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The Use of Peukert Equation in Lead Acid vs Lithium Ion Battery

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Abstract: The limiting capacity of a battery generally varies with respect to the discharge current. As a result, it is necessary to model the limiting capacity as a function of discharge current. In lead acid battery, the capacity significantly decreases as the discharge current increases and is often modelled using Peukert equation. In this paper, we will investigate the use of Peukert equation in modelling the capacity of lead acid and lithium ion battery. Experimental data will be used to investigate curve-fitting of the capacity of the two battery types. Analysis of the data shows Peukert equation provides a good fit at intermediate level current for lead acid battery. On the other hand, analysis shows Peukert equation is not suitable for modelling the capacity of lithium ion battery. Alternatively, a fifth order polynomial with the aid of piece cubic Hermite interpolation polynomial (PCHIP) has been proposed as an alternative method for modelling the battery capacity. Unlike Peukert equation, the polynomial equation with PCHIP can be used to model the capacity at low, intermediate and high discharge current for the two battery types.

Key words: Peukert Equation · Battery Capacity · Lead Acid Battery · Lithium Ion Battery

INTRODUCTION

In portable batteries powered applications (for examples mobile phones, laptops and medical devices) users often would like to know how much longer the devices can operate before they shut down [1]. Whereas in renewable energy applications ones would like to how long the battery can continue to provide a continuous predictable supply of electricity to the power grid given the known state of charge (SoC) of the battery [2]. For example, in wind and solar renewable applications batteries are used to store the intermittent energy been generated and supply to the power grid when needed [3, 4]. In such a system, an energy management system (EMS) is used to monitor and inform the system of how much longer the battery system can continue to supply electricity to the power grid. Now, the analytical calculation of *remaining battery usage time* (t_r) would be straight forward if the battery capacity remains constant regardless of discharge current and temperature. Unfortunately, this is not the case and as a result it is necessary to model the capacity as a function of discharge current and temperature [1].

The capacity of a battery can be expressed as a product of two functions Q(I) and f(T) as shown in equation (1). In the equation, the limiting capacity Q(I) is a function of discharge current and f(T) is the scaling factor, where T is the temperature of the battery. The equation of f(T) can be modelled using empirical equations [5, 6] such as a look-up table [7]. Given that f(T) is known, the focus then is to obtain an equation for Q(I).

$$C(I, T) = Q(I) * f(T) \quad (1)$$

Now, the relationship between the limiting capacity (Q) with respect to discharge rate (I) is typically modelled using the Peukert equation [8-11]. This is a nonlinear relationship, which states that

$$Q(I) = k*I^{1-\alpha} \quad (2)$$

- Q is the limiting capacity
- I is the constant discharge rate
- k and α are constants

Noticeably, if α=1 then the limiting battery capacity Q remains constant and independent of the discharge rate I. As a result, the value of α can be viewed as a measure of the level of dependency of the battery capacity on the discharge current I in the Peukert model.

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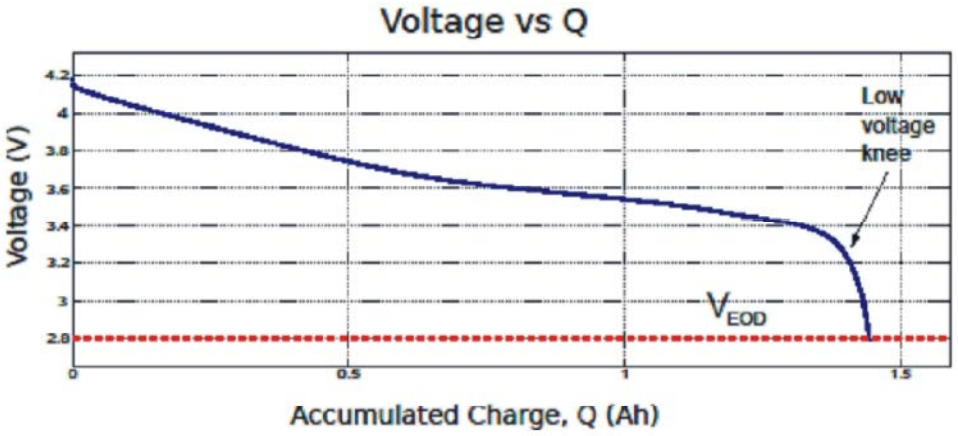

Fig. 1: A plot of how end of discharge voltage is defined

Table 1: Three LG lithium ion batteries.

