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Experimental and Modeling Technique for Liquid Holdup Between Spherical Particles

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Abstract: The semi-empirical model is presented to investigate the liquid bridge formed in liquid between two spherical particles. Also, the prediction of the maximum volume of liquid holdup between two spherical particles with respect to particle size, liquid characteristics and body force is one of the objectives of this paper. In order to calculate the volume of the liquid bridge, the previous methods used a combination of the variables such as filling angle, interface curvature and the liquid bridge neck diameter for measurement. However, the filling angle and interface curvature are difficult to measure in practice. In this paper, some equations from previous publications are transformed into functions of a single variable, which is the ratio between liquid bridge neck diameter and particle size. Then a comparison is made between the result based on these equations and some experimental results. It is also assumed that the liquid contact angle is zero and the Maple computer software was adopted to modeling. The main difference between the current research and the previous ones is that the gravity effect is not neglected and hence it might be applicable to centrifugal processes.

Key words: Liquid holdup · Liquid bridge · Spherical particles · Contact angle · Maple software · Gravity effect

INTRODUCTION

The investigation of maximum liquid holdup between particles is closely related to the liquid drainage in a packed bed or porous media. It has diverse applications in different fields such as centrifugal separation [1,2], sintering [3], landfill saturation [4], oil recovery from reservoirs [5, 6, 7], rheological behavior of aerated palm kernel oil-water emulsions [8] and in the removal of heavy metals from wastewater [9].

There had been many attempts to study the liquid bridge formed between two particles. However, the main theme was the force required to separate two particles joined by a liquid bridge [10-17]. The main assumption in all those papers was neglecting the body force (gravitational/centrifugal) due to small particle size. Moreover their consideration of the liquid bridge volume concerned only the volume and not the maximum liquid holdup at equilibrium. Therefore they are not applicable in a separation process such as centrifugation where the body force plays an important role. Three forms of liquid holdup involved in a draining bed can be identified as dynamic liquid holdup, static liquid holdup and free surface liquid holdup. It is believed that the static liquid holdup between the particles makes the major contribution toward the carryover of impurities in the sugar crystal centrifugal separation process. Thus, this study is focused on static liquid holdup.

MATERIALS AND METHODS

In these experiments, molasses of four different dilutions, oil and water have been used as the liquid media. The pairs of spheres of sizes 0.01, 0.02, 0.03 and 0.04 meters in diameter were used.

Experimental Setup: The experimental setup to measure the static liquid holdup between two spheres is shown in Figure 1. It consists of two equal size spherical glass particles mounted vertically one above the other. The experiments are carried out by lifting the upper particle about half a centimeter. Then, while placing it back, a syringe is used to fill the gap between the particles with a liquid. A traveling microscope is then used to measure the liquid bridge neck diameter at different times till it reaches as a steady value.

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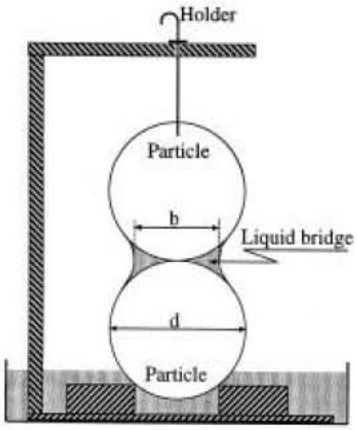

Fig. 1: Setup for measuring static liquid holdup

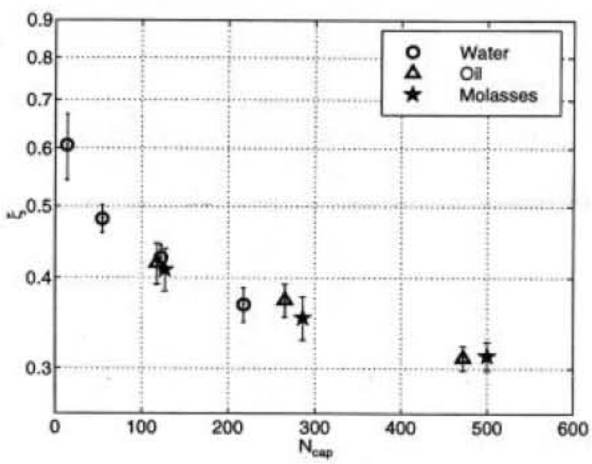

Fig. 2: Experimental results for static liquid holdup

Experimental Error: Experimental errors consist of three parts in the following:

