

## Chaotic Nature of Energy Demand

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**Abstract:** Energy forecasting plays a dominant role in the sustainable development economic optimization, resource planning and secure operation of electric power systems. The variation in energy demand is a major source of uncertainty in planning for future capacity enhancement, resource needs and operation of existing generation resources. Electric utilities need monthly peak and yearly demand forecasting for budgetary planning, maintenance scheduling and fuel management. The software tools currently used by the utilities and the various regulatory organizations indicate that 'official long term energy demand' forecasts of these systems are based, at best, on some form of linear or log-log linear regression (econometric) models with the parameters often estimated using the ordinary least-squares method. These approaches aim to develop mathematical models on the basis of available data. Most of the utilities have attempted to build energy forecasting models based on different behavioural assumptions about the shape of the demand curve. The parameters of these models are estimated by a variety of techniques. However, the in-depth study of energy demand data carried out by the authors has revealed its chaotic nature. Hence, the conventional modeling techniques have resulted in limited success in the forecasting and modeling of energy demand. This paper aims to depict the chaotic nature of energy demand by carrying out experimental studies on U S energy data and comparing its distinguishing features such as Correlation Dimension, Embedding Dimension and False Nearest Neighbours, Lyapunov Exponents and Predictive Power, Return Map and Power Spectral Density with those of a benchmark chaotic systems, viz., Box and Jenkins gas furnace data. The results clearly indicate that energy demand should be treated as chaotic system for better efficacy in its modeling and prediction.

**Key words:** Energy demand . chaotic systems . Box and Jenkins gas furnace data . correlation dimension . embedding dimension . false nearest neighbours . Lyapunov exponents . predictive power . return map . power spectral density . Alyuda Forecaster XL

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### INTRODUCTION

The last few decades have witnessed a revolution in scientific thought, which has inspired scientists, mathematicians and philosophers to reconsider previous assumptions concerning the nature of the physical world. This is due to the realization that it is the intrinsic nature of certain physical systems to defy their prediction and complete understanding. They show an inherent limitation to scientific experimentation. Most of the natural phenomena exhibit such nonlinear and dynamic behaviour.

The real world nonlinear dynamical systems are capable of producing radically different patterns from two almost indistinguishable initial conditions-a fundamental property of chaos known as sensitive dependence on initial conditions [1]. Chaotic systems make nearby trajectories diverge exponentially and keep the motion bounded by globally folding the

trajectories [2]. Small errors are quickly amplified, making the system unpredictable. However, periodic orbits are embedded inside the chaotic regions and when followed for a long time, most of the trajectories end up in a dynamical equilibrium called an attractor [3]. Thus, there are three fundamental characteristics of chaos.

- Irregular periodicity,
- Sensitivity to initial conditions and
- A lack of predictability.

There have been a number of interesting applications of chaos theory in recent years, following the theoretical prediction that chaotic systems can be controlled by changing their bifurcation parameters [4]. Some of these systems whose behaviour can be described as chaotic nonlinear dynamics are:

- Macroeconomic systems, where many factors have to be considered and the state of a system depends in a nonlinear fashion on its previous states.
- Financial systems, for example, exchange rates.
- Market systems, for example, the stock market and the electricity market.
- Biological systems, for example, the brain, the heart beat when a heart attack occurs.
- Geographical and ecological systems, for example, the weather of a place, the ozone hole movement.
- In general, the overall energy demand of the systems is continually increasing due to industrial growth, increase in electricity consumption and other related factors.
- The energy demand changes cyclically in response to the seasonal variations.
- The demand follows a definite pattern for each day of the week, but changes considerably over the weekends and public holidays. Day ahead, weather-based, forecasting has become the most crucial factor.

Estimating the future energy demand is an important factor in calculating the cost of energy investment projects and energy production [5].

Most of the utilities have attempted to build energy forecasting models based on different behavioural assumptions about the shape of the demand curve. The parameters of these models are estimated by a variety of techniques. However, the in-depth study of energy demand data carried out by the authors has revealed its chaotic nature. Hence, the conventional modeling techniques have resulted in limited success in the forecasting and modeling of energy demand.

This paper aims to depict the chaotic nature of energy demand by carrying out experimental studies on U S energy data and comparing its distinguishing features such as Correlation Dimension, Embedding Dimension and False Nearest Neighbours, Lyapunov Exponents and Predictive Power, Return Map and Power Spectral Density with those of a benchmark chaotic systems, viz., Box and Jenkins gas furnace data. The results clearly indicate that energy demand should be treated as chaotic system for better efficacy in its modeling and prediction.

#### **STATE OF ART**

Energy forecasting plays a dominant role in the sustainable development economic optimization, resource planning and secure operation of electric power systems. The variation in energy demand is a major source of uncertainty in planning for future capacity enhancement, resource needs and operation of existing generation resources. Electric utilities need monthly peak and yearly demand forecasting for budgetary planning, maintenance scheduling and fuel management.

The following characteristics make electrical energy forecasting a challenging task [6, 7]:

- Actual load of the electrical power system at any point in time depends on a number of factors, all of which cannot be accurately predicted.

- Forecast errors have significant implications for profits, market shares and ultimately shareholder value.
- Effect of price on demand is required to be considered in terms of volatility of the market and incentives for consumers.
- Information is becoming proprietary due to the deregulation.
- Distributed and embedded generation is increasing due to captive plants of large industrial consumers.

