

Evaluating the Volatility Forecasting Performance of EGARCH (1, 1) Models in USDNGN and USDZAR Exchange Rates

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Abstract: This paper aims at evaluating the volatility of forecasting performance for the US Dollar/Nigerian Naira and US Dollar /South African Rand exchange rates obtained via an EGARCH(1, 1) model under six error distributional assumptions: the normal distribution(norm), the skew normal distribution(snorm), the student –t distribution(std), the skew the student –t distribution(sstd), the generalized error distribution(ged) and the skew generalized error distribution(sged). We make use of daily data to evaluate the in-sample parameters and out-of-sample volatility estimates. Our comparison and forecasting performance in both in-sample and out sample were based on Akaike Information Criterion(AIC), log likelihood for in sample and MSE and MAE for out sample. The results show that the six error distributions may perform quite well with slight advantage to EGARCH(1, 1)-ged in fitting the two data sets considered in this paper. While the EGARCH(1, 1)-std and EGARCH(1, 1)-sstd give the best performance models in out sample USDNGN and USDZAR exchange rates data respectively. We carry out paired –t test between the best fitting and the best forecasting (performance) models using their MSEs (MAEs) and the test prove to be statistically insignificant which, clearly demonstrates that it is reliable to use the best fitted model for volatility forecasting.

Key words: AIC • EGARCH models • MSE • Student-t distribution • Volatility

INTRODUCTION

The need for accurate volatility forecasting in financial markets cannot be over emphasized as regards financial management, risk management, investment, monetary policy making etc [1]. The relevance of volatility forecasting in risk management on the short run was pointed out by [2]. The ‘stylized features’ exhibited by financial time series (FTS) are very crucial in applied economic analysis [3]. The stylized characteristics of asset returns common to FTS which include heavy tails, volatility clustering, excess kurtosis, leverage effect, etc were examined by [4]. Autoregressive conditional Heteroskedasticity (ARCH) model with normal innovations that captured some of the stylized features of FTS was introduced by [5]. The parsimonious Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model was proposed by [6] which further improved the ARCH modelling process. Since then many researchers have adopted the ARCH (GARCH) frame work for studying and explaining FTS and stock volatilities.

[7] argue that no reasonable evidence can be found that would allow to conclude to the inferiority of the GARCH(1, 1) as compared to more Complicated models. Some researchers focus on the estimation of the stock returns volatility and the persistence of shocks to volatility [8]. The works, of [9] and [10] examined the volatility in US and found that the 1987 crash help to support the fact that lower than average returns induce more speculative activity and therefore increased market volatility. An estimation of the volatility of stock returns in key mature markets was considered by [11], while [12] measured the financial volatility of the Athens Stock Exchange from 1998 to 2008. The GARCH (1, 1) model outperforms other models considered, not including asymmetric models, in estimating volatility of foreign exchange rate [7]. [13] have shown that a GARCH model with an underlying leptokurtic asymmetric innovation distribution outperforms one with an underlying Gaussian innovation distribution, for modelling volatility of the Chinese Stock Market. Also studies by [14] have demonstrated that the use of fat tailed error distributions

within a GARCH(1, 1) framework leads to improved volatility forecasts. The former considered nine possible error distributions to model the volatility of the Standard & Poor's 500 with the leptokurtic distributions being preferred. The Mean Absolute Error and Heteroscedasticity-adjusted MAE were used to evaluate the forecasts. [15] also finds out that GARCH model with a student's t innovation distributional assumption produces better forecasts as compared to the GARCH models Exponential distribution and a mixture of Normal distributions. GARCH model, the GARCH in Mean is used by [16] to assess the impact of market, interest rate and foreign exchange rate risks on the sensitivity of Australian bank Stock Returns. [17] investigates the volatility of Nigerian Naira / Dollar Exchange rate by fitting six univariate GARCH models using monthly data and concludes that the best performing models are the Asymmetric Power ARCH and TS - GARCH. The models used in this study use Student's t innovations. In Ghana, [18] examines the impact of Exchange rate volatility on the local Stock Exchange. The authors make use of an Exponential GARCH for their purpose and observe the negative relationship between the exchange rate volatility and stock market returns. [19] compares the forecast - ability of symmetric and asymmetric GARCH models. The US Dollar / Deutsche Mark returns series is filtered using an AR(1) process and the GARCH(1, 1), GJR-GARCH(1, 1) and EGARCH(1, 1) volatility equations are used. The author concludes that the EGARCH performs better in producing out of sample forecasts with the GARCH(1, 1) closely following whereas the GJR-GARCH fares worst. The list of work on the forecast ability of asymmetric GARCH models include 3 Forecasting Volatility of USD/MUR Exchange Rate Using a GARCH (1, 1) Model with GED and Student's-t errors the research by [20] where the authors compare the performance of the classical GARCH(1, 1) versus other asymmetric variations using the out sample. The asymmetric models are found to produce better forecasts. However, the GARCH (1, 1) is seen to outperform other GARCH models not taking into consideration the asymmetry property. [21] Consider asymmetric GARCH models for measuring the volatility for the AUD/USD, GBP/USD and JPY/USD exchange rates.