- In accuracy in traveling microscope.
- Variation in media temperature which affects the surface tension by about 3% to 5%.
- The volume of the liquid bridge holdup between spheres of 0.01 to 0.04 m diameter varies from 0.04 to 0.25 cm^3 respectively. This shows that even minute variation in total liquid holdup can affect the result. Therefore, if the experiment has not been conducted in a well-sealed container, the variation of air temperature and humidity can cause evaporation of the liquid holdup. This is more obvious in the first set of experiments (Figure 2 at the dimensionless liquid bridge neck diameter (ξ) is 0.6), where part of the experiment has not been conducted in a completely sealed container.

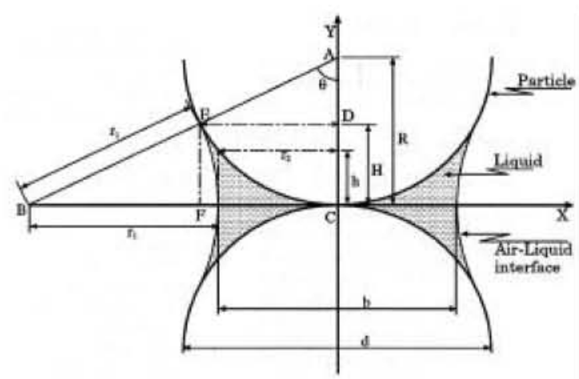

Fig. 3: Liquid holdup between two vertically attached spheres

Phenomenological Aspect: The technique of feeding the liquid between the particles and also initial draining flow rate also may affect the final result.

Modeling: Modeling the system, consider two spherical particles vertically positioned and kept in contact (zero gaps). In this research for modeling, it is assumed that the air-liquid interface has a circular shape and is symmetrically balanced around both X and Y axes (Figure 3). It is assumed that the liquid contact angle is zero. In the present paper, a force balance considered for the liquid bridge and in turn a semi-empirical model predicts the dimensionless liquid bridge neck diameter (ξ) as a function of capillary number (N_{cap}).

The model is presented in Figure 3. Consider two identical spheres of radius R in contact vertically with each other. At the equilibrium, there will be maximum liquid held up between the particles.

Consider the triangle ABC where $r_2 = b/2$ is the liquid bridge neck radius along the X -axis which passes through the sphere's contact point and r_1 is the liquid bridge air-liquid interface radius in XY plane. The angle EAC (θ) is called the filling angle and is the angle formed between the Y-axis and the line passing through the center point of the particle and the air-liquid-solid interface point. Then the volume of the liquid bridge is calculated as a function of (ξ). For the calculation of the volume of the liquid bridge, three of the previously reported models [16, 18 and 19] were used. These models were functions of θ, r_1 and r_2 (Figure 3). However, θ and r_1 are difficult to measure practically. Therefore, they were transformed to be only the function of ξ which is in practice easily measurable.

RESULTS AND DISCUSSION

Prediction of ξ as a Function of N_{cap}: In this research, as it is assumed that the air-liquid interface is of a circular cross section and thus is symmetrical about the BC axis, the forces acting on the liquid bridge are F_s (surface tension), F_c (hydrostatic) and F_g (gravity). Therefore, the force balance can be written as:

$$F_s+F_c=F^g \tag{1}$$

Where

$$F_s=2\pi\sigma R\sin^2\theta \tag{2}$$

$$F_c=\pi\sigma\left(\frac{1}{r_1}-\frac{1}{r_2}\right)R^2\sin^2\theta \tag{3}$$

$$F_g=2\pi\rho g\overline{DE}H^2 \tag{4}$$

The liquid bridge neck diameter (b) and particle size (d) are the practically measurable variables. Therefore, it is important to express all other variables in terms of the dimensionless number ξ which is the function of r_2 and R.

$$\xi=\frac{b}{d}=\frac{r_2}{R} \tag{5}$$

Where b = liquid bridge neck diameter and d = particle diameter.