The software tools currently used by the utilities and the various regulatory organizations indicate that official long term energy forecasts are based, at best, on some form of linear or log-log linear regression (econometric) models with the parameters often estimated using the ordinary least-squares method [8]. These approaches aim to develop mathematical models on the basis of available data. Most of the studies have attempted to build models based on different behavioural assumptions about the energy shape. The parameters of these models are estimated by a variety of techniques. These include:

- Linear Regression Models based on different criteria such as Least Square Error or Least Absolute Error.
- State Space Model.
- Kalman Filter Model.
- Time Series Models such as, ARMA, ARIMA, ARMAX, etc. and Box-Jenkins Methodology.

The classical forecasting methods, when applied to fast developing utilities with a period characterized by fast and dynamic changes do not produce proper results due to ingrained uncertainty. New techniques with due consideration of uncertainty and system dynamics are, hence, being developed for better long term forecasting of energy. Modern methods for energy forecasting are based on emerging techniques, such as expert systems, artificial neural networks, fuzzy logic and wavelets. These have been applied recently and

show encouraging results [9]. Among them, Artificial Neural Networks (ANN) [10] and Fuzzy Logic models are particularly attractive, as they have the ability to model nonlinear relationships between energy and the factors affecting it, directly from historical data and without having to design a mathematical model. Many types of ANNs have been applied to energy forecasting; for example: Multi-layer Feed-Forward Network, Recurrent Neural Networks, Radial Basis Function Networks etc. The fuzzy logic based forecasting methods are capable of dealing with uncertainty effectively, whereas neural network based techniques provide efficient models for representing energy data sets. Hence, a combination of the two to form neurofuzzy [11] technique is expected to provide the best forecasting approach. It is notable that several neurofuzzy techniques have been developed recently for different applications.

### **CHAOTIC SYSTEMS**

Chaotic systems modelling and simulation has paved the path for in-depth study and forecasting of the real world systems found in nature. The mathematical exploration of chaotic systems has opened the door to a greater understanding of real-world phenomena. Besides being applicable to the natural sciences and engineering, the findings of chaos theory have also been used in philosophical and theological contexts. The emphasis on the foundation for scientific determinism laid by Newton and his contemporaries has been challenged by chaos theory. This theory was revolutionary in the sense that it acknowledged the fact that the motion of a system or its evolution through time may not be only stable or periodic, but also chaotic and unpredictable. As stated by Poincare, "Classical science emphasized order and stability; now, in contrast, we see fluctuations, instability, multiple choices and limited predictability at all levels of observation" [12].

A chaotic dynamical system is in continuous evolution and its basic characteristics are formed by a large number of constituent units, where each unit interacts with other units and responds to signals received from the others in a nonlinear manner [13]. The system is hierarchical i.e., a feature may be treated at several different levels before reaching the centre of action. Further, the local interactions may have global effect: they may produce considerable global change in the system. The system is adaptative in its evolution and has memory [14]. Hence, it is hard to be modelled mathematically. Some characteristics of the systems are randomly distributed. Sometimes the system develops a self-organization structure in its evolution i.e., from an

extremely disordered state, order emerges. The system may have several attractors and fractal structures also appear in many complex systems.

There are two major classes of tests for chaos within data. The first is to examine the paths or trajectories of the data when the system's initial conditions are adjusted slightly. This is done by estimating Lyapunov exponent. This is a measure of the average divergence or convergence between trajectories generated by experimental data of systems with infinitesimally small changes in their initial conditions. If the trajectories of a deterministic model deviate exponentially from each other, it has a positive Lyapunov exponent. If their paths converge back to a steady state, then the Lyapunov exponent is negative. A positive exponent signals that the system must have sensitive dependence to initial conditions and is chaotic.

The second type of test for chaos examines the dimensionality of the system. It may seem easy to explain that a square has two dimensions and a line has one, but it is significantly more complicated for chaotic systems since they have non-integer dimensionality. The "fractional" dimensionality has resulted in the use of the term "fractal" for shapes generated by chaotic data. Hence, if a system does not have an integer dimension, then it could be chaotic [15].

Chaotic system analysis is usually carried out by use of time series based modelling techniques. The measurement of input-output pairs of data at a common instant of time at regular intervals forms the basis of the time series analysis. The time series data is nowadays commonly used for modelling of complex industrial processes which cannot be represented by classical mathematical models. This technique is particularly useful when the process shows nonlinear dynamics and chaotic behaviour. A time-series model postulates a relationship amongst a number of temporal sequences or time series.

Chaotic time series is considered to be the output of a deterministic, autonomous dynamical system. The complexity of the time series is linked to the high order and nonlinear nature of the dynamical system and not to the exogenous random excitation, as in the linear case. In this approach, the model system must either be nonlinear or linear with time varying parameters, because linear time invariant systems have trivial autonomous dynamics.

The behaviour of chaotic time series is characterized by the observation that over a time period, there is neither a clear tendency to return to a fixed value nor a linear trend. Such time series usually have seasonal variations, long-term and short-term fluctuations, which may not be limited only to the mean of the series but may also affect its overall variance

structure. Typically, such series are characterized by patterns like trends and localized abrupt changes, also known as volatility clustering. An important concept in time series analysis is multiscaling, in which time series exhibit several phenomena, each occurring at different time horizons. Some time series are also characterized by a persistence of autocorrelations much longer than expected (long memory processes) and hence cannot be explained appropriately by prevalent statistical modelling techniques. Since these series do not follow well-established theoretical phenomena, they are increasingly analyzed by employing the various 'model free' soft computing and intelligent techniques.

On the complexity scale, chaotic time series spans the gap between periodic signals and random noise. Chaotic systems offer the ultimate difficulty for developing a model from a time series, because the signal is time varying and highly complex with a flat continuous spectrum. It is therefore the perfect environment to test new modelling approaches since a minor error in the model parameters is amplified by the natural divergence of the trajectories in phase space. Its potential usefulness is also enormous, because many real world phenomena are chaotic. Time series produced by chaotic systems that were previously considered to be random, in fact have deterministic structure which can be modelled.

