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Application of Project Evaluation, Review Technique and Critical Path Method (PERT-CPM) Model in Monitoring Building Construction

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Abstract: The need to control large-scale projects cannot be over emphasized, especially in construction companies that face numerous challenges. A tool for monitoring and controlling projects is the Project Evaluation and Review Technique -Critical Path Method (PERT-CPM). In this paper, PERT-CPM model was applied in monitoring the construction of a five bed-rooms duplex building. Activities of the building project considered in this study were translated to a network diagram. Critical activities were identified. The activity times were observed to have been Weibully distributed. Tchebyshev's inequality method was used to calculate the lower and upper bound estimates in days for critical activities of the project. The overall time for the completion of the Building was obtained. The probability of completing the project in a given time was obtained. The PERT-CPM method applied in this study reduced the actual time the company spent in executing the entire project by 97 days and this has a serious implication on the profit margin for the Company.

Key words: Critical path Method · Project Evaluation and Review Technique · Building construction · Monitoring · Weibull distribution · Tchebyshev's inequality

INTRODUCTION

Residential and non-residential buildings are structures, which serve as shelters for man, his properties and activities. So, building projects must be properly planned, designed and erected to obtain desired satisfaction from the environment. Such building project involves a single non-representative scheme typically undertaken to accomplish a premeditated result within a time bound and financial plan. However, because of the individuality of each project, its outcome can never be predicted with an absolute confidence as observed by [1].

The management of large-scale project of constructing a five bed room building project poses numerous challenges. These challenges have led to widespread use of project management techniques such as Critical Path Method (CPM) and PERT (Project Evaluation and Review Technique). These project management techniques provide managers with a systematic quantitative framework for planning, scheduling and coordination of numerous interrelated activities associated with the successful on-time completion of large building projects made up of smaller tasks some of which can be started straight away while some need to await the completion of other activities or can be done in parallel before they eventually commence as observed by [2]. The task of planning, scheduling (or organizing) and control are considered to be basic managerial functions and CPM-PERT has been rightfully accorded due importance in the literature of Operations Research and Quantitative Analysis for handling the planning, scheduling and control of building projects.Hence, the goal of CPM-PERT and related techniques is to facilitate the management, coordination and control of the various activities involved in a project so that the project itself may be completed successfully because PERT-CPM can answer the following important questions: How long will the entire project take to be completed? What are the risks involved? Which are the critical activities or tasks in the project which could delay

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the entire project if they were not completed on time? Is the project on schedule, behind schedule or ahead of schedule? If the project has to be finished earlier than planned, what is the best way to do this at the least cost? The questions posed above are answered by CPM-PERT by structuring the activities of the building construction work into a network and examining the time requirements and precedence relationships associated with the activities as observed in [3].

It is pertinent to observe that PERT-CPM provided a focus around which managers could brain-storm and put their ideas together. It has been demonstrated that it is a great communication medium by which thinkers and planners at one level could communicate their ideas, their doubts and fears to another level and so it is most importantly a useful tool for evaluating the performance of individuals and teams in a project. There are many variations of CPM-PERT which have been useful in planning costing, scheduling manpower and machine time. [4], studied project planning and scheduling using PERT and CPM techniques with linear programming. He investigated the trade-off between the cost and minimum expected time that will be required to complete a building project and observed that the use of CPM-PERT reduced the time for the building construction from 79 days to 40 days which is very substantial but this increased the cost of the building project.

When the activity times in the projects are deterministic and known, Critical Path Method (CPM), a mathematically ordered network for planning and scheduling project has been demonstrated to be a useful tool in managing such projects in an efficient manner to meet the challenge of meeting dead-lines in project execution. It has been used extensively in project management because of its simplicity and for easy identification of critical activities in the system. CPM is also applied in computation of project duration by identifying sequence of activities that must not be delayed in execution of the project if the dead-lines must be met. The critical path is obtained by computing early start and finish times using a forward-pass algorithm while late start and finish time are computed using a backward pass algorithm.

However, there are many cases where the activity times may not be presented in a precise manner, such problems have been demonstrated by researchers to be solved using the Project Evaluation and Review Technique which is based on the probability theory. Project Evaluation and Review Technique (PERT) emphasizes the relationship between the time each activity takes in the completion of the entire project. PERT is a technique involving three estimates. These estimates are identified as Optimistic, most likely and Pessimistic and are made for each activity of the overall project. The optimistic time estimate is the minimum time the activity will take by assuming that everything surrounding the execution of the project goes right the first time. The reverse is the pessimistic estimate, or maximum time estimate for completing the activity. The third is the most likely estimate or the normal or realistic time an activity requires. PERT was developed in 1958 by a research team to help measure and control the progress of the U.S.A Navy's Polaris Fleet Ballistic Missile Program [5]. PERT was introduced to complement CPM by incorporating a probabilistic concept for modeling activity durations through estimating the uncertainty in a project network and by computing the probability to complete the project within a specified time.