Despite the fact that many models have been proposed to the ARCH family, the underlying distributional assumption for the innovations in these models is usually the normal (Gaussian) distribution. However, GARCH-type models with normality assumption

may not be useful because financial time series returns are found to violate this assumption in real life, hence non normal distributions such as skewed normal, student-t, skewed student-t, generalized error distribution, skewed generalized error distribution, etc are given consideration in modeling conditional variance in returns of financial time series such as exchange rates. One problem in modeling financial time series data is that of identifying an appropriate model that will be suitable for a particular scenario. Again, it is an observable fact that little attention has been paid in GARCH-type studies in developing economies such as Nigeria and South Africa and to the best of our knowledge, empirical research on the topic of volatility forecasting of exchange rate is fragmented as there is no work that compares the ability of different volatility forecasting models in African countries. This study is an attempt to abridge the paucity of empirical research literature in exchange rate volatility in two biggest African developing economies, following the work of Awartani and Corradi (2005). We shall achieve this by analyzing and comparing the volatility forecasting performance of EGARCH model with different error distributions, which include normal (norm), skew normal (snorm), student-t (std), skew student-t (sstd), generalized error distribution (ged) and the skew generalized error distribution (sged), for Nigeria and South Africa exchange rate returns during the sample period. The reason for choosing these six error distributions is to take into account the skewness, excess kurtosis and heavy -tails usually associated with financial time series return distributions. The comparison and the forecasting ability of the models would be considered both in-sample and out-of-sample using the AIC, Loglikelihood, Mean Square Error(MSE), Mean Absolute Error(MAE) as matrices.

The rest of the article is structured as follows: Methodology section 2, Data and empirical properties are presented in section 3, while the in-sample parameter estimations are given in section 3.1. Section 4 discusses evaluation volatility forecasts and section 5 presents the conclusions.

MATERIALS AND METHODS

Volatility Model: The statistical package used in this study is R version 3.1.2 (2014-10-31) for the purpose of this study make use of a GARCH (1, 1) model for conditional variance and MA(1) (Moving Average) model for the mean equation the MA(1) model is used a filter for the returns series. We define the following.

$$r_t = \mu + \theta \sigma_t z_t \quad (2)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 r_{t-1}^2 + \beta \sigma_{t-1}^2 \quad (3)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 r_{t-1}^2 + \beta \sigma_{t-1}^2 \quad (4)$$

where ϵ_t is the innovation, σ_t is the volatility measure and z_t is an i.i.d variable such that $z_t \sim F$ where F is some distribution with mean zero. In this study, we consider an asymmetric GARCH(1, 1) model called exponential Generalized Autoregressive conditional Heteroskedasticity (eGARCH), F will be the normal, skewed normal, student's t , skewed student's t , GED and skewed GED error distributions.

The Distribution of Error: This paper considered six different types of error distributions

Normal Distribution:

$$f(z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right), -\infty < z < \infty$$

Skewed Normal Distribution:

$$f(z) = \frac{1}{\omega\pi} \exp\left(-\frac{(z-\xi)^2}{2\omega^2}\right) \int_{-\infty}^{\frac{z-\xi}{\omega}} \exp\left(-\frac{t^2}{2}\right) dt, -\infty < z < \infty$$

where ξ denotes the location; ω denotes the scale and α denotes the shape of density

Student-t distribution:
$$f(z) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi} \Gamma\left(\frac{\nu}{2}\right)} \left(1 + \frac{z^2}{\nu}\right)^{-\frac{\nu+1}{2}} \quad (5)$$

where ν denotes the number of degrees of freedom and Γ denotes the Gamma function.

• Skewed Student- t Distribution:

$$f(z; \mu, \sigma, \nu, \lambda) = bc \left(1 + \frac{1}{\nu-2} \left(\frac{b\left(\frac{z-\mu}{\sigma}\right) + a}{1-\lambda}\right)^2\right)^{-\frac{\nu+1}{2}} \quad \text{if } z \leq \frac{\mu-a}{b}$$

or,

$$f(z; \mu, \sigma, \nu, \lambda) = bc \left(1 + \frac{1}{\nu-2} \left(\frac{b\left(\frac{z-\mu}{\sigma}\right) + a}{1+\lambda}\right)^2\right)^{-\frac{\nu+1}{2}} \quad \text{if } z > \frac{\mu-a}{b}$$

if $z < \frac{\mu-a}{b}$

where ν is a shape parameter with $2 < \nu < \infty$ and λ is a skewness parameter with $-1 < \lambda < 1$. The

constants a , b and c are given as: $a = 4\lambda\epsilon\left(\frac{\nu-2}{\nu-1}\right)$, $b = 1 + 3\lambda^2 - a^2$, $c = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi(\nu-2)}\Gamma\left(\frac{\nu}{2}\right)}$ μ and σ^2 are the mean and

variance of the Skewed Student- t distribution.