### **BEHAVIOUR OF CHAOTIC SYSTEMS**

It is necessary to visualise the significant behavioural aspects of dynamic systems prior to the in-depth study of chaotic systems. It is important because, a dynamical system is a member of the parameterised family of chaotic systems with control parameters fixed at a certain value. Mathematically, it signifies that the equations expressing these nonlinear systems are extremely sensitive to initial inputs. Although such systems normally show some regularity, it is impossible to predict their future behaviour with a high degree of certainty. However, by adjusting their parameters, nonlinear models can capture long term memory behaviours and internal dynamics of chaotic systems to a reasonable extent [16].

The noise like behaviour of chaos includes very important information. Understanding of a chaotic behaviour is a crucial step towards its prediction and control. In order to better understand chaotic behaviour, it is necessary to become familiar with some basic notions that characterize every chaotic process [17]. Some of these are discussed as follows:

The phase space of a chaotic process is the feature space where the process is traced over time. The traces of points recorded over time are called trajectories. If a

process periodically repeats the same points in the phase space after a definite time period, it is periodic and not chaotic. A chaotic process does not repeat the same trajectory pattern over time. Choosing the phase space is very important for understanding any process. The most useful image of a dynamical system is a collection of its trajectories called as portrait which presents the state space divided into basins by separatrices, with one attractor indicated in each basin.

A chaotic process usually goes around areas or points on its phase space, but without repeating the same trajectory. Such areas or points, in the phase space are called chaotic attractors. A chaotic process may have many attractors in the phase space and many unstable fixed points [18].

The portrait of a chaotic system changes when its control parameters are varied. When the controls are moved smoothly and gradually, the portrait may be seen to also change smoothly and gradually. Sometimes, however, the portrait undergoes a radical change even when the controls are moved very gently. Such an event is called a bifurcation. Bifurcations are certainly the most important features of a nonlinear dynamical system and locating them is possible by looking at some exemplary cases. They fall into three categories [19]:

- Subtle bifurcations, in which the change is not immediately striking
- Catastrophic bifurcations, in which a basin suddenly appears or disappears
- Explosive bifurcations, in which an attractor suddenly expands or contracts.

Expansions and contractions of a chaotic process due to bifurcation may be observed in its phase space. Moreover, sensitivity to initial conditions is an important characteristic of a chaotic process. A slight difference in the initial values of some parameters that characterize the chaotic process will result in quite different trends in its future development.

It can hence be concluded that chaotic processes seem random, but they are not completely so. They are not periodic, but they do repeat almost the same patterns in their phase space. They are predictable, but only for a short time in the future [20]. Hence, it is important to realize that the presence of chaos in a system does not necessarily imply randomness [21]. A chaotic system does display some element of order despite its apparent disorder and is in fact governed by deterministic equations. Mathematical chaos inhabits a middle ground between order and disorder: "chaotic randomness so understood is a tertium quid; it is mathematically distinguishable from both strict

randomness and the complete absence of randomness” [22]. A mathematical model of the way in which randomness and order may coexist within a system is the “strange attractor” a chaotic set which attracts nearby points to it. So the presence of chaos in a system may give the appearance of randomness, even though the system is governed by clearly defined dynamics [23]. This is not randomness in the true sense of the word, as it does not imply complete disorder, but only eventual unpredictability. This lack of complete randomness is necessary for a proper understanding of chaos, since randomness is not usually associated with rational modelling. Thus, both sides of the spectrum, strict determinism and complete randomness, seem incompatible with the notion of making rational and creative decisions [24].

The following parameters appear to be the most useful in the context of characterizing chaotic behaviours:

**Correlation dimension:** A non integer result for the correlation dimension indicates that the data is probably fractal. For too low or too high values of  $r$ , the results are inaccurate. The formula used for finding out the correlation dimension is as follows:

$$D = \lim_{r \rightarrow 0} \frac{\log C(r)}{\log r} \quad (1)$$

$C(r)$  is the probability of two points being closer than  $r$ .

**Embedding dimension and false nearest neighbours:** The estimation of false nearest neighbours provides a method of choosing the minimum embedding dimension of a one-dimensional time series. This method finds the nearest neighbour of every point in a given dimension, then checks to see if these are still close neighbours in one higher dimension. The percentage of false nearest neighbours should drop to zero when the appropriate embedding dimension has been reached. If the number of false nearest neighbours is not zero but is very small, the embedding dimension may still be suitable for most forms of analysis.

**Lyapunov exponents:** The most widely accepted definition of chaos is that a chaotic system is one which has at least one positive Lyapunov exponent. The Lyapunov exponents of a system describe the exponential rates of convergence or divergence of nearby trajectories. For an  $n$ -dimensional system there are  $n$  exponents. Chaos arises from the exponential growth of infinitesimal perturbations. Each exponent is the average rate of growth of infinitesimal perturbations along any direction.

**Predictive power:** The Lyapunov exponent provides the average rate of divergence or convergence for a chaotic process. Based on the Lyapunov exponent, another parameter  $q$  called predictive power can be calculated for any chaotic process, as  $q = 1/L$ , where  $L$  is the largest Lyapunov exponent. This parameter can be used, to evaluate roughly how many points can be predicted in the future, based on the current value and on some previous values of the index [25].

**Power spectral density:** The spectral density of the time series, when multiplied by an appropriate factor, gives the power carried by the time series, per unit frequency. Thus, the amount of power per unit (density) of frequency plotted as a function of the frequency (spectral) is known as the power spectral density of the signal. Power Spectral Density (PSD) is commonly expressed in watts per hertz (W/Hz) or dBm/Hz. Here power can be the actual physical power, or more often, for convenience with abstract signals, can be defined as the squared value of the signal, that is same as the actual power if the signal was a voltage applied to a 1-ohm load.