Considering equations (2-4), the values of r_1, $\sin\theta$, H and DE are to be expressed in dimensionless form as a function of ξ.

$$\varsigma=\frac{r_1}{R}=\frac{\xi^2}{2(1-\xi)} \tag{6}$$

And

$$\frac{H}{R}=\frac{\xi^2}{(2-2\xi+\xi^2)} \tag{7}$$

Now to define DE as a function of

$$\frac{\overline{DE}}{R}=\left(\frac{\xi(2-\xi)}{2-2\xi+\xi^2}\right) \tag{8}$$

The value of $\sin\theta$ is:

$$\sin\theta=\left(\frac{\xi(2-\xi)}{2-2\xi+\xi^2}\right) \tag{9}$$

After substituting the equations 2-9 in equation 1 and simplification, we will arrive at the following:

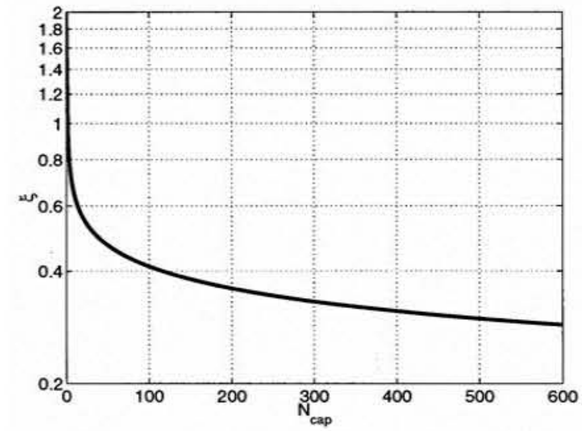

Fig. 4: Dimensionless liquid bridge neck diameter as a function of N_{cap}

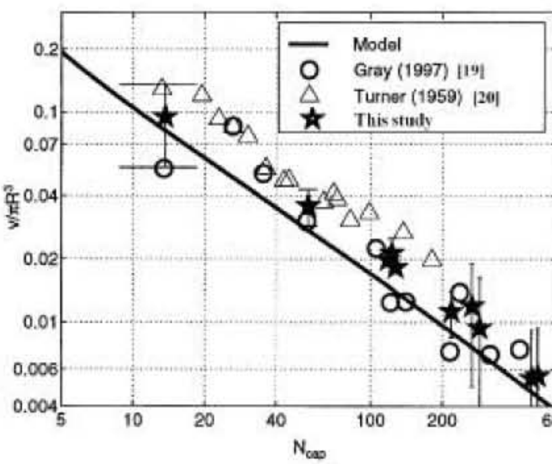

Fig. 5: Dimensionless liquid bridge neck diameter as a function of N_{cap}

$$N_{cap}=\frac{2(2-\xi)(2\xi^3-3\xi+2)}{\xi^4} \tag{10}$$

Where

$$N_{cap}=\frac{\rho g d^2}{\sigma} \tag{11}$$

Where N_{cap} is called the Capillary Number or alternatively the Bond Number and consists of known liquid physical characteristics (density ρ and surface tension σ), body force (gravitational/centrifugal g) and particle size (diameter d).

For a given N_{cap}, ξ can be predicted and from that, the liquid bridge volume. Figure 4 illustrates the value of N_{cap} calculated when ξ varies from 0.29 to 2.00.

For a particular system within the value of N_{cap}, the maximum liquid bridge diameter (ξ) can be found from Figure 5 [20, 21].

Prediction of Volume as a Function of ξ: It is important to be able to calculate the liquid bridge volume as a function of ξ. For this study, equations presented by Fisher [16] and Simons *et al.* [18] for horizontally arranged particles and Smith *et al.* [19] for vertically arranged particles have been considered. In all these models, the particles were equal size spherical and liquids with zero contact angles were only considered. Also, the gravitational force is considered to be zero.

These equations are first transformed to non-dimensional form and then changed to become only a function of ξ (which can also easily be measured experimentally).

