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Comparison of A New Modified McDougall-Wotherspoon Method with "Modified Cauchy Methods Using Padé Approximation" to Solvenonlinear Equations

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Abstract: The roots of nonlinear algebraic equations is an important problem in Applied science and engineering. Many mathematical models in physics, engineering and applied science, are applied with nonlinear equations. The efficient methods to find the roots of nonlinear equations has been developed in recent years, NatasaGlisovicet al developed the method of T J. McDougall and J. Wotherspoonwhich have derived a multistep iterative method with memory as a new modificationof the classical Newton's method, Nasr Al Din IDE give a new modified of T J. McDougall and J. Wotherspoonmethod by using Geometry Mean Concept. In this paper we give a comparison of a new modified of Nasr Al Din IDE of "the modified of T J. McDougall and J. Wotherspoon" with the optimal forth-order family of modified Cauchy methods given by Tianbao L. *et al*. Finally we verified on a number of examples and numerical results obtained the efficiency of this comparison which show us the efficiency of the new modified method of Nasr Al Din IDE of "the modified of T J. McDougall and J. Wotherspoon method.

Key words: Nonlinear equations · Newton's Method. TJ. McDougall and J. Wotherspoon method · Geometric Mean

INTRODUCTION

One of the most important problem in scientific and engineering applications is solving nonlinear equations (1). There are several well-known methods for solving nonlinearalgebraic equations of the form:

$$f(x)=0 \tag{1}$$

where f denote a continuously differentiable function on [a, b]$\subset R$ and has at least one root α, in [a, b] Such as Newton's Method, Bisection method, Regula Falsi method, NonlinearRegression Method and several another methods see forexample [1-12]. In this paper we will compare the new modified of T J. McDougall and J. Wotherspoonmethod by using Geometry Mean Concept given by Nasr Al Din IDE [13] with The optimal family of Modified "Cauchy methods using Padé Approximation" for Solving Nonlinear Equations (1), studied by Tianbao L. *et al.*, 2019, [14].

The New Modified Methodof T J. Mcdougall and J. Wotherspoon by Using Geometry Mean Concept Given by Nasr Al Din IDE [13]: Consider a nonlinear equation (1), consider the following iterative method proposed by T J. McDougall and J. Wotherspoon Which have derived a multistep iterative method with memory [2],

$$y_0 = x_0 \tag{2}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'\left(\frac{1}{2}(y_0 + x_0)\right)} = x_0 - \frac{f(x_0)}{f'(x_0)} \tag{3}$$

followed by (for n$\geq$1)

$$y_n = x_n - \frac{f(x_n)}{f'(\frac{1}{2}(x_{n-1} + y_{n-1}))} \tag{4}$$

$$X_{n+1} = x_n = \frac{f(x_n)}{f'(\frac{1}{2}(x_n + y_n))} \tag{5}$$

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Glisovic *et al*. [1], replace in this method of T J. McDougall and J. Wotherspoon, harmonic mean by arithmetic mean of xn and yn, then new iterative scheme obtained for n $\geq$ 1, preserving y_0 and x_1.

$$y_n = x_n - \frac{f(x_n)}{f'\left(\frac{2x_{n-1}.y_{n-1}}{x_{n-1}+y_{n-1}}\right)} \quad (6)$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(\frac{2x_n.y_n}{x_n+y_n})} \quad (7)$$

Ide. N, [13] replace arithmetic mean of x_n and y_n, by geometric mean, then heobtain's the following New scheme,

$$y_0 = x_0 \quad (8)$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(\sqrt{x_0.y_0})} = x_0 - \frac{f(x_0)}{f'(x_0)} \quad (9)$$

followed by (for n $\geq$ 1)

$$y_n = x_n - \frac{f(x_n)}{f'\left(\sqrt{x_{n-1}.y_{n-1}}\right)} \quad (10)$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'\left(\sqrt{x_n.y_n}\right)} \quad (11)$$

Algorithm of the Method: Give x_0 initial value (number real), give the tolerance number ε (for stopping) and take $y_0 = x_0$.

Calculus of $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

Calculus (for n $\geq$ 1): $y_n = x_n - \frac{f(x_n)}{f'\left(\sqrt{x_{n-1}.y_{n-1}}\right)}$ and $x_{n+1} = x_n - \frac{f(x_n)}{f'\left(\sqrt{x_n.y_n}\right)}$

Calculus of stopping condition: if $\left|\frac{x_{n+1}-x_n}{x_{n+1}}\right| \leq \varepsilon$ then stop, else,

Take n = n + 1 and return to (3).