The power spectral density, PSD, describes how the power (or variance) of a time series is distributed with frequency. Mathematically, it is defined as the Fourier Transform of the autocorrelation sequence of the time series. An equivalent definition of PSD is the squared modulus of the Fourier transform of the time series, scaled by a proper constant term.

**Return map:** Plot of a time series as a function of the current and of the previous values is known as return map. In the case of chaotic systems, the return map is confined to a definite region of state space.

The authors now present the case studies of Box and Jenkins Gas Furnace time series, which is a benchmark chaotic system and U S Energy Demand time series and compare their parameters to ascertain the chaotic nature of energy demand.

#### CASE STUDY OF A BENCHMARK CHAOTIC SYSTEM (BOX AND JENKINS GAS FURNACE TIME SERIES)

The gas furnace data (series J) of Box & Jenkins is a nonlinear dynamical chaotic system. This data set is readily available on the internet and in the seminal work of the two researchers, reproduced in the book by Box, Jenkins and Reinsel [26]. It represents the data recorded from the combustion process of a methane air mixture. It is well known and frequently used as a benchmark example for testing, modelling and prediction algorithms. The input  $u(t)$  is methane

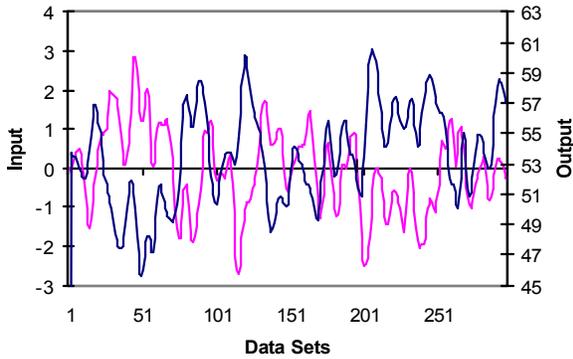


Fig. 1: Box and Jenkins time series input and output plotted on same time base

concentration in the gas flow into the furnace and the output  $y(t)$  is  $\text{CO}_2$  concentration in outlet gas. During the process the portion of methane was randomly changed, keeping a constant gas flow rate. The data set consists of 296 pairs of input output measurements. The sampling interval is 9 sec.

The plot of Box and Jenkins time series output (blue) and input (pink) after suitable scaling has been shown in Fig. 1. The plot reveals that there is a definite inverse relationship between the input and output along with the feature of time delay.

Since Box and Jenkins time series is a well known benchmark that exhibits chaotic nature, the basic parameters that are commonly used for ascertaining chaotic characteristic have been evaluated for it. These include the following:

**Correlation dimension:** The plot of correlation dimension of Box and Jenkins time series, i.e.,  $\log(C)$  versus  $\log r$  is shown in Fig. 2. It can be seen that the logarithmic plot of correlation dimension of the data is fairly linear, with a slope of nearly unity. The correlation dimension plot clearly exhibits the saturation phenomenon, as expected for chaotic system.

**Embedding dimension and false nearest neighbours:** The plot of false nearest neighbours versus embedding dimension for Box and Jenkins series remains high until embedding dimension = 3 as shown in Fig. 3. It suddenly drops to a minimum for embedding dimension = 4 showing that the system is chaotic with the number of false nearest neighbours falling to nearly zero at this value of embedding dimension.

**Lyapunov exponents:** The plot of Lyapunov exponent for 100 iterations of Box and Jenkins time series data is shown in Fig. 4. The value of Lyapunov exponent is positive throughout, showing the divergent nature of time series. However, beyond 50 iterations its value remains constant at 0.462.

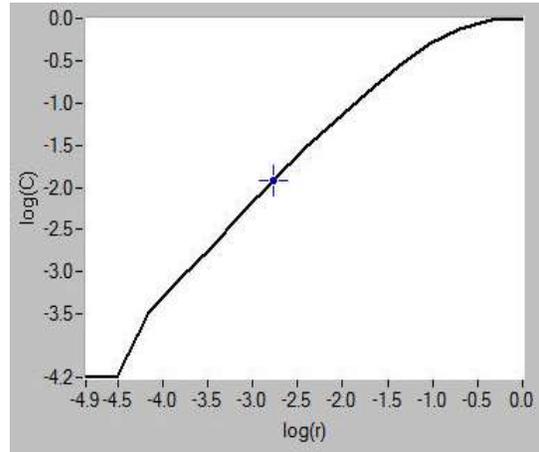


Fig. 2: Plots of correlation dimension of Box and Jenkins time series

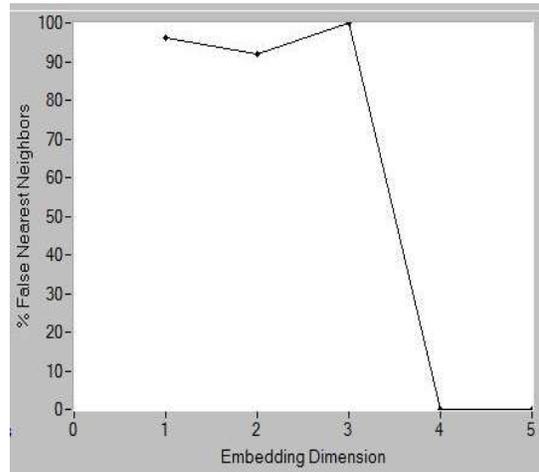


Fig. 3: Plots of false nearest neighbours of Box and Jenkins time series

**Predictive power:** The predictive power of Box and Jenkins time series is 1.667.

**Power spectral density:** The power density spectrum of the time series is shown in Fig. 5. The power spectrum shows a continuous broad range and the absence of discrete peaks clearly indicates that this time series cannot be modelled by use of methods based on superposition of regular oscillations, such as in Fourier analysis.