Fisher's Equation: The basic equation to calculate the liquid bridge volume with a symmetrical interface given by Fisher [14] is:

$$V = 2\pi R^3 (\sec\theta - 1)^2 \left\{1 - \left(\frac{\pi}{2} - \theta\right)\tan\theta\right\} \tag{12}$$

This is an approximate model and it is assumed that the total curvature to be constant and hence the meridian curve to be circular. With this assumption, the volume is calculated by merely geometrical consideration in terms of particle size (R) and filling angle (θ). The variables $\sec\theta$, θ and $\tan\theta$ in equation 12 can be transformed to be functions of ξ only.
Thus,

$$\sec\theta = \frac{1}{\cos\theta} = \frac{2-2\xi}{2-2\xi+\xi^2} \tag{13}$$

Now by using equation 9, the value of θ as a function of ξ can be written as;

$$\theta = \sin^{-1}\left(\frac{\xi(2-\xi)}{2-2\xi+\xi^2}\right) \tag{14}$$

By using equations 9 and 13, $\tan\theta$ can be written as a function of ξ

$$\tan\theta = \frac{\xi(2-\xi)}{2-2\xi} \tag{15}$$

By substituting equation 13, 14 and 15 into equation 12 and simplifying, the liquid bridge volume can be written in dimensionless form:

$$V = 2\left(\frac{\xi^2}{2-2\xi}\right)^2 \left\{1 - \left[\frac{\pi}{2} - \sin^{-1}\left(\frac{\xi(2-\xi)}{2-2\xi+\xi^2}\right)\right] \times \left(\frac{\xi(2-\xi)}{2-2\xi}\right)\right\} \tag{16}$$

Where

$$V = \frac{v}{\pi R^3} \tag{17}$$

Equation 16 can be further simplified;

$$V = \frac{\xi^4\left\{4 - 4\xi - 2\pi\xi + \pi\xi^2 + \theta\left(2\xi^2 - 4\xi\right)\right\}}{8(1-\xi)^3} \tag{18}$$

If the value of ξ is known, the value of θ can be calculated from equation 14. Then ξ and θ can be substituted into equation 18 to calculate the dimensionless liquid holdup volume (V).

Simons *et al.*'s Equation: Simons *et al.* [18] followed the toroidal analysis of Jacques et al. [22] for equal size particle and zero contact angle liquid. Toroidal approximation involves treating the meridional profile of the air-liquid interface as an arc of a circle. Simons *et al.*'s [18] equation can be used to calculate the volume of the liquid bridge with a gap between the particles. However, equation 19 is the non-dimensional form of Simons *et al.*'s [18] equation for the calculation of the volume of the liquid bridge when there is no gap between the particles.

$$V = \frac{v}{\pi R^3} = 2\left\{\begin{array}{l} (1-\cos\theta)\times\left[\sin\theta + (1-\cos\theta)\tan\theta\right]^2 \\ -(1-\cos\theta)^2\times\left[\sin\theta + (1-\cos\theta)\tan\theta\right]\tan\theta \\ -\dfrac{\left[\sin\theta + (1-\cos\theta)\tan\theta\right](1-\cos\theta)^2\left(\frac{\pi}{2}-\theta\right)}{\cos\theta^2} \\ +\dfrac{(1-\cos\theta)^3}{\cos\theta^2} - (1-\cos\theta)^2 \end{array}\right\} \tag{19}$$

Substituting for the values of $\cos\theta$, $\sin\theta$ and $\tan\theta$ from equations 9, 13 and 15 respectively into equation 19 and using Maple computer software to further simplify gives

$$V = \frac{\xi^4\left\{4 - 4\xi - 2\pi\xi + \pi\xi^2 + \theta\left(2\xi^2 - 4\xi\right)\right\}}{8(1-\xi)^3} \tag{20}$$

At zero gap between the particles, equation 20 is exactly the same as equation 18 which has been derived from Fisher's model [16]. This shows that Simons *et al.*'s model is an extension to Fisher's model for calculating the volume of the liquid bridge formed between two particles kept at a distance from each other. If the maximum value of ξ is known, equation 20 can be used to calculate the dimensionless maximum liquid holdup volume (V).