Batteries Name	Chemical Notation	Label
Lithium cobalt oxide	$LiCoO_2$	ICR
Lithium manganese oxide	$LiMn_2O_4$	IMR
Lithium nickel manganese cobalt oxide	$LiNiMnCoO_2$	INR
Lithium nickel cobalt aluminium oxide	$LiNiCoAlO_2$	NCA

Table 2: Data of 126Ah Lead acid battery with V_{EOD} = 10.2V, obtained from Odyssey battery datasheet

Current (A)	Capacity (Ah)
6.3	126.0
11.4	114.0
13.8	110.4
20.6	103.0
25.0	100.0
32.0	96.0
45.2	90.4
80.2	80.2
100.9	75.7
137.9	69.0
185.8	61.3
227.1	56.8
296.4	50.4
443.8	37.0
671.6	22.4

Table 3: Experimental data of LG lithium ion 18650 single cell battery with V_{EOD} = 2.8 V

Current (A)	INR (Ah)	IMR (Ah)	ICR (Ah)	NCA (Ah)
0.2	2.865	2.409	2.450	3.128
0.5	2.824	2.388	2.443	3.113
1.0	2.806	2.371	2.419	3.081
2.0	2.793	2.349	2.387	3.046
3.0	2.763	2.334	2.372	3.035
5.0	2.743	2.314	2.370	3.027
7.0	2.728	2.297	2.371	2.970
10.0	2.702	2.276	2.365	2.896
15.0	2.650	2.227	2.327	
20.0	2.570	2.147	2.252	
30.0	1.963	1.870	1.987	

Let us now examine the definition of the battery capacity and how it is calculated. It is computed by considering a fully charged battery and calculating the total amount of energy being discharge (at a constant current) until the voltage falls below a prescribed end of discharge voltage (V_{EOD}). The selection of V_{EOD} is not standardized and it is different amongst manufacturers, even for the same battery type. The inconsistency of the value used for V_{EOD} influences the resulting battery capacity calculation. A typical method is to choose a V_{EOD} value after a sharp fall in the voltage, which is sometimes referred to as the low voltage knee as illustrated in Figure 1.

Let us now examine the application of the capacity equation C(I, T) and how it is used to calculate the *remaining battery usage time* (t_r). Given a known value for SoC, t_r can be calculated using equation 3.

$$t_r = (1 - SoC) * \frac{C(I,T)}{I} \quad (3)$$

Now, if the scaling factor f(T) is also known, what is left is the curve fitting of Q(I), which is needed to analytical calculate t_r. As a result, this paper will focus on Q(I) and investigate the use of Peukert equation for modelling the capacity of lead acid and lithium ion battery. Ideally, the equation for Q(I) should be applicable for a wide range of discharge current [9]. That is, for low, intermediate and high-level discharge current.

Experimental Data: Two sets of data will be examined, one is of lead acid battery obtained from Odyssey battery PC2250 datasheet and the other is of LG lithium ion batteries. For the LG batteries, four different types of lithium ion batteries will be analyzed, which can be seen in Table 1 (i.e. INR, IMR, ICR and NCA). Moreover, the LG batteries were discharged to the maximum allowed current specified in their respective datasheets.

The data shown in Table 2 and 3 are the results of discharging a fully charged battery at a specified current until its battery voltage falls below V_{EOD} as illustrated in Figure 1. It should also be noted that the experiments were conducted at room temperature.

Data Fitting Using Peukert Equations: To fit the data shown in Table 2 and 3 onto the Peukert equation, we need to calculate the optimal values of k and á that minimize the residual. That is,

Minimize $\|y_i - Q(x_i)\|_2$

where x_i is the discharge rate data point and y_i is the corresponding capacity data point.