Generalized Error Distribution (GED):

$$f(z, \mu, \sigma, \nu) = \frac{\sigma^{-\lambda} \exp\left(-0.5\left(\frac{z-\mu}{\sigma}\right)^\lambda\right)}{\lambda 2^{\lambda-1} \Gamma\left(\frac{\lambda}{\lambda-1}\right)}, 1 < \lambda < \infty, \nu > 0$$

degree of freedom or tail thickness parameter and $\lambda = \sqrt{2\left(\frac{\lambda}{\lambda-1}\right)\Gamma\left(\frac{\lambda}{\lambda-1}\right)/\Gamma\left(\frac{\lambda}{\lambda-1}\right)}$ If $\nu=2$, the GED yields the normal

Distribution. If $\nu < 1$, the density function has thicker tails, than the normal density function, whereas for $\nu > 2$ it has thinner tails.

Skewed Generalized Error Distribution: (SGED)

$$f(z, \nu, \xi) = \nu \left[2\theta\Gamma\left(\frac{1}{\nu}\right)\right]^{-1} \exp\left(-\frac{|z-\delta|^\nu}{[1-\text{sign}(z-\delta)\xi]^\nu\theta^\nu}\right)$$

where $\theta = \Gamma\left(\frac{1}{\nu}\right)^{-0.5} \Gamma\left(\frac{2}{\nu}\right)^{-0.5} S(\xi)^{-1}$, $\delta = 2\xi AS(\xi)^{1-\frac{1}{\nu}}$,

$S(\xi) = \nu(1 + 3\xi^2 - 4A^2\nu^2)$

$A = \Gamma\left(\frac{2}{\nu}\right) \Gamma\left(\frac{1}{\nu}\right)^{-0.5} \Gamma\left(\frac{3}{\nu}\right)^{-0.5}$ where $\nu > 0$ is the

shape parameter controlling the height and heavy-tail of the density function, while ξ is a skewness parameter of the density with $-1 < \xi < 1$ in the empirical section of this study, all parameter in the above distribution are that default values in R package, location, scale and skewness parameter are equal to 0, 1 and 1.5 respectively shape parameter is equal to 5 for students t and skewed student's t distributions and equal to 2 for GED and skewed GED distributions the parameters of the model in (4) α_0 , α_1 and β are non-negative with $\alpha + \beta < 1$ to ensure stationarity. The parameters of the model are estimated using the R package 3.1.2, the set of 1825 returns is estimation sample comprising 1725 observations for in-sample evaluation and 100 forecasting sample called out of sample data used to investigate the p.

$$\ln(\sigma_t^2) = \alpha_0 + \beta_1 \ln(\sigma_{t-1}^2) + \alpha_1 \left|\frac{x_{t-1}}{\sigma_{t-1}}\right| + \gamma \frac{x_{t-1}}{\sigma_{t-1}} \quad (5)$$

For a positive shock $\frac{x_{t-1}}{\sigma_{t-1}} > 0$, (5) becomes

$$\ln(\sigma_t^2) = \alpha_0 + \beta_1 \ln(\sigma_{t-1}^2) + (\alpha_1 + \gamma) \frac{x_{t-1}}{\sigma_{t-1}} \quad (6)$$

But for a negative shock $\frac{x_{t-1}}{\sigma_{t-1}} < 0$, (5) becomes

$$\ln(\sigma_t^2) = \alpha_0 + \beta_1 \ln(\sigma_{t-1}^2) + (\alpha_1 - \gamma) \frac{x_{t-1}}{\sigma_{t-1}} \quad (7)$$

The term $\frac{x_{t-1}}{\sigma_{t-1}}$ in the equation (5) represents the asymmetric effect of shocks. A negative shock leads to higher conditional variance in the following period which is not the case with a positive shock, Poon and Granger (2003). When compared to the GARCH (p, q) models, the parameters of the EGARCH models are not restricted to be non-negative. The EGARCH models have the ability to produce positive conditional variance independently of the signs of the estimated parameters in the models and no restrictions are needed. This comes into play when the restriction in the GARCH model encounters problems when the estimated parameters violate its inequality constraints. We note that the exponential nature of the EGARCH(p, q) model ensures that the conditional variance is always positive even if the coefficients are negative. We can determine the presence of leverage effect by testing the hypothesis that $\gamma = 0$ against $\gamma \neq 0$. If $\gamma \neq 0$, the impact is asymmetric. By including the parameter, β in the EGARCH (1, 1) model the persistence of volatility shocks is captured. Apart from the parametric relaxation of the positive constraints, another advantage of the EGARCH (p, q) model over the symmetric GARCH(p, q) model is that it incorporates the asymmetries in returns volatilities. The parameters α and γ capture two important asymmetries in conditional variances. If $\gamma < 0$, negative shocks increase the volatility more than positive shocks of the same magnitude. Because the parameter α is expected to be positive, large shocks of any sign will have larger impact compared to small shocks. Based on these advantages, we apply the EGARCH model for estimating volatility of the Nigeria and South Africa exchange rate returns.

