta + \exp(-i\omega_2 t)\sin^2\theta. \quad (9\text{-}2)$$

Thus as an effect of time evolution, the state $|\nu_e(t)\rangle$ "rotate" as shown by equation (9-2). However at t = T we have,

$$\langle\nu_e(0)|H|\nu_e(T)\rangle = \exp(-i\varphi) = \exp(i\beta_e)\exp(-i\omega_{ee}T), \quad (10\text{-}2)$$

Which indicate that $|\nu_e(T)\rangle$ differ from $|\nu_e(0)\rangle$ by a geometrical phase that part of it known as Berry phase. Other part of this phase refer to dynamical origin. Generally, for time, $t = T + \tau$ we have,

$$\langle\nu_e(0)|H|\nu_e(t)\rangle = \exp(i\varphi)\langle\nu_e(0)|H|\nu_e(\tau)\rangle \quad (11\text{-}2)$$
$$= \exp(-i2\pi\sin^2\theta)\ \exp(-i\omega_{ee}T)\left[\exp(-i\omega_1\tau\cos^2\theta + \exp(-i\omega_2\tau)\sin^2\theta\right].$$

Also for different states in time $t = T + \tau$ we have,

$$\langle\nu_\mu(0)|H|\nu_e(t)\rangle = \frac{1}{2}\exp(i\varphi)\exp(-i\omega_1\tau) \quad (12\text{-}2)$$
$$\sin 2\theta\left[\exp(-i(\omega_2 - \omega_1)\tau) - 1\right].$$

Wears for $t + T$ will be equal to zero. Also in a given t, states with different flavor are orthogonal

$$\langle\nu_\mu(t)|H|\nu_e(t)\rangle = 0. \quad (13\text{-}2)$$

Equation (12-2) show that the oscillation of $|\nu_e(t)\rangle$ related to flavor of muon and Berry phase. To develop calculations for n cycle, for example relations (4-2) and (5-2) can be rewrite as,

$$\beta_e = \int_0^{nT} \langle\nu_e(t)|H - \omega_1|\nu_e(t)\rangle dt = 2\pi\sin^2\theta,$$
$$\beta_\mu = \int_0^{nT} \langle\nu_\mu(t)|H - \omega_2|\nu_\mu(t)\rangle dt = 2\pi\cos^2\theta. \quad (14\text{-}2)$$

These results indicate that the berry phase acts as a "counter" of neutrino oscillation.

Berry Phase for Three-Flavor Oscillating Neutrinos: The Berry geometric phase also can be computed in the case of three-flavor mixing. In this case, we can apply the method used for two-flavor neutrino system so we have [21],

$$\left|v_f\right\rangle = U\left|v_i\right\rangle \tag{1-3}$$

$$U=\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}c_{13}e^{-i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{-i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{12}e^{-i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{-i\delta} & c_{23}c_{13} \end{pmatrix} \tag{2-3}$$

Where S_{ij} = sin θ_{ij}, C_{ij} = cosθ_{ij} and δ is the CP-violating phase of the Cabibbo–Kobayashi –Maskawa (CKM) matrix. However if the CP-violating phase to be equal to zero, the neutrino state at time *t* will be,

$$\begin{aligned}
&\left|v_e(t)\right\rangle = \exp(-i\omega_1 t)C_{12}C_{13}\left|v_1\right\rangle + \exp(-i\omega_2 t)S_{12}C_{13}\left|v_2\right\rangle + \exp(-i\omega_3 t)S_{13}\left|v_3\right\rangle, \\
&\left|v_\mu(t)\right\rangle = \exp(-i\omega_1 t)(-S_{12}C_{23}-C_{12}S_{23}C_{13})\left|v_1\right\rangle + \exp(-i\omega_2 t)(C_{12}C_{23}-S_{12}S_{23}S_{13}\left|v_2\right\rangle \\
&+\exp(-i\omega_3 t)S_{23}C_{13}\left|v_3\right\rangle, \\
&\left|v_\tau(t)\right\rangle = \exp(-i\omega_1 t)(S_{12}S_{23}-C_{12}C_{23}S_{12})\left|v_1\right\rangle + \exp(-i\omega_2 t)(-C_{12}S_{23}-S_{12}C_{23}S_{13}\left|v_2\right\rangle \\
&+\exp(-i\omega_3 t)C_{23}C_{13}\left|v_3\right\rangle.
\end{aligned} \tag{3-3}$$