To accomplish the minimizing above it is more convenience to write *equation 1* in the following form.

$$\ln(k) + \ln(I) * (1 - a) = \ln(Q)$$

$$\begin{bmatrix} 1 & \ln(x_1) \\ 1 & \ln(x_2) \\ \vdots & \vdots \\ 1 & \ln(x_i) \\ \vdots & \vdots \\ 1 & \ln(x_n) \end{bmatrix} \begin{bmatrix} \ln(k) \\ 1-\alpha \end{bmatrix} = \begin{bmatrix} \ln(y_1) \\ \ln(y_2) \\ \vdots \\ \ln(y_i) \\ \vdots \\ \ln(y_n) \end{bmatrix}$$

$$A \quad \begin{bmatrix} \ln(k) \\ 1-\alpha \end{bmatrix} = d$$

$$\begin{bmatrix} \ln(k) \\ 1-\alpha \end{bmatrix} = \left(A^T A\right)^{-1} A^T d \tag{4}$$

Let the result of computing the right hand side of *equation 4* be $[a, b]^T$.

Therefore,

$$k = \exp(a) \tag{5}$$
$$\alpha = 1 - b.$$

Data Fitting Using a Polynomial Equation: The data shown in Table 2 and 3 can be fitted with a polynomial function shown in *equation 6* rather than using the Peukert equation, but what order of polynomial should be used? This question will also be answered in this section.

Firstly, let us consider the notations used in fitting a set of data points (x_i, y_i):

- Let x_{max} be the maximum value of the set of x_i and $x_{min} = 0$.
- Let $\hat{x}_i = x_i/x_{max}$ and consider a new set of z_i with n number of elements.
- Let $x_s = 1.25*x_{max}$ and $h = (x_s - x_{min})/(n-1)$.
- $z_i = x_{min}/ x_{max} + (i-1)*h$ where $i \in N$, $i \in [1, n]$.
- Also, consider a set of f_i with n number of elements.

Secondly, consider how each value of f_i is calculated.

- f_i is calculated using *Piecewise Cubic Hermite Interpolating Polynomial* (PCHIP) [12] on the set of data points $(\hat{x}_i, y_i)$ over a specified set of values z_i.
- The PCHIP function is available in Matlab (i.e. f = pchip(x, y, z)).

$$Q(I) = a_0 + a_1*(I/x_{max}) + a_2*(I/x_{max})^2 + \ldots + a_m*(I/x_{max})^m \tag{6}$$

Incorporating the data points $(\hat{x}_i\, i, f_i)$ with *equation 6*, we can write a matrix system as shown in the equation below. Solution of *equation 7* yields the coefficients of the polynomial in *equation 6*.

$$\begin{bmatrix} 1 & \hat{x}_1 & \hat{x}_1^2 & \ldots & \hat{x}_1^m \\ 1 & \hat{x}_2 & \hat{x}_2^2 & \ldots & \hat{x}_2^m \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \hat{x}_i & \hat{x}_i^2 & \ldots & \hat{x}_i^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \hat{x}_n & \hat{x}_n^2 & \ldots & \hat{x}_n^m \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_i \\ \vdots \\ a_m \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_i \\ \vdots \\ f_m \end{bmatrix}$$

$$A \quad v = d$$

$$v = \left(A^T A\right)^{-1} A^T d \tag{7}$$

Consider now the analysis of polynomial curve fitting of the two battery types. A polynomial degree of 1 through to 15 with corresponding computation of the residual was performed for both batteries. The residual graph of the analysis done on lead acid and lithium ion battery can be seen in Figure 2 and 3 respectively. The two graphs show a decrease in the residual as complexity of the polynomial increases. In each of the graph a 5^{th} order polynomial is sufficient to model the battery capacity. Furthermore, a higher order polynomial can be used but the disadvantages are over-fitting the data and increasing the complexity of the equation, which is undesirable.