Another significant feature of chaotic behaviour is that the return map of the data for both  $y(t)$  and  $x(t)$  is confined to a definite region. It can be clearly observe d from Fig. 6 for the output data and Fig. 7 for the input data that Box and Jenkins time series is confined to a small region of the return map, as expected. It is notable that the output data is more tightly clustered as

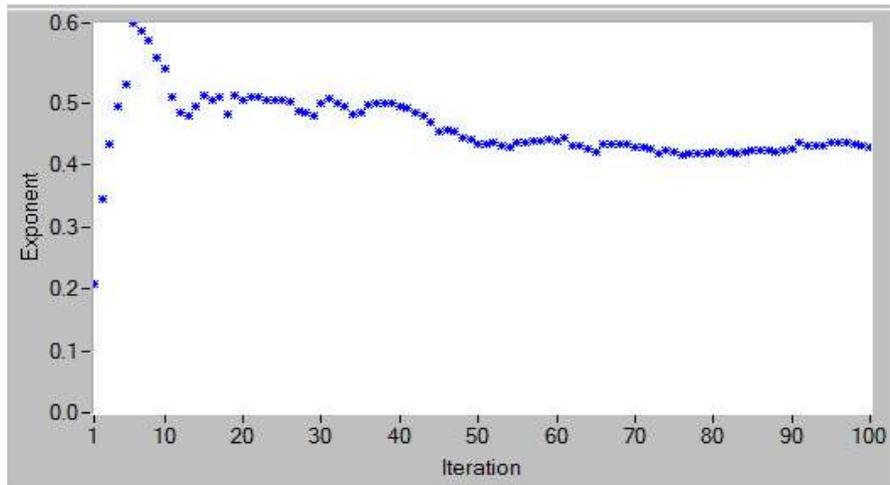


Fig. 4: Plots of Lyapunov exponent of Box and Jenkins time series

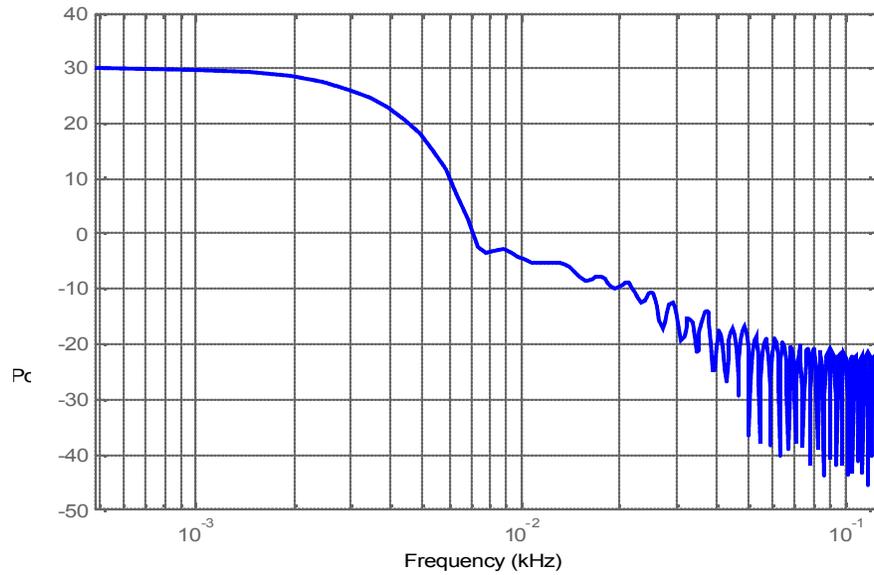


Fig. 5: Plots of power spectral density of Box and Jenkins time series

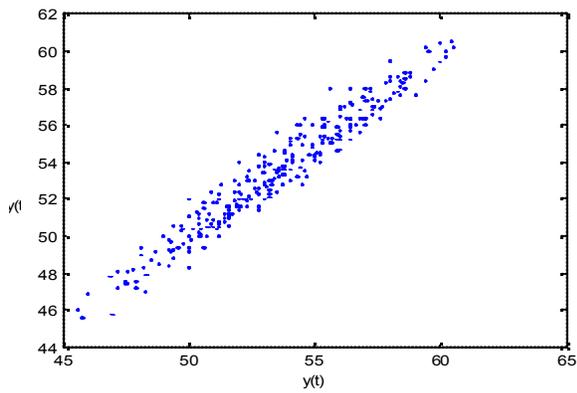


Fig. 6: Return map of Box and Jenkins time series for output variable

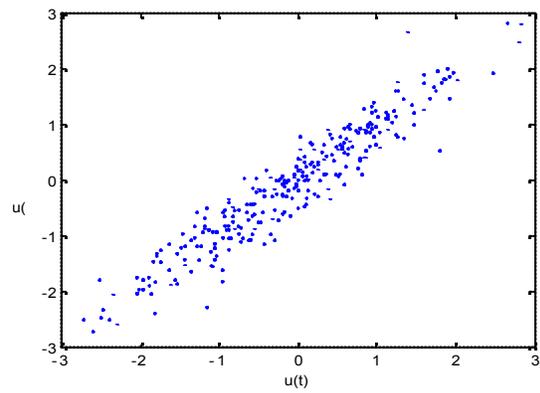


Fig. 7: Return map of Box and Jenkins time series for input variable

Compared to the input data, showing that the output appears to fall around an attractor zone.