The Optimal Family of Modified "Cauchy Methods Using Padé Approximation" of Tianbao L. *et al*.: In this method, Tianbao L. *et al*. derive and analyze a new one-parameter family of modifed Cauchy method free from second derivative for obtaining simple roots of nonlinear equations by using Padéapproximant. The convergence analysis of the family is also considered and the methods have convergence order three.

The classical Cauchy's method [14] is expressed as:

$$x_{n+1} = x_n - \frac{2}{1+\sqrt{1-2L_f(x_n)}}\frac{f(x_n)}{f'(x_n)} \quad (12)$$

where,

$$L_f(x_n) = \frac{f''(x_n)f(x_n)}{f'^2(x_n)} \quad (13)$$

which is a well-known third-order method, this method depends on the second derivatives in computing process. Tianbao L. *et al*. [14] improve that, the family defined by (3) obtain third and optimal fourth order family of second-derivative-free variants of Cauchy's methods by using Padé approximation.

Development of the Third Order Method: In order to avoid the evaluation of the second derivatives f''(x_n) of Cauchy's method (3), he approximant it by the derivative y''(x_n) of the Padé function of second degree:

$$y(t) = \frac{a_1 + a_2(t-w_n) + a_2(t-w_n)^2}{f1+'^2(x_n)} \text{c} \quad (14)$$

where a_1, a_2, a_3 and a_4 are real parameters. Tianbao L. *et al*. [14] gave a new family of iterative methods , with order of convergence three, as follows:

$$x_{n+1} = x_n - \left(\sum_{k\geq 0}^{m}\binom{\frac{1}{2}}{k+1}(-1)^k 2^{k+1} L_{f,\mu}(x_n,w_n)^k\right)\frac{f(x_n)}{f'(x_n)} \quad (15)$$

where $\mu \in$R, and

$$L_{f,\mu}(x_n,w_n) = \frac{2f(w_n)}{\left[f'^2(x_n)f(x_n) + \mu f^2(x_n) - f'^2(x_n)f(w_n)\right]} \quad (16)$$

Tianbao L. *et al*. gave some special cases as follows:

- For μ = 0: $L_{f,0}(x_n,w_n) = \frac{2f(w_n)}{[f(x_n)-f(w_n)]}$

$$x_{n+1} = x_n - \left(\sum_{k \geq 0}^{m} \binom{\frac{1}{2}}{k+1} (-1)^k 2^{k+1} L_{f,0}(x_n, w_n)^k \right) \frac{f(x_n)}{f'(x_n)} \quad (17)$$

where m > 0. For m = 2, he gave a third-order method (LM1):

$$x_{n+1} = x_n - \left(1 + \frac{1}{2} L_{f,0}(x_n, w_n) + \frac{1}{2} L_{f,0}(x_n, w_n)^2 \right) \frac{f(x_n)}{f'(x_n)} \quad (18)$$

For m = 3, he gave a third-order method (LM2):

$$x_{n+1} = x_n - \left(1 + \frac{1}{2} L_{f,0}(x_n, w_n) + \frac{1}{2} L_{f,0}(x_n, w_n)^2 + \frac{5}{8} L_{f,0}(x_n, w_n)^3 \right) \frac{f(x_n)}{f'(x_n)} \quad (19)$$

- For μ = 1: $L_{f,1}(x_n, w_n) = \dfrac{2 f'^2(x_n) f(w_n)}{[f'^2(x_n) f(x_n) + f^2(x_n) - f'^2(x_n) f(w_n)]}$ (20)

For m = 2, he gave a third-order method (LM3):

$$x_{n+1} = x_n - \left(1 + \frac{1}{2} L_{f,0}(x_n, w_n) + \frac{1}{2} L_{f,0}(x_n, w_n)^2 \right) \frac{f(x_n)}{f'(x_n)} \quad (21)$$

- For $\mu = -\frac{1}{2}$: $L_{f,-\frac{1}{2}}(x_n, w_n) = \dfrac{2 f'^2(x_n) f(w_n)}{[f'^2(x_n) f(x_n) - \frac{1}{2} f^2(x_n) - f'^2(x_n) f(w_n)]}$ (22)