In real life practice PERT-CPM has been frequently used for planning, managing and controlling projects in areas such as research projects, road construction, building construction, software development, product development, construction and plant maintenance, various engineering practices among others. The techniques have exceptionally wide applications and their applications have contributed significantly to better planning, control and general organization of many projects. Construction companies which always have a number of construction works awarded to them on contract by the public sector or private and individuals face lots of challenges like varying cost of materials, mechanical breakdown, power failure, strike, water failure etc. Because of these challenges mentioned above, this study applied Tchebyshev's approach to obtain the lower bound and upper bound probability of completing critical activities of the project under consideration because project schedules change on a regular basis. PERT-CPM allows continuous monitoring of the schedule and thereby helps project managers track the critical activities in their projects. The time taken to execute these critical activities guide in determining the length of time the non-critical activities can be delayed within the total float period so that the duration of the construction target will still be met. The trend in recent years has been to merge the two methods- PERT and CPM into what is called the PERT-CPM approach.

This study therefore applied PERT-CPM approach to identify the critical activities that could delay completion of a building construction, duration of the entire project activities and the probability of meeting specified deadlines in the construction of five bed-room duplex.

MATERIAL AND METHODS

Study Location and Data Description: In order to schedule the activities in the network we require an estimate of how much time each activity should take when it is done in the normal way. These estimates as provided by the project manager of Fenlands Associates Limited, a construction company based in Port-hercourt Nigeria. Table 1 shows the break down description of activities involved and the respective precedence relationship of the activities for the construction process of a 5-bed room duplex at No. 6, King Perekule Road, GRA Phase 1 Port-Harcourt, Rivers State Nigeria The construction activity begins with activity 1 and ends with activity 65. The network analysis technique applied in analyzing the data are the critical path method (CPM) and project evaluation review technique (PERT). Traditionally Beta distribution is applied in using PERT in analyzing network problem but in this work we observed that Weibull distribution which does not require approximation of the mean and variance does better than the Beta distribution. Since the Weibull probability distribution can usually accommodate longer right tail probability than the Beta probability distribution. Furthermore, since Weibull probability distribution is continuous, has a finite endpoint, defined mode between its endpoints and it is capable of describing both skewed and symmetric activity time distribution as observed in [6]. In view of this, the Weilbull Distribution function offers an accurate and more realistic description of distribution phenomena.

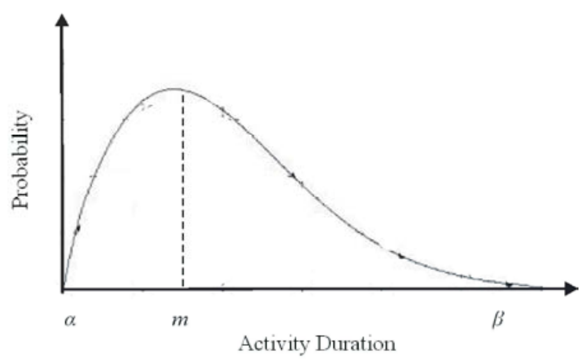

Fig. 1: An Example of Weibull Distribution

The three estimates of time required are:

1. Optimistic time denoted by α
2. Pessimistic time denoted by β
3. Most likely time denoted by m
   According to [7], the range specified by the optimistic time (α) and pessimistic time (β) estimates is assumed to enclose every possible estimates of the duration of the activity. The most likely time (m) estimate must lie somewhere in the range (α,β). That is, the most likely time (m) estimate may coincide with the midpoint $\frac{a+\beta}{2}$. Hence, we assume that the duration of each activity follows a Weibull distribution with its uni-modal point occurring at m and its end points at α and β.
4. In Weibull distribution, the midpoint $\frac{a+\beta}{2}$ is assumed to weigh half as much as the most likely point (m). Thus, the expected or mean $\{E(T)\ or\ \mu\}$ time of an activity, that is also the weighted average of the time estimates, is computed as the arithmetic mean of $\frac{a+\beta}{2}$ and $2m$. That is, expected time to complete an activity is approximated $E(T)\ or\ \mu = \frac{\frac{\alpha+\beta}{2}+2m}{3} = \frac{\alpha+4m+\beta}{6}$. With uncertain activity time, variance can be used to describe dispersion (variation) in the activity time values. The calculations are based on the analogy to the normal distribution where 99% of area under the normal curve is within ±3δ from the mean or fall within the range approximately 6 standard deviation in length, therefore the interval (α,β) is assumed to enclose about 6 standard deviations of a symmetric distribution. Thus if σ_i denotes the standard deviation of the duration of activity i, then
5. So, the variance of the activity time is $\sigma_i^2 = \left\{\frac{\beta_i - \alpha_i}{6}\right\}^2$
6. If we assume that the deviation of the activities are independent random variables, then the variance of the total critical path's duration is the sum of the variances on the critical path. Suppose σ_c is the standard deviation of the critical path, then $\sigma_c^2 = \sum \sigma_i^2$ and $\delta_c = \sqrt{\sum \sigma_i^2}$ where i is an element on the critical path.