Lead ACID Battery: Figure 4 shows a curve fitting comparison of a 5^{th} order polynomial verses Peukert equation for the capacity of lead acid battery. On the horizontal axis is the C rate (unit of current divided by the battery rated capacity) and on the vertical axis is Q(I) divided by the battery rated capacity. The plot shows the residual of the 5^{th} order polynomial is less than half that of the Peukert equation. What this shows is that the 5^{th} order polynomial is a better fit for the battery capacity than the Peukert equation.

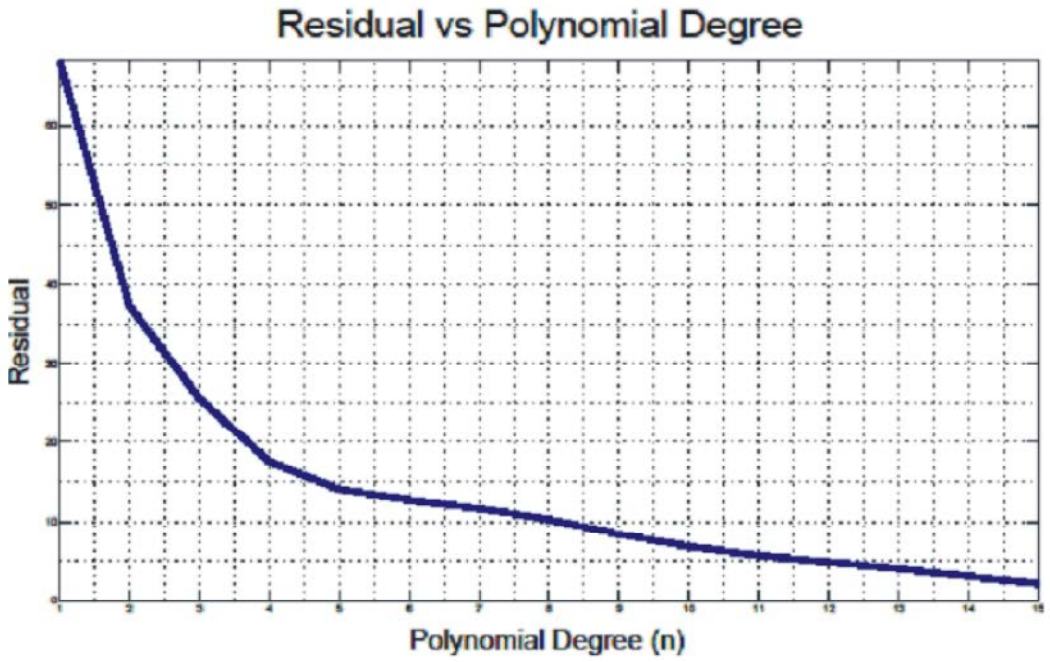

Fig. 2: Residual analysis of lead acid battery curve-fitted using different degree of polynomials