The above analysis confirms that chaotic modelling of the series is necessary.

### CASE STUDY OF ENERGY DEMAND OF US ELECTRICITY DATA

A common forecasting problem in most of the fast growing utilities is the short and insufficient database, unparalleled experience of fast growth and unavailability of forecasting techniques capable of addressing adequately this particular problem [27]. Moreover, there exists a unique dynamic pattern of the energy and demand. The resulting system energy characteristics are too complex to analyze and difficult to predict and hence classical techniques prove to be inadequate. This fact is elaborated in Fig. 8. This figure shows the pattern of variation of net generation in billion kWh by American utilities from selected years. The data has been obtained from the official website of National Energy Information Center of U S government. It pertains to the U S electricity industry. The net generation of electricity (in million kilowatt hours) by the American Utilities has been presented in the form of monthly data for the period January 1973 to December 2008 [28]. The enormous variation in data and the requirement of handling nonlinearity associated with it can be easily seen in this figure. It is clear that this time series is non-stationary and exhibits strong seasonal pattern that is slowly changing in shape. It is chaotic in nature and also depicts a marked increasing trend. This is the normal pattern for a energy forecasting problem, which possesses all the basic characteristics that plague the accuracy of classical methods of forecasting.

An effort was made to analyse the time series of the US energy data for ascertaining whether it exhibits chaotic nature. The basic parameters that are commonly used for determining chaotic characteristic have, hence, been evaluated as follows:

**Correlation dimension:** The plot of correlation dimension, i.e.,  $\log(C)$  versus  $\log r$  is shown in Fig. 9. It can be seen that the logarithmic plot of correlation dimension of the data lies in the third quadrant and is fairly linear, with a slope of nearly unity. The correlation dimension clearly exhibits the saturation phenomenon, as expected for chaotic system.

**Embedding dimension and false nearest neighbours:** The plot of false nearest neighbours versus embedding dimension is also substantially high until embedding dimension = 3 as shown in Fig. 10. It suddenly drops

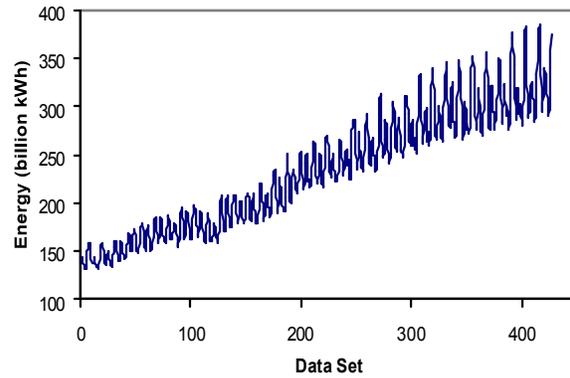


Fig. 8: Time series representation for the month wise energy forecasting data (from Jan 1973 to December 2008)

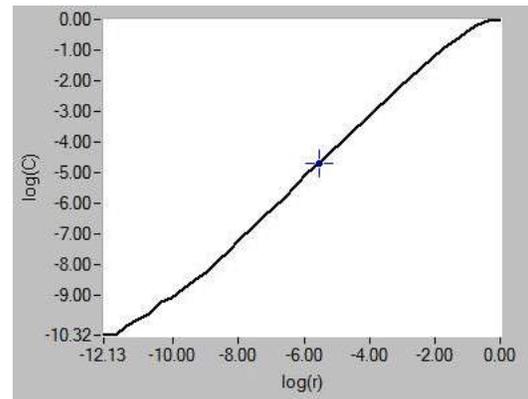


Fig. 9: Plots of correlation dimension of US energy time series

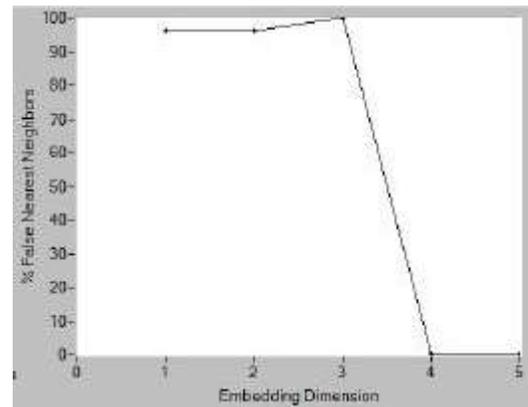


Fig. 10: Plots of false nearest neighbours of US energy time series

for embedding dimension = 4 showing that the system is non-repetitive, i.e., chaotic with the number of false nearest neighbours falling to nearly zero at this value of embedding dimension.