Let consider here a particular case in which the difference of each two frequencies be proportional together: $q=\frac{\omega_3-\omega_1}{\omega_2-\omega_1}$. This condition make the states periodic with rotational number q and period:

$q=\frac{2\pi}{\omega_2-\omega_1}=\frac{2\pi}{\omega_3-\omega_1}$. Therefore we can use the previous definition of Berry phase. As defined previously for the case of two-flavor neutrino system, we Have,

$$\beta_{ee}=\varphi+\int_0^T\left\langle v_e(t)\right|H\left|v_e(t)\right\rangle dt = 2\pi(S_{12}^2S_{13}^2+qS_{13}^2). \tag{4-3}$$

Which is of course reduces to the result of two-flavor neutrino system for θ_{13} = 0. Where $q=\frac{\omega_3-\omega_1}{\omega_2-\omega_1}$ and φ = ((−2π ω $_1$) / (ω$_2$ − ω$_1$ as defind previously for the two-flavor case. For completeness, we have also computed the other eight possible components of Berry phase,

$$\beta_{e\mu}=\varphi+\int_0^T\left\langle v_\mu(t)\right|H\left|v_e(t)\right\rangle dt \tag{5-3}$$

$$=-\frac{2\pi\omega_1}{\omega_2-\omega_1}+2\pi(S_{12}C_{12}C_{23}C_{13}-S_{12}^2S_{13}C_{13}S_{23}+qS_{13}C_{13}S_{23}),$$

$$\beta_{\mu e}=\varphi+\int_0^T\left\langle v_e(t)\right|H\left|v_\mu(t)\right\rangle dt \tag{6-3}$$

$$=\frac{2\pi\omega_1}{\omega_2-\omega_1}+2\pi(-S_{12}C_{12}C_{13}S_{23}-S_{12}^2S_{13}C_{13}C_{23}+qS_{13}C_{13}C_{23})=\beta_{e\mu}$$

$$\beta_{e\tau}=\varphi+\int_0^T\left\langle v_\tau(t)\right|H\left|v_e(t)\right\rangle dt \tag{7-3}$$

$$=-\frac{2\pi\omega_1}{\omega_2-\omega_1}+2\pi(-S_{12}C_{12}C_{13}S_{23}-S_{12}^2C_{13}S_{13}C_{23}+qS_{13}C_{13}C_{23})=\beta_{\tau e}$$

$$\beta_{\mu\mu}=\varphi+\int_0^T\left\langle v_\mu(t)\right|H\left|v_\mu(t)\right\rangle dt$$

$$=2\pi\left[(C_{12}C_{23}-S_{12}S_{23}S_{13})^2+q(C_{13}C_{23})^2\right], \tag{8-3}$$

and

$$\beta_{\tau\tau}=\varphi+\int_0^T\left\langle v_\tau(t)\right|H\left|v_\tau(t)\right\rangle dt$$

$$=2\pi\left[(-C_{12}S_{23}-S_{12}S_{23}S_{13})^2+q(C_{13}C_{23})^2\right]. \tag{9-3}$$

Note that $\beta_{ee}+\beta_{\mu\mu}+\beta_{\tau\tau}=2\pi\,)1-q)$ which shows that β is not completely free from dynamical parameters since the appearance in it of the parameter q.

Although because of this, β is not purely geometric, nevertheless it is interesting that it does not depend on the specific frequencies, but on the ratio of their differences only. This means that we have now (geometric) classes labeled by q.

SUMMARY AND CONCLUSION

In summary, we have calculated Berry phase for both two and three-flavor mixing neutrino oscillating system. Results in two-flavor neutrino system shows that the

Berry phase associated to neutrino oscillations is a function of the mixing angle only. Since geometrical (Berry) phase is an observable, the mixing angle can be (at least in principle) measured directly i.e. independently from dynamical parameters as the neutrino masses and energies.

In the case of three-flavor neutrino mixing oscillation system, the components of Berry phase related to ratio of differences of frequencies and not related directly to frequencies. Therefore in this case the Berry phase is not a pure geometrical parameter and it can not be related to measurements directly.

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