Data and Empirical Properties: The data used in this study are the daily closing values of Nigerian naira/US Dollar South Africa/US Dollar exchange rates. The data were collected from OANDA SOLUTIONS FOR BUSINESS webservices@oanda.com. The data span the period from 29th December, 2010 to 27th December, 2015 and comprise 1825 observations of the spot price and are

converted for the needs of fitting the model to a logarithmic return series. If the price series is denoted by $\{X_t\}$.

$$r_t = \ln\left(\frac{x_t}{x_{t-1}}\right) \quad (8)$$

From Tables 1 and 2, it is observed that the means of USDNGN USDZAR exchange rate returns are 0.000151 and 0.000454 respectively which are very close to zero. It may be seen also that the returns exhibit positive skewness and excess kurtosis. The Jarque-Bera test performed at 5% level of significance rejected the null hypothesis test of zero skewness and zero excess kurtosis ($\chi^2_{test} = 12.321, 13.123 > 5.99$). This suggests departure from normality assumption.

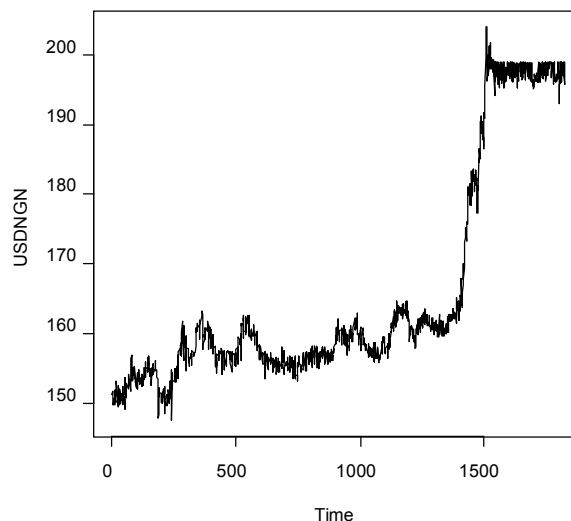


Fig. 1: Displays plots of original USDNGN exchange rates

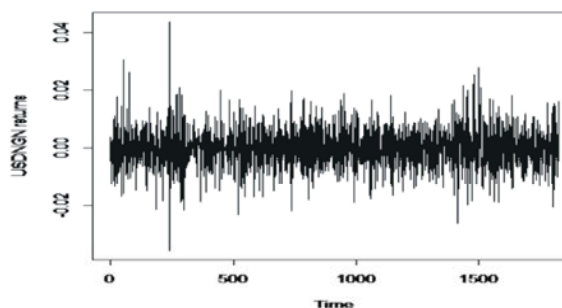


Fig. 2: Displays the time series of prices and log returns of USDNGN. The plot of the returns series suggests the presence of heteroscedasticity. In fact, we observe clusters of periods of high volatility as well as those of low volatility

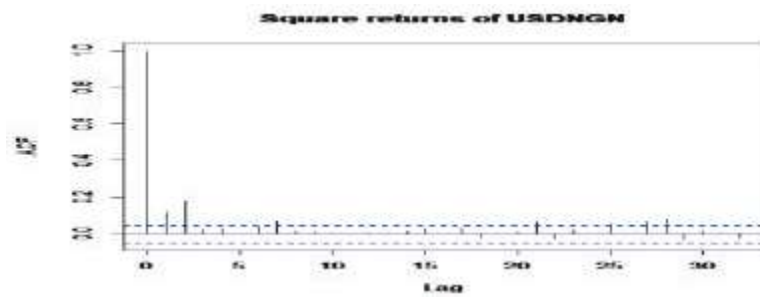


Fig. 3: ACF of square returns of USDNGN exchange rates for the period 29/12/2010 to 27/12/2015

