

A New Aitken Type Method by Using Geometric Mean Concept

Nasr Al-Din Ide

Department of Mathematics, Faculty of Science, Aleppo University, Aleppo, Syria

Abstract: Finding the roots of nonlinear algebraic equations is an important problem in science and engineering. Many mathematical models in physics, engineering and applied science, are applied with nonlinear equations. later many methods developed for solving nonlinear equations. The efficient methods to find the roots of nonlinear equations has been developed in recent years [1-24], Pankaj. J, Kriti. S [1] developed the method of Aitken which converges better than the method of Aitken studied by Pavaloiu and Catina [1]. In this paper we develop these methods by using Geometric Mean instead of the arithmetic mean. We verified on a number of examples and numerical results obtained show that the present method convergences better than of the modification of Pankaj. J, Kriti. S.

Key words: Nonlinear equations • Newton's Method • Method of Pankaj. J • Kriti. S • Method of Pavaloiu and Catina • Geometric Mean • Arithmetic mean

INTRODUCTION

Solving nonlinear equations (1), is one of the most important problem in scientific and engineering applications. There are several well-known methods for solving nonlinear algebraic equations of the form:

$$f(x) = 0 \quad (1)$$

where f denote a continuously differentiable function on $[a, b]$ and has at least one root α , in $[a, b]$ Such as Newton's Method, Bisection method, Regula Falsi method, Nonlinear Regression Method and several another methods see for example [1-24]. Here we describe a new method by using geometric Mean of x_n and y_n instead of the harmonic mean used by Pankaj. J, Kriti. S [1]. We verified on a number of examples and numerical results obtained show that the present method convergences better than of the modification of Pankaj. J, Kriti. S.

The Present Method: Consider a nonlinear equation (1), consider the following iterative method proposed by Pankaj. J, Kriti. S Which have derived a multistep iterative methods with memory [1, 2],

$$y_0 = x_0 \quad (2)$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(\frac{1}{2}(y_0 + x_0))} = x_0 - \frac{f(x_0)}{f'(x_0)} \quad (3)$$

followed by(for n=1)

$$y_n^* = x_n - \frac{f(x_n)}{f'(\frac{1}{2}(x_{n-1} + y_{n-1}^*))} \quad (4)$$

$$y_n = x_n - \frac{f(x_n)}{f'(\frac{1}{2}(x_n + y_n^*))} \quad (5)$$

$$z_n^* = y_n - \frac{f(y_n)}{f'(\frac{1}{2}(y_{n-1} + z_{n-1}^*))} \quad (6)$$

$$z_n = y_n - \frac{f(y_n)}{f'(\frac{1}{2}(y_n + z_n^*))} \quad (7)$$

$$x_{n+1} = Z_n - \frac{(f(z_n))}{[y_n, z_n; f]} \quad (8)$$

where $[y_n, z_n; f]$ denotes the first order divided difference of f on x and y [8].

We replace in this method of Pankaj. J, Kriti, arithmetic mean by geometric mean of x_n and y_n , then we obtain the following New scheme,

$$y_0 = x_0$$

$$(9) \quad z_n^* = y_n - \frac{f(y_n)}{f'(\sqrt{y_{n-1}, z_n^*})}$$

$$x_1 = x_0 = \frac{f(x_0)}{f'(\sqrt{x_0, y_0})} = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$(10)$$

followed by (for $n=1$)

$$y_n^* = x_n - \frac{f(x_n)}{f'(\sqrt{x_{n-1}, y_{n-1}})}$$

$$(11)$$

$$Z_n = y_n - \frac{f(y_n)}{f'(\sqrt{y_n, z_n^*})}$$

$$y_n = x_n - \frac{f(x_n)}{f'(\sqrt{x_{n-1}, y_{n-1}^*})}$$

$$(12)$$

$$z_n^* = y_n - \frac{f(y_n)}{f'(\sqrt{y_{n-1}, x_{n-1}})}$$

$$(13)$$

$$z_n = y_n - \frac{f(y_n)}{f'(\sqrt{y_n, z_n^*})}$$

$$(14)$$

$$x_{n+1} = Z_n - \frac{f(z_n)}{[y_n, z_n; f]}$$

$$(15)$$

$$\left| \frac{x_{n+1} - x_n}{x_{n+1}} \right| \leq \text{then stop, else,}$$

- Take $n=n+1$ and return to (3).