Upper Bound and Lower Bound of a Set of Real Numbers: The understanding of upper bound and lower bound of a set of real numbers is needed since its application is inevitable in this section.

Table 1: Description of activities involved in the building construction under study

Activity	Immediate Precedence	Activity description	Estimated Duration
1	-	Site clearing	3
2	1	Sinking of borehole	2
3	2	Bringing in of materials(sand & chippings) to site & molding of blocks	18
4	1,3	Setting out of building	2
5	4	Excavation of foundation trench & column base	3
6	4,5	Blinding of column base	1
7	6	Preparation of baskets & column starter	1
8	7	Placement of baskets and column starter	1
9	8	Placing of concrete in column base & foundation strip	2
10	8	Block laying to dpc	4
11	10	Formwork to sides of column starter	2
12	11	Placing of concrete in column starter	1
13	12	Removal of formwork from column starter	1
14	13	Backfilling, making up levels, ramming & consolidating	12
15	14	Formwork to sides of bed	1
16	15	Placing of concrete in bed	2
17	16	Block laying to lintel level	9
18	17	Positioning & tying of reinforcement in columns	2
19	18	Formwork to sides of columns to lintel level	2
20	8,9	Placing of concrete in columns	2
21	20	Removal of form work from sides of columns	1
22	21	Formwork to sides & bottom of lintels	4
23	22	Placing & tying of reinforcement in lintels	3
24	22,23	Placing of concrete in lintels	3
25	24	Removal of form work from sides of lintel	2
26	25	Block laying to first floor, beam level	5
27	26	Formwork to sides, underside of 1st floor, beams & staircase	9
28	27	Preparation & placement of reinforcement in beams & staircase	8
29	25	Electrical & plumbing piping in first floor	6
30	29	Placing of concrete in first floor, slab, beams & staircase	4
31	30	Block laying to lintel level	11
32	28,31	Positioning & tying of reinforcement in columns	2
33	19,32	Formwork to sides of columns to lintel level	3
34	33	Placing of concrete in columns	2
35	34	Removal of formwork from sides of columns	1
36	35	Formwork to sides & bottom of lintels	4
37	36	Placing & tying of reinforcement in lintels	3
38	37	Placing of concrete in lintels	3
39	38	Removal of form work from sides of lintel	1
40	39	Laying of blocks/beam level(level the building)	4
41	1, 40	Construction of doors & windows/protectors	10
42	40	Completing of electrical & plumbing piping	2
		Roofing	
43	40	Woodwork or skeleton(noggins)	7
44	43	Roof covering	4
45	41,42,44	Ceiling finish(covering with asbestos)	3
46	45	Fixing of frames(doors & windows)	2
47	46	Plastering of under deck & walls (in & out)	30
48	46	Fixing of doors & windows	10
49	46	Dressing work	8
50	46,47	Floor finish/tilling	13
51	48	Electrical & mech. Fixtures	9

Table 1: Continued

Activity	Immediate Precedence	Activity description	Estimated Duration
		Soak Away Pit and Septic Tank	
52	49,51,50	Excavation	6
53	52	Blinding of the foundation	1
54	53	Installation of reinforcement in columns	1
55	53,54	Casting of concrete in the foundation	1
56	55	Laying of blocks	3
57	56	Form work to sides of columns	2
58	57	Placing of concrete in columns	1
59	58	Form work to sides & bottom of beam & slab	5
60	59	Reinforcement to the beam and slab	2
61	60	Placing of concrete on the beam and slab	1
62	61	Removal of form work from sides & bottom of beam & slab/plastering of inside septic tank	3
63	62	Plumbing fixture from building to soak away pit	1
64	63	Painting of house inside and outside	15
65	64	Clean up of building/site	2

Bounded Subset: A subset S of real numbers is said to be bounded from below (or simply bounded below) if there exist $\alpha \varepsilon \Re$ such that $\alpha \leq X$ for all xεS. A subset S is said to be bounded from above (or simply bounded above) if there exists $\beta \varepsilon \Re$ such that $X \leq \beta$ for all xεS. The subset S is said to be bounded if it is bounded both from above and from below [8].

Lower and Upper Bound: If a subset S of real numbers is bounded below by α, then α is called a Lower bound of S and if S is bounded above by β then β is called an Upper bound of S. [8]. It was proved in a theorem by [9], that the set of all arcs in a given network possesses the upper bound and lower bound probability. This work will imitate Kleindorfer by using Chebyshev's inequality in calculating the lower bound and upper bound probabilities for the critical activities of the project.