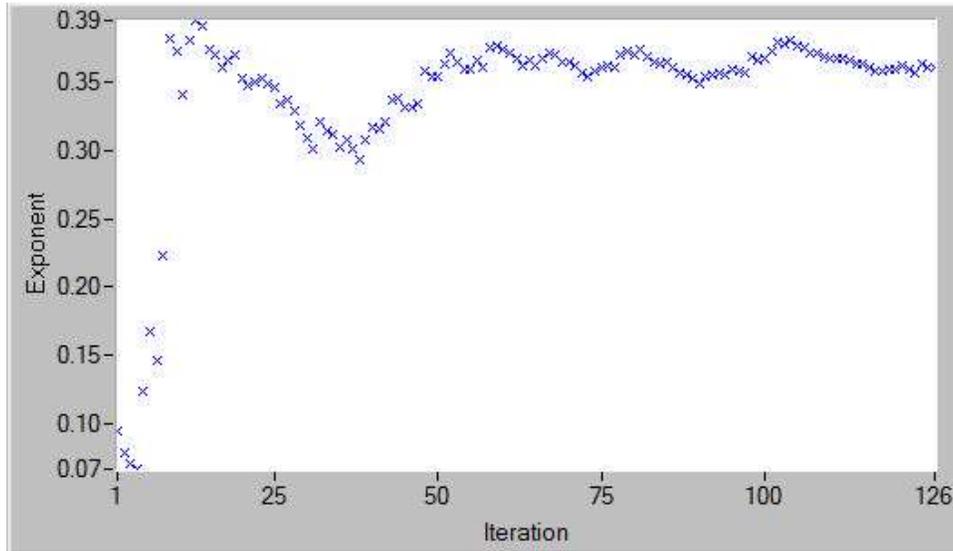


Fig. 11: Plots of Lyapunov exponent of US energy time series

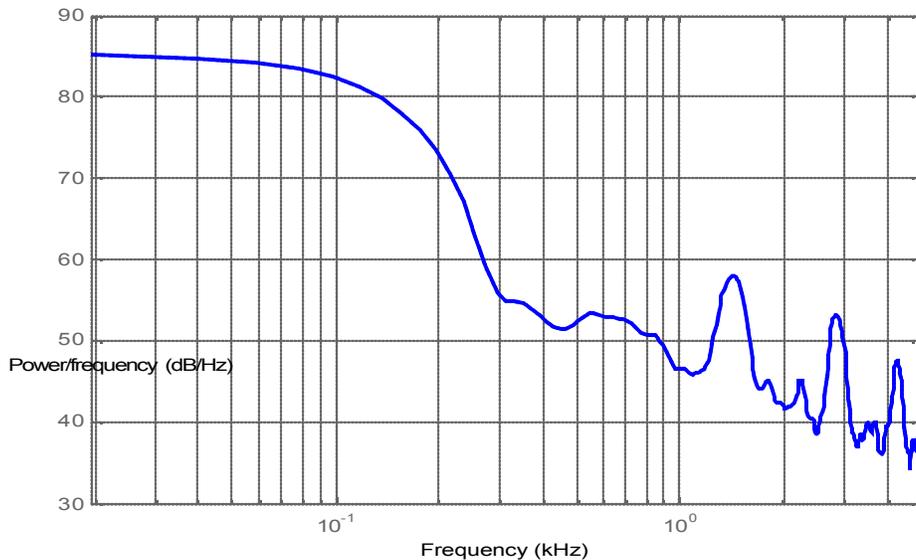


Fig. 12: Plots of power spectral density of US energy time series

**Lyapunov exponents:** The plot of Lyapunov exponent for 126 iterations is shown in Fig. 11. The value of Lyapunov exponent is positive throughout, showing the divergent nature of time series. However, beyond 100 iterations its value remains constant at 0.362.

**Predictive power:** The predictive power of Box and Jenkins time series is 2.56.

**Power spectral density:** The power density spectrum of the time series is shown in Fig. 12. The power spectrum shows a continuous broad range and the absence of discrete peaks clearly indicates that this time

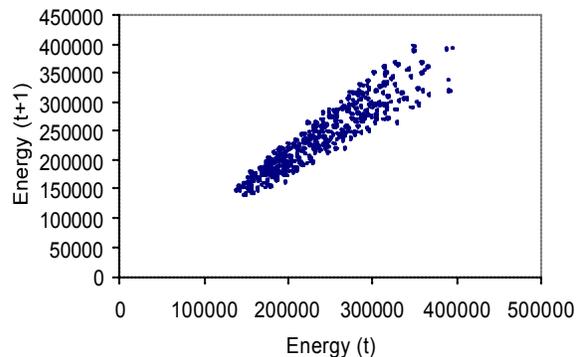


Fig. 13: Return map of U S energy time series

Table 1: Comparison of chaotic parameters

|                           | Box and jenkins gas furnace | Energy demand          |
|---------------------------|-----------------------------|------------------------|
| Correlation dimension     | Unity                       | Unity                  |
| Embedding dimension       | 4                           | 4                      |
| Maximum lyapunov exponent | 0.6                         | 0.39                   |
| Predictive power          | 1.667                       | 2.56                   |
| Power spectral density    | Continuous broad range      | Continuous broad range |
| Return map                | Tightly clustered           | Tightly clustered      |

series cannot be modelled by use of frequency domain methods based on superposition of regular oscillations.

Another significant feature of chaotic behaviour is that the return map of the data for Energy (t) is confined to a definite region. It can be clearly observe from Fig. 13 for the Energy data that is confined to a small region of the return map, as expected. It is notable that the output data is more tightly clustered as compared to the input data, showing that the output appears to fall around an attractor zone.

### RESULTS

It has been observed in this experimental study that there is a striking similarity in the parameters of both of the studied systems. This has been summarized in Table 1.

It can be seen from the above table that the energy demand clearly depicts chaotic nature, as reflected in its parameters, which are quite close to those of the benchmark chaotic system. While correlation dimension and embedding dimension are exactly the same for both the datasets, power spectral density depicts a continuous broad range in different zones for both. It is also notable that even though energy demand is more predictable (with the predictive power of 2.56) as compared to Box and Jenkins gas furnace data (with prediction power of 1.667), the prediction of energy for more than three steps (months) ahead is expected to yield less reliable results. This has been reaffirmed by use of neural network based modeling tool Alyuda Forecaster XL.

The description of Forecaster [29] claims electrical load forecasting and energy demand forecasting as its potential areas of application. It is a neural network forecasting add-on for the MS Excel and is frequently used for forecasting of market price indices. This software developed neural network model for time series data as per details given in the Table 2.