Examples: In this section, we shall check the effectiveness of present method (9)-(15). Numerical comparison for the following test examples given in [1].

Example 1: $f_1(x) = \sin^2 x - x^2 + 1$, $x_0=1$ and compare present method (PM) with the method of Pankaj. J, Kriti (PK). S which implement the method of Wang [1, 9] of order 12, Table 1.

Example 2: $f_2(x) = x^2 \sin x - \cos x$, , $x_0 = 1.5$ and compare present method (PM) with the method of Pankaj. J, Kriti (PK). S [1], Table 2.

Example 3: $f_3(x) = x^3 + 4x^2 - 10$, , $x_0=3$ and compare present method (PM) with the method of Pankaj. J, Kriti (PK). S [1], Table 3.

Algorithm of the Present Method:

- Give x_0 initial value (number real), give the tolerance number \square (for stopping) and take $y_0=x_0$.
- Calculus of $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$
- Calculus (for $n \geq 1$):

$$y_n^* = x_n - \frac{f(x_n)}{f'(\sqrt{x_{n-1}, y_{n-1}})}$$

Table 1: Numerical results for $f_i(x)$

Method	Number of iteration (i)	x_i	$ f(x_i) $
PK	1	1.4072530700	0.0068699926
	2	1.4044916480	$2.4202861940.10^{-13}$
	3	1.4044916480	$3.3306690740.10^{-16}$
	4	1.4044916480	$3.3306690740.10^{-16}$
	5	1.4044916480	$3.3306690740.10^{-16}$
PM	1	1.392644905935806582	0.029136584941025196
	2	1.404491657276815449	$2.249486355227198073.10^{-8}$
	3	1.404491648215341226	$7.534572540496599332.10^{-33}$
	4	1.404491648215341226	$9.483303708084636955.10^{-133}$

Table 2: Numerical results for $f_2(x)$

Method	Number of iteration(i)	x_i	$ f(x_i) $
PK	1	0.9023469634	0.0192246037
	2	0.8951770213	0.0000777429
	3	0.8952060449	1.2703803564. 10^{-9}
	4	0.8952060453	1.1102230246. 10^{-16}
	5	0.8952060453	1.1102230246. 10^{-16}
	6	0.8952060453	1.1102230246. 10^{-16}
PM	1	0.897603798298897518	6.433594492423406785. 10^{-3}
	2	0.895206045380674949	9.527595757738839007. 10^{-12}
	3	0.895206045384231850	4.784424207605521936. 10^{-47}
	4	0.895206045384231850	2.751165877947192966. 10^{-86}

Table 3: Numerical results for $f_1(x)$

Method	Number of iteration(i)	x_i	$ f(x_i) $
PK	1	2.4788706270	0.0192246037
	2	2.0864736370	0.0000777429
	3	1.8034714550	8.8758448120
	4	1.6064524160	4.4685121220
	5	1.4449994650	1.3692865880
	6	1.5745885450	3.8212395230
	7	1.3921047750	0.4496602138
	8	1.3465447930	0.3057365137
	9	1.3621951380	0.0500415762
	10	1.3651247990	0.0017373659
	11	1.3652298790	2.2150939110. 10^{-6}
	12	1.3652300130	3.6077807410. 10^{-12}
PM	1	1.516432039132775308	4.461988357768831619. 10^{-4}
	2	1.365272620656446084	7.036050932439070998. 10^{-4}
	3	1.365230013414096846	4.464874878673165263. 10^{-18}
	4	1.365230013414096845	7.240383541666225342. 10^{-75}
	5	1.365230013414096846	5.741115264847827838. 10^{-85}
	6	1.365230013414096846	5.741115264847827838. 10^{-85}

CONCLUSIONS

In this work, we have proposed a new iterative method by using the geometric mean. The efficiency of this method is shown for some test problems, comparison of the obtained results given with the Aitken method given by Pankaj. J, Kriti(PK). It is shown that the present method(PM) is more efficient than the existing Aitken method given by Pankaj. J, Kriti(PK) and this method has lowest number of iteration and converges faster than this method.