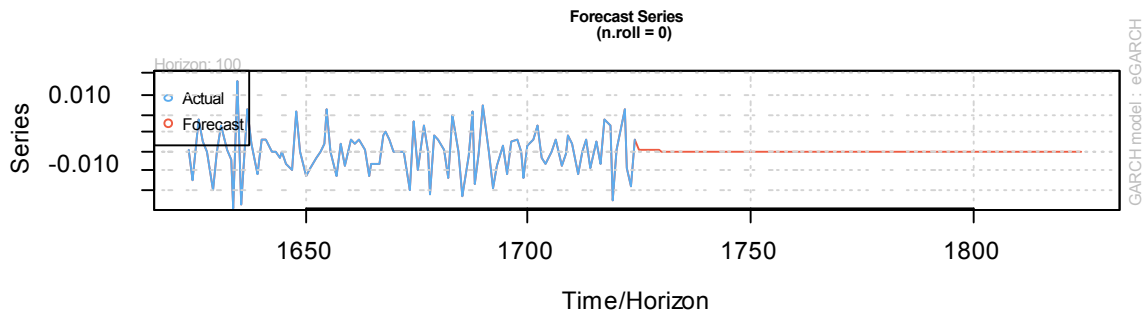


Fig. 4: Out-Sample Volatility forecast EGARCH (1, 1)-ged

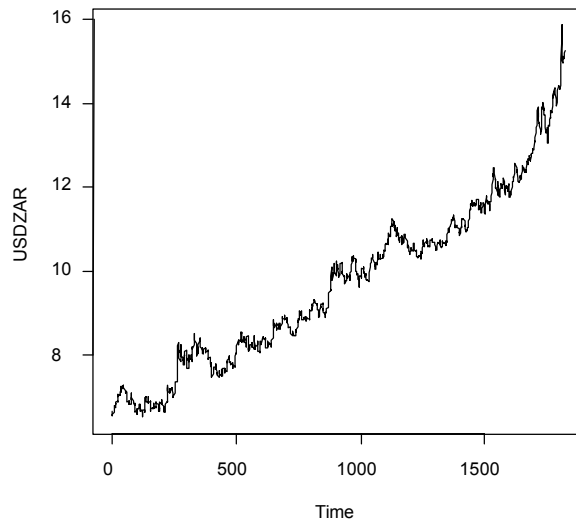


Fig. 5: Displays plots of original USDZAR exchange rates

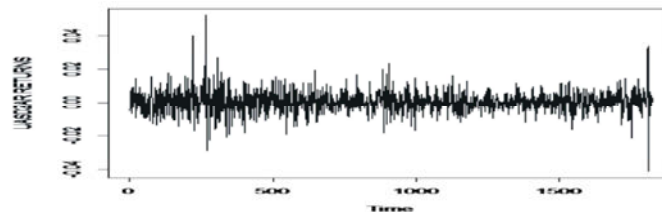


Fig. 6: Displays the time series of prices and log returns of USDZAR. The plot of the returns series suggests the presence of heteroscedasticity. In fact, we observe clusters of periods of high volatility as well as those of low volatility

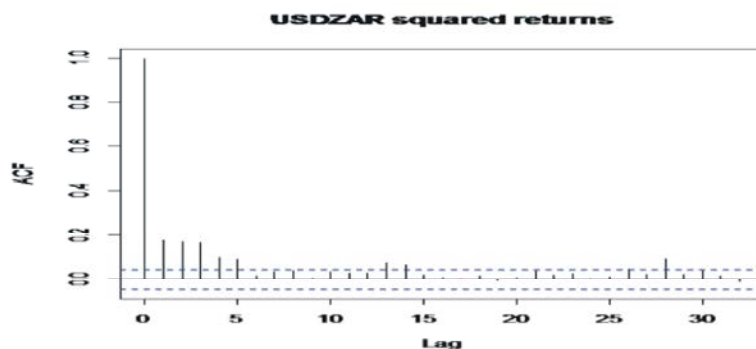


Fig. 7: ACF of square returns of USDZAR exchange rates for the period 29/12/2010 to 27/12/2015.

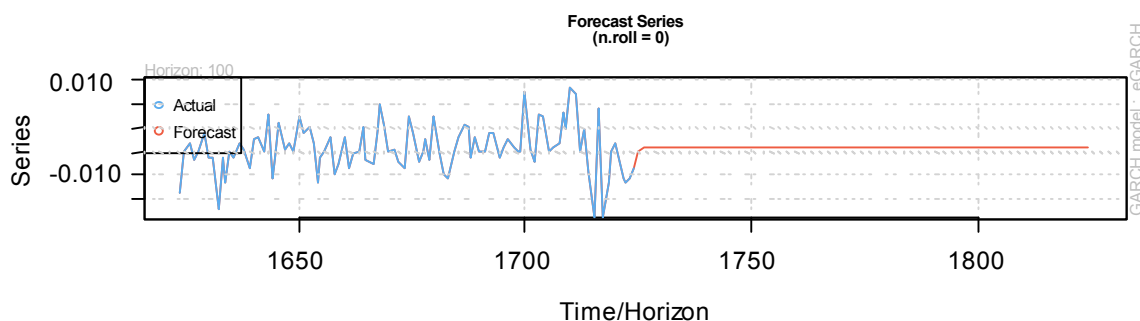


Fig. 8: Out-Sample Volatility forecast EGARCH (1, 1)-ged

Table 1a: Descriptive statistics for USDNGN exchange rate

Statistic	value
mean	0.000151
standard deviation	0.0067
range	0.079011
skewness	0.08378
kurtosis	5.22672
Jarque-Bera	12.321
LBP	119.2
ARCH -test	11.458

Table 1b: Descriptive statistics for USDZAR exchange rate

Statistic	value
mean	0.000454
standard deviation	0.006347
range	0.010714
skewness	0.367086
kurtosis	8.382439
Jarque-Bera	12.321
LBP	131.2
ARCH -test	13.123

The Ljung-Box-Pierce test at 5% significance level and with up to lags allows us to deduce the lack of randomness in the data (presence of autocorrelation).

The critical value for the test is 11.0705. In the same vain Engle's ARCH test at 5% level of significance with up to five lags also rejected the null hypothesis that returns form a random sequence of normal disturbances thus, the presence of heteroscedasticity with the critical value-11.0705. These features support the use of a GARCH model for capturing the time varying volatility.

In-Sample Parameter Estimation

Table 2a: Parameter Estimation of the eGARCH(1, 1)-Normal for USDNGN exchange rate

Parameter	Robust T Statistics	P-Value
α_0	-1.548838	0.016611
α_1	0.021065	0.280273
B_1	0.846994	0
γ_1	0.25082	0.000034

The parameters mean and variance equations are estimated for each distribution. The values of the parameters robust T statistics and p-values are shown in Tables 2a to 4b using the p-values obtained, we may deduce that parameters of the models are significant at the 5% level of significance.