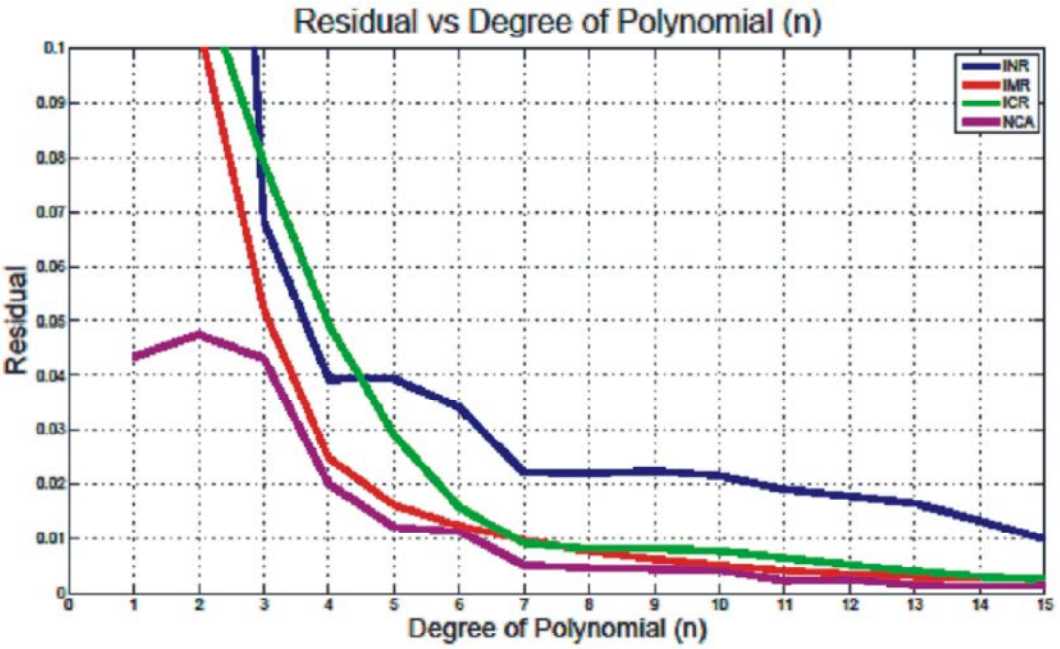

Fig. 3: The residual for different degree order of polynomial curve fitting of LG lithium ion batteries

The plot of the data in Figure 4 also shows that the battery capacity approaches a maximum theoretical capacity as the current I approaches 0 from the right-hand side. In constrast, the Peukert equation does not model this feature and as $I \to 0^{+}$ $C \to 8$ in the Peukert equation. On the other hand, a finite capacity as $I \to 0^{+}$ can be modelled using the polynomial curve. Additionally, over the interval from 0 to 6C the capacity of lead acid battery had reduced by more than 80%. What this shows is that it is important to model the battery capacity when dealing with lead acid battery.