Smith *et al.*'s Equation: For calculating the volume of the liquid bridge formed between equal size vertically contacted spheres and with the assumptions of zero gravity and for a liquid with zero contact angle, Smith *et al.* [19] has given the following equations:

$$V = 2\left(\frac{\varsigma}{1+\varsigma}\right) f(\xi,\varsigma) \tag{21}$$

Where

$$\varsigma = \frac{r_1}{R} \tag{22}$$

$$(\xi+\varsigma)^2 - \varsigma(\xi+\varsigma)\left\{1-\frac{1}{(1+\varsigma)^2}\right\}^{\frac{1}{2}} - \varsigma(\xi+\varsigma)(1+\varsigma)\times\sin^{-1}\left(\frac{1}{(1+\varsigma)}\right) + \varsigma^2 - \left(\frac{\varsigma}{(1+\varsigma)}\right) \tag{23}$$

Equation 21 is the function of r_2 and r_1 and avoided use of filling angle (θ). However, practically ζ can not be measured easily.

Substituting for the values of (equation 6) and f (ξ,ζ) (equation 21) in equation 23 and using maple software to simplify yields the following:

$$V = 0.25\xi^4 \left\{\begin{matrix} -8+24\xi^2-48\xi^2+56\xi^3-30\xi^4+6\xi^5 \\ -\theta\left(-8\xi+20\xi^2-24\xi^3+16\xi^4-6\xi^5+\xi^6\right)\end{matrix}\right\}\left(\begin{matrix}-4+20\xi-44\xi^2+56\xi^3 \\ -45\xi^4+23\xi^5-7\xi^6+\xi^7\end{matrix}\right)^{-1} \tag{24}$$

Knowing ξ, the above equation 24 can be used to calculate the maximum liquid holdup volume (V).

Theoretical Drawback
This Model Has a Number of Drawbacks:

- With the presence of gravitational force, the air-liquid interface can not be exactly circular cross sectional shape.
- The air-liquid interface may not be balanced about both X and Y axis.
- The force balanced is considered to be only half of the liquid bridge and avoided the effect of the other half.

CONCLUSION

The results simplified by Maple for Smith *et al.* looks different from the Fisher and Simons *et al.*'s simplified result. However, if ξ is plotted against V for the above models, all the three models will show complete agreement with one another, irrespective of their orientation.

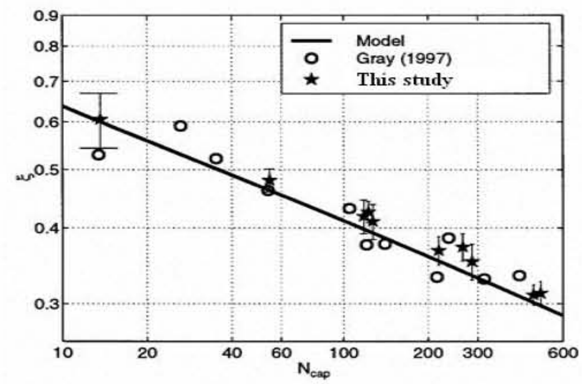

Fig. 6: Dimensionless liquid bridge neck diameter as a function of N_{cap}

This shows that Smith *et al.*'s model could be an extension to Fisher's model which avoided using the filling angle for calculating the volume of the liquid bridge. Nonetheless, since Fisher, Smith *et al.* and Simons *et al.* all assumed zero body force, it was predictable that they could achieve the same results, irrespective of the particle orientation. Using equation 10, N_{cap} can be calculated for a range of values of ξ and plotted against each other. Then, while knowing the value for N_{cap} for a particular system, the maximum liquid bridge diameter (ξ) can be found from Figure 5. If ξ is known, it is possible to calculate the volume of the liquid bridge from any of the equations 18, 20 and 24. Now, the liquid bridge neck diameters and its volume (V) can be compared with the experimental values of this study, Gray, Turner and the model. The results show that the values of ξ predicted conformity very well with experimental results (Figure 6). Then regarding the volume (V), there is an agreement between the model and the experimental results of this study and Gray. However, the model underestimates the Turner's results.

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Abbreviations:

b	liquid bridge neck diameter
d	diameter of the particle
F_c	hydrostatic force
F_g	body force

F_s surface tension force
g gravity
H as shown in Figure (3)
N_{cap} $\rho g d^2/\sigma$
R radius of the particle
r_1 Vapor-liquid interface curvature
r_2 liquid bridge neck radius
v volume of the liquid holdup
V $v/\pi R^3$
ρ density
σ surface tension
ξ filling angle r_2/R or b/d
ζ r_1/R
θ filling angle

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