Table 2b: Parameter Estimation of the eGARCH(1, 1)-Normal (USDZAR)

Parameter		Robust T Statistics	P-Value
α_1	-0.14486	3.4122	0.085772
α_1	0.069731	4.3011	0
B1	0.985649	8.928	0
γ_1	0.107379	5.2181	0.001251

Table 3a: Parameter Estimation of the eGARCH (1, 1)-snormal (USDNGN)

Parameter		Robust T Statistics	P-Value
α_0	-1.505505	2.6347	0.00842
α_1	0.020838	4.2319	0.29817
β_1	0.85126	8.928	0
γ_1	0.248526	5.2181	0.00005
Skew	0.989725	32.3933	0

Table 3b: Parameter Estimation of the eGARCH (1, 1)-snormal (USDZAR)

Parameter		Robust T Statistics	P-Value
α_0	-0.136876	2.6347	0.099441
α_1	0.068578	4.2319	0
β_1	0.85126	8.928	0
γ_1	0.98641	5.2181	0
Skew	1.034705	32.3933	0.001930

Table 4a: Parameter Estimation of the eGARCH (1, 1)-std (USDNGN)

Parameter		Robust T Statistics	P-Value
α_0	-1.64876	2.4471	0.014399
α_1	0.034836	3.819	0.000134
β_1	0.85126	8.928	0
γ_1	0.248526	5.2181	0.000051
Skew	8.473814	5.0434	0

Table 4b: Parameter Estimation of the eGARCH (1, 1)-std (USDZAR)

Parameter		Robust T Statistics	P-Value
α_0	-0.13484	2.4471	0.098006
α_1	0.068076	3.819	0.000002
β_1	0.986915	8.928	0
γ_1	0.097735	5.2181	0.002171
Skew	9.443984	5.0434	0.00000

Forecast Evaluations: We make use of two metrics for forecast evaluation both in and out of samples. We consider mean square error (MSE), the Root Mean Square Error (PMSE) and Mean Absolute Error defined as follows:

$$MSE = \frac{1}{n} \sum_{t=1}^n (r_t^2 - \sigma_t^2)^2 \quad (9)$$

$$MAE = \frac{1}{n} \sum_{t=1}^n |r_t^2 - \sigma_t^2| \quad (10)$$

where r_t^2 is used as a substitute for the realized or actual variance [12] and [20] employed r_t^2 as proxy for realized volatility and σ_t^2 is the forecasted variance.

Table 5a: Parameter Estimation of the eGARCH (1, 1)-sstd (USDNGN)

Parameter		Robust T Statistics	P-value
α_0	-1.61724	2.4763	0.089978
α_1	0.033768	3.8639	0.17173
β_1	0.840864	7.6851	0
γ_1	0.259994	5.2181	0.002202
Skew	0.989451	5.0434	0
Shape	7.792897	5.045	0

Table 5b: Parameter Estimation of the eGARCH (1, 1)-sstd (USDZAR)

Parameter		Robust T Statistics	P-value
α_0	-0.13332	2.4763	0.102729
α_1	0.068098	3.8639	0.000003
β_1	0.987042	7.6851	0
γ_1	0.097299	5.2181	0.002434
Skew	1.013357	5.0434	0
Shape	0.987042	5.045	0.000006

The AIC, loglikelihood and the above two symmetric statistical loss functions are among the most popular methods for evaluating the forecasting power of a model given their simple mathematical structure.

Tables 7a and 7b show the AIC, loglikelihood, MSE and MAE for the forecasted volatilities for the two data sets. We observe that the six models seem to produce relatively accurate forecasts given the quite small values of AIC, MSE, and MAE and large value of loglikelihood. From Tables 7a and 7b, we can see that the true model is always the best fitted model in terms of the AIC but the true model does not necessarily provide the minimum values of MSE and MAE and might not produce the best forecasting volatility. Our result seems to agree with Shamiri and Isa (2009) argument that there are several plausible models that we can select to use for our forecast and we should not be fooled into thinking that the one with the best fit is the one that will forecast the best. The smallest AIC value produced by the EGARCH (1, 1)-GED and the largest score of log-likelihood by the GARCH(1, 1)-GED indicate that the two models perform slightly better than the EGARCH models with Normal, Skewed Normal, Student- t and Skewed Student- t errors even though the EGARCH model with Normal error happened to produce the least values of MSE, and MAE Figures 2, 4, 5 and 8 display the actual returns versus the forecasted volatilities within the evaluation sample.