For m = 2, he gave a third-order method (LM4):

$$x_{n+1} = x_n - \left(1 + \frac{1}{2} L_{f,-\frac{1}{2}}(x_n, w_n) + \frac{1}{2} L_{f,-\frac{1}{2}}(x_n, w_n)^2 \right) \frac{f(x_n)}{f'(x_n)} \quad (23)$$

- For μ =-1, m = 2, he gave a third-order method (LM5):

$$x_{n+1} = x_n - \left(1 + \frac{1}{2} L_{f,-1}(x_n, w_n) + \frac{1}{2} L_{f,-1}(x_n, w_n)^2 \right) \frac{f(x_n)}{f'(x_n)} \quad (24)$$

- For $\mu = \frac{1}{2}$, m = 1, he gave a third-order method:

$$x_{n+1} = x_n - \frac{2}{2 - L_{f,\frac{1}{2}}(x_n, w_n)} \frac{f(x_n)}{f'(x_n)} \quad (25)$$

For m = 2, he gave a third-order method (LM6):

$$x_{n+1} = x_n - \frac{4}{4 - 2L_{f,\frac{1}{2}}(x_n, w_n) - L_{f,\frac{1}{2}}(w_n, w_n)^2} \frac{f(x_n)}{f'(x_n)} \quad (26)$$

For m = 3, he gave a third-order method:

$$x_{n+1} = x_n - \frac{4}{4 - 2L_{f,\frac{1}{2}}(x_n, w_n) - L_{f,\frac{1}{2}}(w_n, w_n)^2 - L_{f,\frac{1}{2}}(x_n, w_n)^3} \frac{f(x_n)}{f'(x_n)} \quad (27)$$

Examples: In this section, we shall check the effectiveness of the method(8)-(11). First numerical comparison for the following test examples taken in [14], we compare themethod (NMMW), (8)-(11) [14] with the Newton's method (NM), the Weerakoon-Fernando method (wf) [15], method of Potra and Ptak (PP) [16], Chebyshev's method (CHM) [17–18], Halley's method (HM) [17] and the methods (LM1), (LM2), (LM3), (LM4), (LM5) and (LM6). given in Table 2, the number of iterations (IT), the absolute residual error of the corresponding function value $|f(x_n)|$,, We used theMapleProgram to approximate the exact root and we take the test functions given in Table 1 [14, 19].

Table 1: The test functions, the initial values and the computed approximate zero x^*

$f_1(x) = x^3+4x^2-10$,	x_0=1 and,	x_0=2,	x^* = 1.3652300134140969
$f_2(x) = x^2-e^x-3x+2$,	x_0=0 and,	x_0=0.5,	x^* = 0.25753028543986076
$f_3(x) = \sin(x)e^x+ln(1+x^2)$,	x_0=1 and,	x_0=0.5,	x^* = 0
$f_4(x) = (x-1)^3-1$,	x_0=2.5 and,	x_0=3.5,	x^* = 2
$f_5(x) = cos(x)-x$,	x_0=0 and,	x_0=1,	x^* = 0.73908513321516067
$f_6(x) = sin^2(x)-x^2+1$,	x_0=1 and,	x_0=2.5,	x^* = 1.4044916482153411
$f_7(x) = x^*$ -1,	x_0=3.25 and	x_0=3.45,	x^* = 3.

Table 2: Comparison of various third-order methods, Newton method and the New Modified McDougall-Wotherspoon method (NMMW)

$f_1(x)$: x_0=1

Method	Number of iteration(n)	$\|f(x_n)\|$
NM	5	2.126987475037367e-011
WF	3	2.284722713019605e-006
PP	4	1.558753126573720e-013
CHM	4	1.643130076445232e-014
HM	3	3.698649917449615e-007
LM_1	3	7.656778435505274e-006
LM_2	3	6.519974116159233e-009
LM_3	3	6.545952378145259e-006
LM_4	4	2.220446049250313e-016
LM_5	4	2.220446049250313e-016
LM_6	3	1.582211815787105e-006
NMMW	2	1.442789334888179e-013
	3	9.713322670685396e-071

