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Iterative Method by Using Tchybcheve Integral for Solving Nonlinear Algebraic Equations

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Abstract: This paper presents an iterative numerical method to solve non linear algebraic equations of the form f(x)=0. This method uses the Newton theorem by approximating the integral by the Tchybcheve method, this method likes the method of S. Weeraksoon and Fernand [1] gives also by H.H.H. Homeir [2], also likes widely methods as the method of Xing-Guo Luo [3] and Nasr Al-Din Ide [4], [6], [8-18]. By considering two examples we confirm that this new iterative method do's not converge much more quickly than Newton, Hybrid iteration method [3], new Hybrid iteration method [4] and a new modified Newton methods [6].

Key words: Algebraic equations · Newton method · Non linear equation · Iteration method

INTRODUCTION

In [S. Weeraksoon and T.G.I. Fernando, A variant of Newton's method with accelerated third-order convergence], given also by [H.H.H. Homeier, On Newton-type methods with cubic convergence] a new iteration method for solving algebraic equations has been proposed, by the Newton theorem:

$$f(x) = f(x_n) + \int_{x_n}^{x} f'(t).dt \qquad (1)$$

by approximating the integral by the rectangular, trapezoidal rules.

Frontini and Sormani [5] also generalized the approach of Weerakoon and Fernando by using general interpolator quadrate rules of order one at least.

In this paper we proposed a new iteration method for solving algebraic equations by using the approximating integral of the Newton theorem (1) by the Tchybcheve method.

The principle of the new iteration method.

We consider the non linear algebraic equation (2)

$$f(x) = 0 \qquad (2)$$

then, let us use the Newton theorem (1), by approximating the integral by Tchybcheve rule according to

$$\int_{x_n}^{x} f'(t)dt \approx \frac{x-x_n}{2}\left[f'\left(\frac{x_n+x}{2}+\frac{x_n-x}{2}x_n\right)+f'\left(\frac{x_n+x}{2}+\frac{x_n-x}{2}x\right)\right] \qquad (3)$$

leads to the following:

$$f(x) = f(x_n) + \frac{x-x_n}{4}\left[f'(x_n + x + x_n^2 - x.x_n) + f'(x_n + x + x_n^2.x - x^2)\right] \qquad (4)$$

by using f(x) = 0, also the substitution of x by x_{n+1} and by considering;

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = y_n \qquad (5)$$

in the right-side of the following result,

$$x_{n+1} = x_n - \frac{4.f(x_n)}{f'(x_n+y_n+x_n^2-x_n.y_n)+f'(x_n+y_n+x_n^2.y_n-y_n^2)} \qquad (6)$$

Analysis of Convergence: Theorem1. If n > 0, then the formula defined in equation (6) converges to the simple zero of *f* defined by equation (2) which is called the root of the equation.

Proof: To prove the result it suffices to prove, as, $n \to \infty$, $f(x_n) \to 0$. Since, it is a iterative process, so $n \to \infty$ meant $x_{n+1} \approx x_n$, but as we consider (Newton method) $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = y_n$, hence, as $n \to \infty$, we have $f(x_n) \to 0$ that proves the required result.

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Numerical Results and Discussion: The following examples illustrate the result obtained by suggested method defined by (6) and verify the validity of the new iteration method to solve non-linear equations. Some other proposed methods are presented to compare with the suggested method defined by equation (6). The comparison appear that this method dos not converge much more quickly than the Newton, Hybrid iteration method [3], new Hybrid iteration method [4] and a new modified Newton methods [6].

Some Examples:

Examples (1)

Consider the following equation [1]:

$$f(x) = x^3 - e^{-x} = 0. \quad (7)$$

To find a positive root near x = 1, we start with $x_0 = 1$ and obtain x = 0.7728.831 by 15 iterations.

The results obtained by Newton iteration, hybrid iteration [3], New Hybrid iteration [4] and present iteration are shown in Table 1, Table 2 Table 3 and Table 4.