Table 6a: Parameter Estimation of the eGARCH (1, 1)-ged (USDNGN)

Parameter		Robust T Statistics	P-value
α_1	-0.16678	2.5212	0.080926
α_1	0.028061	3.6321	0.261384
β_1	0.836047	7.0889	0
γ_1	0.258954	5.2181	0.001767
skew	1.415876	18.8191	0

Table 6b: Parameter Estimation of the eGARCH (1, 1)-ged for USDZAR exchange rate

Parameter		Robust T Statistics	P-value
α_1	-0.124394	2.5212	0.127797
α_1	0.065715	3.6321	0.000001
β_1	0.987969	7.0889	0
γ_1	0.096168	5.2181	0.004645
skew	1.46516	18.8191	0.000869

Table 7a: Parameter Estimation of the eGARCH (1, 1)-sged for USDNGN exchange rate

Parameter		Robust T Statistics	P-value
α_0	-1.61081	1.3028	0.192634
α_1	0.026477	2.1481	0.031709
β_1	0.84166	4.0232	0.000057
γ_1	0.256677	5.2181	0.001767
skew	0.988617	31.3254	0
shape	1.415834	13.8237	0

Table 7b: Parameter Estimation of the eGARCH (1, 1)-sged for USDNGN exchange rate

Parameter		Robust T Statistics	P-value
α_0	-0.12124	1.3028	0.134021
α_1	0.065503	2.1481	0.000001
β_1	0.988243	4.0232	0
γ_1	0.094903	5.2181	0.005416
skew	1.015829	31.3254	0
shape	1.466812	13.8237	0

Table 8a: AIC, LOGLIKELIHOOD, MSE and MAE for the fitted distributions (USDNGN)

	Distribution					
	norm	snorm	std	sstd	ged	sged
AIC	-7.34	-7.3391	-7.3606	-7.3598	-7.361	-7.3602
MSE(1step)	5.9043	5.1413	5.1895	5.1785	5.1553	5.1564
MSE(10step)	5.6436	5.3096	5.4315	5.3180	5.3216	5.1851
MAE(1step)	7.2	7.91	7.164	7.33	7.18	7.18
MAE(10step)	7.46	7.74	7.348	7.51	7.36	7.45
LOGLIKE	6333.106	6333.315	6351.852	6352.118	6352.519	6352.216

Notes: The minimum values of AIC, Log like, MSE and MAE in the same row are in bold. The reported MSE is multiplied by ($\times 10^{-5}$) and MAE by ($\times 10^{-3}$)

Table 8b: AIC, LOGLIKELIHOOD, MSE and MAE for the fitted distributions (USDZAR)

	Distribution					
	norm	snorm	std	sstd	ged	sged
AIC	-7.5774	-7.577	-7.5939	-7.5929	-7.5977	-7.5967
MSE(1step)	2.8999	2.7974	2.915	2.9031	2.9301	2.9301
MSE(10step)	2.6853	2.8452	2.7433	2.8346	2.7563	2.8612
MAE(1step)	1.5881	1.5891	1.4851	1.3991	1.4131	1.4131
MAE(10step)	1.4216	1.3964	1.4281	1.3185	1.3895	1.3639
LOGLIKE	6538.76	6539.393	6553.983	6554.064	6557.348	6557.209

Notes: The minimum values of AIC, Log like, MSE and MAE in the same row are in bold. The reported MSE is multiplied by ($\times 10^{-5}$) and MAE by ($\times 10^{-3}$)

Evaluation of Volatility Forecasts: To investigate how much difference between the best forecast and best fitted models, each sample size of the two data sets is divided into two 1785 observations for in-sample data and 100 sample observations for out-sample data. For each sample set, we fit the data by models with the six different error distributions respectively and then calculate the value of MSE and MAE for different horizons. We carry out paired -t test using the following hypothesis:

$$H_0: \mu_a - \mu_b = 0$$

$$H_1: \mu_a - \mu_b > 0$$

where μ_a denotes the mean of MSE (MAE), given by the best fitted model. μ_b denotes the mean of MSE (MAE) given by the best performance model.

The reason for this test is to find out if in fact, the mean of MSE and MAE from the best fitted model are

statistically significantly different from the mean o MSE and MAE from the best performance model. If the null hypothesis cannot be rejected, it means that statistically, the best performance model does not provide better volatility forecast than the best fitted model in terms of statistical loss functions of MSE and MAE values. The p- values of the positive one tailed paired t-test for step-1 and 10-step ahead forecasts are given in Table 9.

Table 9: The p-values of paired –t test result between the best fitted model and the best performance model from EGARCH (1, 1)

DISTR	USDNGN		USDZAR	
	MSE	MAE	MSE	MAE
norm	0.2156	0.1528	0.2051	0.2534
snorm	0.4317	0.8754	0.7894	0.8451
std	0.6182	0.1514	0.6523	0.7893
sstd	0.1124	0.2502	0.2658	0.1218
ged	0.0731	0.1001	0.1214	0.1184
sged	0.2583	0.3626	0.2635	0.2318

Table 10: Percent Error of MSE

Percent Error of MSE				
	USDNGN		USDZAR	
	Difference	PE%	Difference	PE%
Step 1	0.014	1.4%	0.015	1.5%
Step 10	0.012	1.2%	0.013	1.3%

The percentage performance of the fitted models and best performance models are calculated thus:

$$PE = ((A-B)/A) \times 100\%$$

A=MSE (MAE) of best fitted model and B=MSE (MAE) best performing model. The PE values are reported in tables 10 and 11. The tables show that PE values are very small which suggests that the MSE(MAE) given by the best fitted models are not statistically different from those given by the best performance models. The inference is that, we can still use the best fitted model for USDNGN and USDZAR volatility forecasting.