The forecasts shown in the various columns are for 1, 2, 3, 4, 5 and 6 months ahead. The training dataset was created from previous years data from Jan 1973 to the corresponding last month of 2007. Since there is a high correlation in datasets for 1 month ahead, it shows

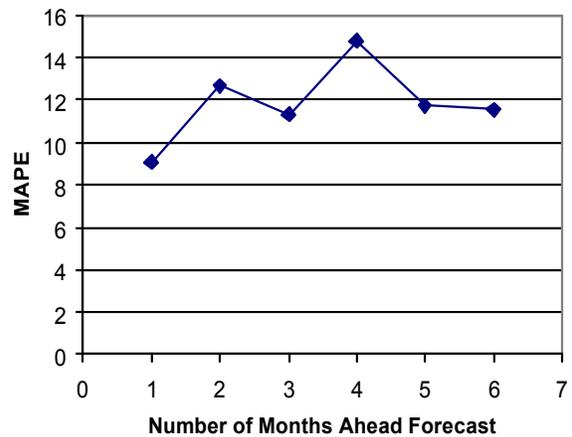


Fig. 14: Forecaster MAPE with respect to months ahead for energy time series

the minimum MAPE of 9.02 %. Complete predictions for 12 months are obtained for 2, 3 and 4 months ahead forecast with slight increase in MAPE. It is in close range, 11.29 % for 3 months ahead, 12.67 % for 2 months ahead and 14.80 % for 4 months ahead. The smaller MAPE values for 5 and 6 months ahead forecasts are deceptive.

The Forecaster was expected to produce forecast for 12 data points, out of which it forecasted for only 11 months successfully for the 5 and 6 months ahead forecasts. The MAPE for these columns for 11 month is 11.71794 and 11.55973 respectively. Thus, the limit of accurate forecasts of U S energy demand is 3 months ahead, which agrees well with its predictive power of 2.56. This shows that the Forecaster, which is among the most successful forecasting tool commercial available in the market, strengthens our concept of need for application of predictive power and chaotic analysis in the modeling and forecasting of energy demand. Thus the forecasting of energy data several months ahead is not so accurate and it should be proper care should be taken to obtain accurate forecasts with the help of chaotic analysis. The plot of error with respect to the number of months ahead forecasts is shown in Fig. 14.

Table 2: Comparison of MAPE using Alyuda Forecaster XL for months ahead

| Month       | Actual energy | Forecast model (Month ahead) energy (Million kWh) |                  |                 |                |                  |                  |
|-------------|---------------|---|------------------|-----------------|----------------|------------------|------------------|
|             | (Million kWh) | One   | Two              | Three           | Four           | Five             | Six              |
| Jan 08      | 362142.474    | 305357.805  | 295311.16        | 270926.1        | 289146         | No Forecast      | 332315.1         |
| Feb 08      | 324274.772    | 303922.818  | 287539.20        | 326324.3        | 289506.3       | 310904.9         | No Forecast      |
| Mar 08      | 323931.760    | 336599.528  | 293771.16        | 265809.3        | 271665.6       | 291715           | 318272           |
| Apr 08      | 304333.531    | 337638.925  | 341604.98        | 276855.3        | 289240.7       | 331154.5         | 298603.1         |
| May 08      | 324589.071    | 326175.049  | 344187.35        | 356191.9        | 289076.8       | 302497.2         | 273528.3         |
| June 08     | 372442.921    | 335373.342  | 312243.51        | 357738.9        | 290482.6       | 316896.4         | 274429.8         |
| July 08     | 402088.175    | 316036.760  | 338578.08        | 304177.2        | 290547.4       | 292122.3         | 305662.9         |
| Aug 08      | 387974.725    | 367112.705  | 298020.08        | 354533.5        | 312806.3       | 292437.6         | 291898.1         |
| Sep 08      | 337258.801    | 390734.846  | 312884.52        | 280721.9        | 290421.9       | 283170.1         | 293228.3         |
| Oct 08      | 318231.559    | 296806.404  | 304952.18        | 296654.3        | 288973         | 291868.7         | 298256.3         |
| Nov 08      | 309929.807    | 300309.806  | 284875.18        | 289783.8        | 289182.9       | 298371.7         | 291540.5         |
| Dec 08      | 343061.285    | 313674.689  | 274227.30        | 322667.1        | 288864.9       | 357713.8         | 306036.1         |
| <b>MAPE</b> | -----         | <b>9.024104</b>                                   | <b>12.672610</b> | <b>11.28626</b> | <b>14.8007</b> | <b>11.71794*</b> | <b>11.55973*</b> |

\*These values have been evaluated after discarding the skipped forecasts

Thus, it can be concluded that the forecasting horizon for the energy demand should be determined carefully for guarantee of correct forecasts. It is, hence, necessary to model energy data using chaotic system modeling techniques, rather than the hitherto commonly employed linear/nonlinear mathematical methods. Thus, chaotic modeling of the energy demand and consequently the determination of predictive power is expected to be very useful for prevention from pitfalls of wrong forecast.

### CONCLUSION

The analysis of various properties of the benchmark chaotic system Box and Jenkins gas furnace and the energy demand system under consideration, viz., U S energy demand has been carried out in this paper. The results clearly indicate that the application of chaotic time series models is important for researchers working in the field of modelling and control of electrical energy demand, since it often shows random disturbances and irregular periodic oscillations. However, the response of the overall system tends to become stable, as if drawn into attractors. Another notable feature is its sensitivity to the small changes in initial conditions and energy. Hence, power system researchers have ample scope for the application of the concepts of chaotic systems and provide a fertile ground for soft computing based modelling and prediction. The authors of this paper are presently engaged in developing soft computing based methods for modeling and prediction of chaotic systems. These methods are expected to be of great utility in the accurate forecasting of energy demand.

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