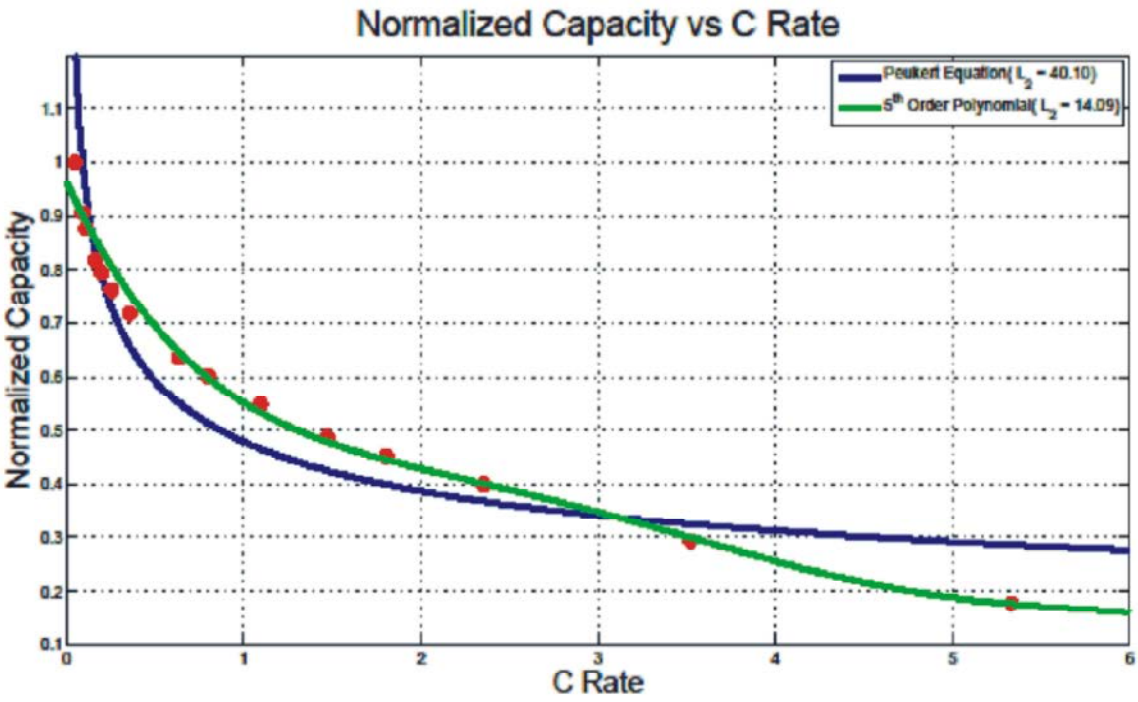

Fig. 4: A plot of the capacity of lead acid battery versus C rate with a comparison between curve fittings using peukert equation and 5th order polynomial equation

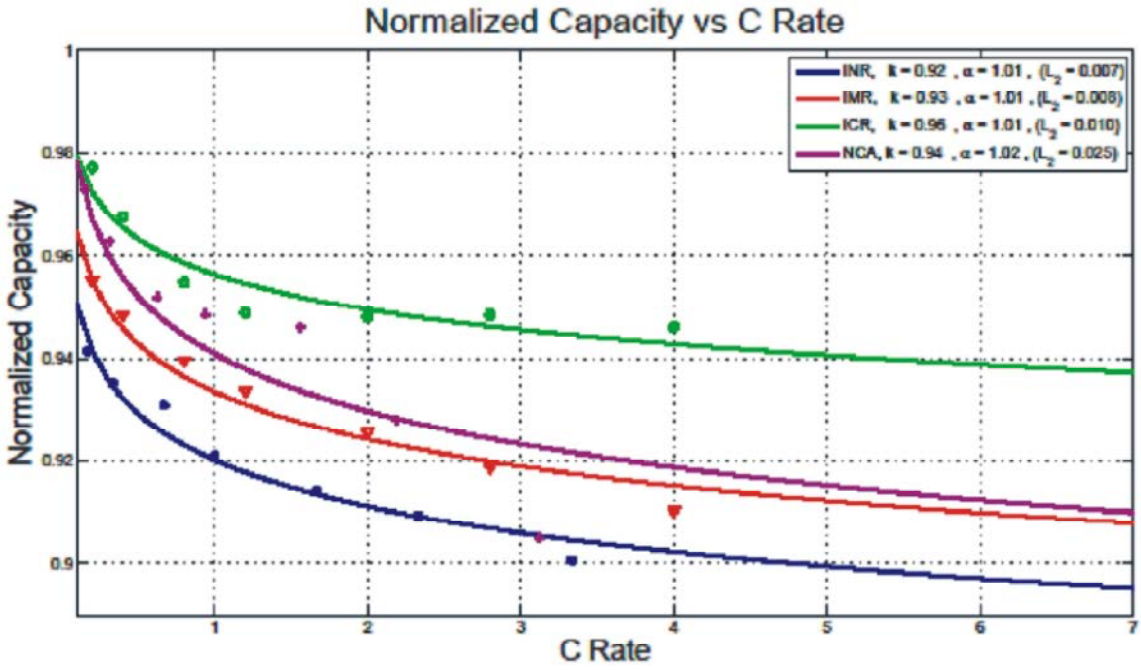

Fig. 5: Plots of Peukert equation curves fitted on LG lithium ion batteries to the limit of 7C discharge rate

Lithium Ion Battery: Peukert analysis of lead acid battery done over the interval from 0 to 6C was repeated for LG lithium ion batteries over the interval of 0 to 7C as shown in Figure 5 below. Over this interval the capacity of the lithium ion batteries had decreased by less than 15% and α is close to 1. What this shows is that over this interval