Table 11: Percent Error of MAE

Percent Error of MAE				
	USDNGN		USDZAR	
	Difference	PE%	Difference	PE%
Step 1	0.016	1.6%	0.014	1.4%
Step 10	0.012	1.2%	0.011	1.1%

CONCLUSIONS

The accurate measurement and forecasting of exchange rate returns is very important for the African

economies. Firstly, Africa depends heavily on imported goods, secondly, there is significant increase in the amount of foreign investments into the continent and lastly, important national reserves are held in foreign currencies, mostly in US dollars. This is the reason the study of fluctuations in foreign exchange rates in Nigeria and South Africa is a key issue. R package was used to find the best fitted EGARCH model among the different error distributional EGARCH models considered for in-sample data and the estimation of parameters in the model. The error distributions used are the Normal, Skewed Normal, Student-*t* distribution, Skewed Student-*t* distribution, Generalized Error distribution (GED) and Skewed Generalized Error distribution. The results obtained indicate that all the models performed fairly well in capturing the volatility fluctuation of Nigerian exchange rate returns with slight advantage to the models with GED and SGED errors for forecasting out-of-sample volatility. The two models have the lowest AIC and the highest log-likelihood values. The AIC and log-likelihood values given by different models with different error distributions are reported in Table 8. The empirical evidence show that MSE (MAE) given by the best fitted models in both data sets considered in this paper are not statistically different from those given by the best performance models. Given the fact that it is impracticable in real life, to identify the best performance model, this research clearly demonstrates that it is reliable to use the best fitted model for volatility forecasting.

REFERENCES

1. Poon, S. and C. Granger, 2003. Forecasting Volatility in Financial markets: A Review, *Journal of Economic Literature*, 41: 478-539.
2. Christoffersen, P.F. and F.X. Diebold, 1998. How relevant is volatility forecasting for financial risk management? NBER working papers 6844, 1998, National Bureau of Economic Research, Inc.
3. Liu, H.C. and J.C. Hung, 2010. Forecasting S&P-100 stock index volatility: The role of volatility asymmetry and distributional assumption in GARCH models, *Expert Systems with Application*, 37: 4928-4934.
4. Cont, R., 2001. Empirical properties of asset returns: stylized facts and Statistical issues, *Quantitative Finance*, 1: 223-236.
5. Engle, R.F., 1982. Autoregressive Conditional heteroskedasticity with estimates of the variance of U.K. inflation. *Econometrica*, 50: 987-1007.

6. Bollerslev, T., 1986. Generalized auto-regressive conditional heteroskedasticity. *Journal of Economics*, 31: 307-321.
7. Hansen, P.R. and A. Lunde, 2005. A forecast comparison of volatility models: does anything beat a GARCH (1, 1)? *Journal of Applied Econometrics*, 20: 873-889.
8. Choudhry, T., 1996. Stock Market volatility and the crash of 1987: evidence from six emerging markets. *Journal of International Money and Financial*, 15: 981-996.
9. Chou, R., 1988. Volatility persistence and stock valuation: some empirical evidence using GARCH. *Review of Economics and Statistics*, 69: 542-547.
10. Richard, B. and R. Degennaro, 1990. Stock returns and volatility, *Journal of Financial and Quantitative Analysis*, 25: 203-214.
11. Alexander, C. and E. Lazar, 2004. The equity index skew, market crashes and asymmetric normal mixture GARCH. ISMA center Discussion Papers, 14.
12. Drakos, A.A., G.P. Kouretas and L.P. Zarangas, 2010. Forecasting Financial volatility of the Athens Stock Exchange daily returns: An application of asymmetric normal mixture GARCH model, *International Journal of finance and Economics*, pp: 1-4.
13. Hung-Chung, L., L. Yan-Hsien and L. Ming-chihm, 2009. Forecasting China stock market volatility via GARCH models under skewed. GED Distribution. *Journal of Money, Instrument and Banking*, pp: 542-547.
14. Wilhelmsson, A., 2006. GARCH forecasting performance under different distribution assumptions. *Journal of Forecasting*, 25: 561-578.
15. Chuang, V., J.R. Lu and P.H. Lee, 2007. Forecasting Volatility in the financial market: a comparison of alternative distributional assumptions. *Applied Financial Economics*, 17: 1051-1060.
16. Worthington, A.C. and S.K. Ryan, 2004. Market interest rate and foreign exchange rate risk in Australian banking: A GARCH-M approach, *International Journal of Applied Business and Economic Research*, 2: 81-103.
17. Olowe, R., 2009. Modelling naira/dollar exchange rate: Application of garch and asymmetric models. *International Review of Business Research Papers*, 5: 377-378.
18. Ajasi
19. Balaban, E., 2004. Forecasting Exchange rate volatility. Working paper, 2004, URL: <http://ssrn.com/bstract=494482>.
20. Awartani, B.M. and V. Corradi, 2005. Predicting the volatility of the S & P -500 stock index via GARCH models: the rate of asymmetries, *International Journal of Forecasting*, 21: 167-183.
21. Wang, J. and M. Yang, 2009. Asymmetric Volatility in the foreign exchange markets. *Journal of International Finance Markets, Institutions and Money*, 19: 597-615.