$f_1(x)$: x_0=2

Method	Number of iteration(n)	$\|f(x_n)\|$
NM	5	5.020497351182485e-010
WF	4	4.440892098500626e-016
PP	4	7.949196856316121e-014
CHM	4	2.065014825802791e-014
HM	3	3.107350415199051e-006
LM_1	3	1.870204660026076e-007
LM_2	4	2.220446049250313e-016
LM_3	3	3.063923381674272e-008
LM_4	3	3.636202978718472e-007
LM_5	4	6.449765455052159e-007
LM_6	4	2.220446049250313e-016
NMMW	2	4.613795697126499e-013
	3	3.248220190184235e-068

$f_2(x)$: x_0=0

Method	Number of iteration(n)	$\|f(x_n)\|$
NM	4	2.665312415217613e-012
WF	3	7.801814749797131e-012
PP	3	1.219191414492116e-012
CHM	3	8.906764215055318e-013
HM	3	7.374600929921371e-012
LM_1	3	1.014743844507393e-013
LM_2	3	1.497690860219336e-013
LM_3	3	3.035904860837491e-013
LM_4	3	2.620348382720295e-012
LM_5	3	1.656591530618812e-011
LM_6	2	1.015871229748111e-005
NMMW	2	6.826962274600170e-007
	3	1.109727185565823e-037

$f_2(x)$: x_0=0.5

Method	Number of iteration(n)	$\|f(x_n)\|$
NM	4	1.791899961745003e-013
WF	3	6.424749621203318e-012
PP	3	4.607425552194400e-014
CHM	3	3.087480271446452e-011
HM	3	4.208039472430869e-011
LM_1	3	1.054711873393899e-015
LM_2	3	7.216449660063518e-016
LM_3	3	1.497135748707024e-013
LM_4	3	9.942047185518277e-014
LM_5	3	7.234768339969833e-013
LM_6	3	1.887379141862766e-015
NMMW	2	1.065786137660662e-040

$f_3(x)$: x_0=1

Method	Number of iteration(n)	$\|f(x_n)\|$
NM	7	1.085848323840232e-012
WF	4	4.330310691887267e-006
PP	5	8.806888499109001e-012
CHM	5	2.520356663650445e-011
HM	5	8.459855063117184e-015
LM_1	4	3.479185746468363e-009
LM_2	4	1.804501237019987e-013
LM_3	4	3.732361177500448e-009
LM_4	4	6.861191797005728e-010
LM_5	4	3.650246134545045e-015
LM_6	4	2.539428973634579e-010
NMMW	3	0.5644406618391220e020

$f_3(x)$: x_0=0.5

Method	Number of iteration(n)	$\|f(x_n)\|$
NM	6	1.402992074360412e-010
WF	4	4.200981459101664e-009
PP	4	2.767209186089879e-007
CHM	4	3.059650585008672e-007
HM	3	5.518067735074518e-009
LM_1	4	4.539201791677570e-014
LM_2	4	5.535242064507416e-014
LM_3	4	2.221451036901571e-013
LM_4	3	2.852561908633870e-007
LM_5	4	6.743878120329082e-014
LM_6	3	1.053838990356347e-011
NMMW	3	2.153141690902633e-025

$f_4(x)$: x_0=2.5

Method	Number of iteration(n)	$\|f(x_n)\|$
NM	6	1.154631945610163e-014
WF	4	7.314593375440381e-012
PP	4	4.221685223626537e-010
CHM	4	9.853584614916144e-011
HM	4	4.662936703425658e-014
LM_1	3	1.544537542308433e-008
LM_2	4	2.244870955792067e-013
LM_3	3	6.254473188249676e-007
LM_4	3	1.067152234579538e-006
LM_5	4	4.440892098500626e-016
LM_6	4	2.042810365310288e-014
NMMW	2	3.696590601169957e-010
	3	1.70432239623964e-049

$f_4(x)$: x_0=3.5

Method	Number of iteration(n)	$\|f(x_n)\|$
NM	7	2.877564853065451e-011
WF	5	6.550315845288424e-013
PP	5	4.512221707386743e-010
CHM	5	4.188738245147761e-011
HM	4	4.485352507632712e-006
LM_1	4	1.079692646399622e-008
LM_2	4	7.838174553853605e-013
LM_3	4	3.379705404427114e-010
LM_4	4	1.283696526854783e-008
LM_5	4	8.547096808086963e-009
LM_6	4	8.725183908708800e-009
NMMW	3	4.255829500558037e-018