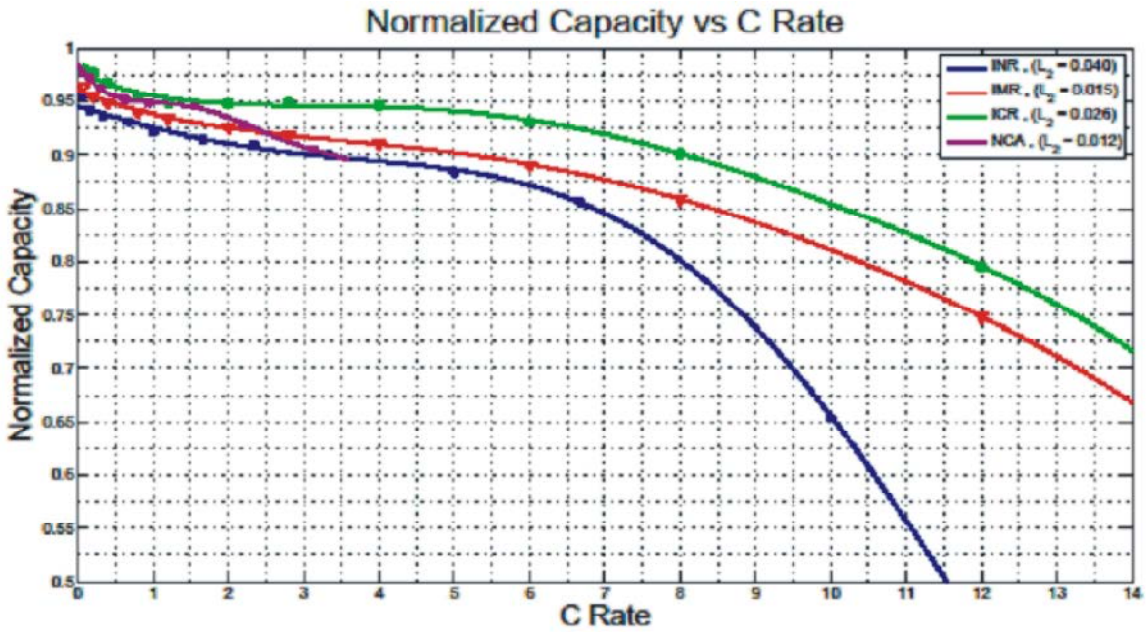

Fig. 6: Plots of 5th order polynomial curve fitting of LG lithium ion batteries

we can treat the capacity as being constant if we are willing to accept a 15% inaccuracy in the prediction of battery capacity. Moreover, one can argue that Peukert equation can only model constant discharge current but in applications the current is variable and as a result a 15% error can be within the margin of error for the applications.

Like lead acid battery, Figure 6 shows lithium ion batteries have a theoretical maximum capacity as discharge rate I approaches zero from the right-hand side. This property can also be modelled using the polynomial equation as shown in *equation 6*.

Additionally, let us now consider the interval where the discharge current is greater than 6C. Noticeably in the case of NCA the discharge rate was limited to 10A due to a fuse placed on the terminal of the battery. In the other cases, the batteries were discharge up to 30A. For I > 6C, capacity of INR, IMR and ICR begins to decrease at a steeper rate as shown in Figure 6. As a result, the Peukert curve is not able to model the capacity effectively at high current (e.g I > 6C).

CONCLUSION

Over the interval of $0 \leq I \leq 6C$ the capacity of lead acid battery significantly decreases with respect to I. At approximately 6C its capacity has decreased by 80%. As a result, equations such as Peukert equation are needed to model the reduction in battery capacity as a function of discharge current for lead acid. Alternatively, the use 5th order polynomial with PCHIP shows a better fit to the battery capacity compared to Peukert equation. The residual of the 5th order polynomial curve is more than half that of the Peukert equation.

In the case of LG lithium ion batteries over the interval of $0 \leq I \leq 7C$, the capacity has decreased by less than 15% and the corresponding value of α associated with Peukert equation is close to 1. What this implies is that Peukert equation is not suitable for modelling the capacity of lithium ion battery. A better approach is to simply model the curve using a polynomial equation as described in this paper. The polynomial equation can be used to model the capacity curve over the entire available data points (e.g. for IMR with $0 \leq I \leq 12C$). Additional, by using the polynomial curve we can model the theoretical capacity at I = 0 and for interval greater I = 6C, which is not possible with the Peukert equation.

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