REFERENCES

- Pankaj, J. and S. Kriti, 2018. Aitken type methods with high efficiency, Transactions of A. Razmadze Mathematical Institute, 172: 223-237.
- Pavaloiu, I. and E. Catina, 2013. On an Aitken-Newton type method, Numer. Algorithms, 62: 253-260.
- McDougall, T.J. and S.J. Wotherspoon, 2014. A simple modification of Newton's method to achieve convergence of order $1+\sqrt{2}$, Appl. Math. Lett. 29: 20-25.
- Weerakoon, S. and T.G.I. Fernando, 2000. A variant of Newton's method with accelerated third-order convergence, Appl. Math. Lett., 17: 87-93.
- Homeier, H.H.H., 2005. On Newton-type methods with cubic convergence, J. Comput. Appl. Math., 176: 425-432.
- Ozban, A.Y., 2004. Some new variants of Newton's method, Appl. Math. Lett., 17: 677-682.
- Wu, X., 1999. A significant improvement on Newton's iterative method, Appl. Math. Mech., 20: 924-927.
- Ide Nasr Al-Din, 2012. A nonstationary Halley's iteration method by using divided differences formula, Applied Mathematics, Scientific Research, USA, 3: 169-171.

9. Wang, P., 2011. A third-order family of Newton-like iteration methods for solving nonlinear equations, *J. Numer. Math. Stoch.*, 3: 13-19.
10. Frontini, M. and E. Sormani, 2003. Some variant of Newton's method with third-order convergence, *Appl. Math. Comput.*, 140: 419-426.
11. Jayakumar, J. and M. Kalyansundaram, 2015. Power means based modification of Newton's method for solving nonlinear equations with cubicconvergence, *Appl. Math. Comput.*, 6: 1-6.
12. Traub, J.F., 1964. Iterative Methods for the Solution of Equations. Prentice-Hall, Englewood Cliffs, NJ.
13. King, R., 1973. A family of fourth order methods for nonlinear equations. *SIAM J. Numer. Anal.*, 10: 876-879.
14. Hou, L. and X. Li, 2010. Twelfth-order method for nonlinear equation. *Int. J. Res. Rev. Appl. Sci.*, 3(1): 30-36.
15. Hu, Z., L. Guocai and L. Tian, 2011. An iterative method with ninth-order convergence for solving nonlinear equations. *Int. J. Contemp. Math. Sci.*, 6(1): 17-23.
16. Jutaporn, N., P. Bumrungsak and N. Apichat, 2015. A new method for finding Root of Nonlinear Equations by using Nonlinear Regression, *Asian Journal of Applied Sciences*, 03(6): 818-822.
17. Neamvonk, A., 2015. A Modified Regula Falsi Method for Solving Root of Nonlinear Equations, *Asian Journal of Applied Sciences*, 3(4): 776-778.
18. Ide Nasr Al-Din, 2008. A new Hybrid iteration method for solving algebraic equations, *Journal of Applied Mathematics and Computation*, Elsevier Editorial, 195: 772-774.
19. Ide Nasr Al-Din, 2008. On modified Newton methods for solving a nonlinear algebraic equations, *Journal of Applied Mathematics and Computation*, Elsevier Editorial., 198: 138-142.
20. Ide Nasr Al-Din, 2013. Some New Type Iterative Methods for Solving Nonlinear Algebraic Equation", *World Applied Sciences Journal*, 26(10): 1330-1334, © IDOSI Publications, Doi: 10.5829/idosi.wasj.2013.26.10.512.
21. Ide Nasr Al-Din, 2016. A New Algorithm for Solving Nonlinear Equations by Using Least Square Method, *Mathematics and Computer Science*, Science PG Publishing, 1(3): 44-47, Published: Sep. 18, (2016).
22. Ide Nasr Al-Din, 2016. Using Lagrange Interpolation for Solving Nonlinear Algebraic Equations, *International Journal of Theoretical and Applied Mathematics*, Science PG Publishing, 2(2): 165-169.
23. Ide Nasr Al-Din, 2018. Bisection Method by using Fuzzy Concept, *International Journal of Scientific and Innovative Mathematical Research*, (IJSIMR), 7(4): 8-11.
24. Ide Nasr Al-Din and Sundus Naji Al-Aziz, 2019. Using the Least Squares Method with Five Points to Solve Algebraic Equations Nonlinear, *International Journal of Scientific and Innovative Mathematical Research*, (IJSIMR), 7(5): 26-30.