$f_5(x)$: x_0=0

Method	Number of iteration(n)	$\|f(x_n)\|$
NM	5	1.701233598438989e-010
WF	3	7.792236328407753e-007
PP	4	1.500558566291943e-010
CHM	4	5.327979279989847e-009
HM	4	1.121325254871408e-014
LM_1	4	3.819167204710539e-014
LM_2	3	8.247395144600489e-008
LM_3	4	1.818811767861917e-011
LM_4	4	8.344436253082677e-013
LM_5	4	8.471505719143124e-010
LM_6	4	2.348121697082206e-013
NMMW	2	4.895792518924954e-012
	3	1.704372784692600e-060

$f_5(x)$: x_0=1

Method	Number of iteration(n)	$\|f(x_n)\|$
NM	4	1.701233598438989e-010
WF	2	2.674277017133964e-005
PP	3	9.809075773858922e-011
CHM	3	1.600380383770528e-009
HM	3	6.624212289807474e-010
LM_1	3	2.252753539266905e-012
LM_2	3	4.671929509925121e-012
LM_3	3	2.668459120336308e-009
LM_4	3	1.749711486809247e-012
LM_5	3	1.375262126401822e-010
LM_6	3	3.148793448204401e-010
NMMW	2	6.516741213543832e-031

$f_6(x)$: x_0=1

Method	Number of iteration(n)	$\|f(x_n)\|$
NM	6	3.059774655866931e-013
WF	4	1.793023507445923e-010
PP	16	1.531728257564424e-007
CHM	5	6.883094094689568e-010
HM	4	2.686739719592879e-013
LM_1	4	1.042735342515755e-008
LM_2	4	7.038286398142191e-009
LM_3	4	3.420013050536852e-008
LM_4	4	1.918714076509787e-010
LM_5	4	1.002852445530778e-007
LM_6	4	6.483733550055604e-010
NMMW	3	9.118511562469905e-023

$f_6(x)$: x_0=2.5

Method	Number of iteration(n)	$|f(x_n)|$
NM	6	1.404654170755748e-012
WF	4	4.229505634611996e-012
PP	4	1.030850205196998e-008
CHM	4	1.475204565171140e-007
HM	4	9.462626682221753e-009
LM_1	4	1.265654248072679e-014
LM_2	3	1.176158348492606e-008
LM_3	4	4.662936703425658e-015
LM_4	4	1.501021529293212e-013
LM_5	4	8.837375276016246e-014
LM_6	4	2.375877272697835e-014
NMMW	2	8.699507369284137e-009
	3	9.891874287586063e-043

$f_7(x)$: x_0=3.25

Method	Number of iteration(n)	$|f(x_n)|$
NM	8	9.720393379097914e-010
WF	6	1.691979889528739e-013
PP	6	1.131490456884876e-010
CHM	6	2.398081733190338e-014
HM	5	3.082423205569285e-012
LM_1	4	1.781058993621798e-007
LM_2	4	5.758273724509877e-008
LM_3	4	2.094845084066321e-007
LM_4	4	1.635068742622536e-007
LM_5	4	1.496285984003976e-007
LM_6	-	-
NMMW	2	5.526763232226664e-007
	3	6.756176357793037e-033

$f_7(x)$: x_0=3.45

Method	Number of iteration(n)	$|f(x_n)|$
NM	11	4.008793297316515e-011
WF	8	7.105427357601002e-015
PP	8	2.160227552394645e-010
CHM	7	2.160227552394645e-010
HM	6	1.694565332499565e-008
LM_1	6	3.221867217462204e-012
LM_2	5	7.682743330406083e-014
LM_3	6	3.313793683901167e-012
LM_4	6	3.313793683901167e-012
LM_5	6	3.125055769714891e-012
LM_6	-	-
NMMW	5	8.185106391098922e-022

CONCLUSIONS

In this work, we have compare the new modified method of T J. McDougall and J. Wotherspoon by using Geometry Mean Concept given by Nasr Al Din IDE (NMMW), with The optimal family of Modified "Cauchy methods using Padé Approximation. The efficiency of the (NMMW) method is shown for some test problems, comparison is given with some methods such as with the Newton's method (NM), the Weerakoon-Fernando method (wf), method of Potra and Ptak (PP), Chebyshev's method (CHM), Halley's method (HM) and the methods (LM1), (LM2), (LM3), (LM4), (LM5) and (LM6) given in Table 2, the number of iterations (IT), the absolute residual error of the corresponding function value $|f(x_n)|$, it is shown that (NMMW) method has lowest number of iteration.

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