Examples (2):

Consider the following equation [6]:

$$f(x) = x^3 + 4x^2 - 10 = 0. \quad (3.3)$$

The results obtained by Newton iteration [7], modified Newton method and present iteration are shown in Table 5, Table 6 and Table 7.

Table 1: Newton iteration for solving $f(x) = x^3 - e^{-x} = 0$

n	x_n	$\|f(x_n)\|$
1	1	0.63212055882855767000
2	0.81230903009738120	0.09216677153431299100(9.2E-2)
3	0.77427654898550025	0.00314482497861333540(3.1E-3)
4	0.77288475620962160	0.00000405008554745892(4.1E-6)
5	0.77288295915220184	0.00000000000674254836(6.7E-12)
6	0.77288295914921012	0.00000000000000006456(6.5E-17)

Table 2: Hybrid iteration [3], for solving $f(x) = x^3 - e^{-x} = 0$

n	x_n	$\|f(x_n)\|$
1	1	0.63212055882855767000
2	0.81230903009738120	0.09216677153431299100(9.2E-2)
3	0.77597366111597321	0.00698556573567983560(7.0E-3)
4	0.77290581952315329	0.00005152207246609381(5.2E-5)
5	0.77288297294415598	0.00000003109000577256(3.1E-8)
6	0.77288295914921590	0.00000000000001299950(1.3E-14)
7	0.77288295914921012	0.00000000000000006456(6.5E-17)

Table 3: New Hybrid iteration for solving $f(x) = x^3 - e^{-x} = 0$

n	x_n	$\|f(x_n)\|$
1	1	0.63212055882855767000
2	0.801305391412732989	0.0657676468308537 (6.5E-2)
3	0.773375282149371597	0.0011100665084369(1.1E-3)
4	0.772883108807313135	3.37288157491135E-7
5	0.772882959149223945	3.11745529564533E-14
6	0.772882959149210113	1.62630325872826E-19
7	0.772882959149210113	1.62630325872826E-19

Table 4: Present iteration for solving $f(x) = x^3 - e^{-x} = 0$

n	x_n	$\|f(x_n)\|$
1	1	0.63212055882855767000
2	0.894012909071994163	0.305536806008939222
3	0.827563679035886136	0.129653683284384780
4	0.794290185343672894	0.049213326778535763
5	0.780562850085326313	0.017431985425972664
6	0.775530143320345827	0.005980673893888940
7	0.773781553293144707	0.002026870128975578
8	0.773186344944415558	0.000683940151777998
9	0.772985199718703875	0.000230443886167608
10	0.772917392491212175	0.000077605736734849
11	0.772894553416456339	0.000026130563055312
12	0.772886862849929352	0.000008797897176088
13	0.772884273463741569	0.000002962106267645
14	0.772883401654634060	0.000000997285657105
15	0.772883108132207173	0.000000335766654325

Table 5: Newton iteration, for solving $f(x) = x^3 + 4x^2 - 10 = 0$. ($x_0$=2):

n	x_n	$\|f(x_n)\|$
1	1.5	2.375000000000000000
2	1.333333333333333330	0.1343454814814815
3	1.365262014874626620	0.0005284611795157
4	1.365230013916146650	0.0000000082905488
5	1.365230013414096850	0.0000000000000000

Table 6: Modified Newton method [6], for solving $f(x) = x^3 + 4x^2 - 10 = 0$. ($x_0$=2):

n	x_n	$\|f(x_n)\|$
1	1.427841634738186460	1.0659129872799756
2	1.365742111478465810	0.0084586028868118
3	1.365230045574601760	0.0000005310792606
4	1.365230013414096970	0.0000000000000021
5	1.365230013414096850	0.0000000000000000

Table 7: Present iteration, for solving $f(x) = x^3 + 4x^2 - 10 = 0$.(x_0=2):

n	x_n	$\|f(x_n)\|$
1	1.363485533564984240	0.028782660352634281
2	1.365497079773124380	0.004410750805975232
3	1.365189680362105870	0.000666022613836371
4	1.365236117374271680	0.000100797431942446
5	1.365229089939084440	0.000015249704512425
6	1.365230153134349980	0.000002307256457038

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