Chebyshev's Inequality: The Chebyshev's inequality enables us to find the upper bound or lower bound for certain probabilities.

Theorem 1: Let u(x) be a non-negative function of the random variable x. If $E\{u(x)\}$ exists, then for every positive constant C; $P\{u(x) \geq c\} \leq \frac{E\{u(x)\}}{c}$

Proof: This proof is given when the random variable X is of the continuous type, but the proof can be adapted to the discrete case if we replace integrals by sums.

Let A $=\{x: u(x) \geq k\}$, $A^* = \{x: u(x) \leq k\}$ and let f(x) denote the probability density function (pdf) of x. Then

$$E\{u(x)\} = \int_{-\infty}^{\infty} u(x) f(x)\, dx$$
$$= \int_A u(x) f(x) dx + \int_{A^*} u(x) f(x)\, dx$$

Since each of the integrals in the extreme right hand member of the preceding equation is non-negative, the left hand member is greater than or equal to either of them.

$$\int_{-\infty}^{\infty} u(x) f(x)\, dx \;\geq\; \int_A \; u(x) f(x)\, dx$$

However, if $x \in A$, then, $u(x) \geq C$ accordingly, the right hand side (RHS) member of preceding inequality is not increased if we replace U(x) by C.

Thus $\{u(x)\} \geq C \int_A \; f(x)\, dx$
Since $\int_A \; f(x) dx = P(x \in A) = P\{u(x) \geq C\}$

It follows that E$\{u(x)\} \geq CP\{u(x) \geq C\}$ which is the desired result.

The preceding theorem is a generalization of an inequality, which is often called the Chebyshev's inequality [10].

The Chebyshev's inequality gives some insight into the fact that the standard deviation of a random variable is a rather natural unit for the probability law of a random variable. The inequality gives a bound on the probability that a random variable will be within k standard deviations of its mean.

Theorem 2: Let the random variable X have a distribution of probability about which we assume only that there is a finite variance δ^2. This implies that if there is a mean (μ), then for K > O; P $\{|x - \mu| \geq k\delta\} \leq \frac{1}{k^2}$ or equivalently

$P\{|x-\mu| < k\delta\} \geq 1 - \frac{1}{k^2}$

Proof: From theorem 1, let $u(x) = (x - \mu)^2$ and $C = k^2\ \delta^2$ Replacing them, we now have

$P\{(x-\mu)^2 \geq k^2\delta^2\} \leq \frac{\{E(x-\mu^2)\}}{k^2\delta^2} = P\{(x-\mu)^2 \geq k^2\delta^2\} \leq \frac{\delta^2}{k^2\delta^2}$

Since the numerator of the RHS of the inequality is δ^2 This implies that δ^2 which is our desired result [10].

The Weibull probability density function, reliability, mode, variance and mean formula are given in equations (1) -(5) from [11].

$$f(x) = \left(\frac{\beta}{\theta}\right)\left(\frac{x}{\theta}\right)^{\beta-1} \exp\left[-\left(\frac{x}{\theta}\right)^{\beta}\right] \quad (1)$$

$$R(x) = \exp\left[-(x/\theta)^{\beta}\right] \quad (2)$$

$$M = \text{mod}\,e = \theta\left(1-\left(\frac{1}{\beta}\right)\right)^{\left(\frac{1}{\beta}\right)} \text{ for } \beta > 1 \quad (3)$$

$$\mu = mean = x_0 + \theta\Gamma\left(1+\frac{1}{\beta}\right) \quad (4)$$

$$\sigma^2 = \theta^2\left\{\Gamma\left(1+\frac{2}{\beta}\right) - \left[\Gamma\left(1+\frac{1}{\beta}\right)\right]^2\right\} \quad (5)$$

where β is the shape parameter, θ is the scale parameter, Γ is the gamma function and x_0 shifts the mean on the x-axis. The addition of a threshold value does not change the basic shape of the distribution, only its location on the x-axis is affected..

Data presentation: Table 1 shows the break down description of activities involved and the respective precedence relationship of the activities for the construction process of a 5-bed room duplex at No. 6, King Perekule Road, GRA Phase 1 Port-Harcourt, Rivers State Nigeria. The construction activity starts with activity 1 and ends with activity 65 as shown in Table 1: The Earliest Occurrence Time and Latest Occurrence Time for All the Activities of the Project are shown in Table 2. These earliest occurrence time and latest occurrence time of each activity was computed using the forward pass and backward pass procedure. The forward pass is obtained with the formula $ET_j = Muximum\ (ET_i + t_{jj})$ while the Latest time is computed with the formula $Lt_j = minimum\ (LT_i + t_{jj})$. The optimistic estimate (α), most likely estimate (m) and pessimistic estimate (β) of the building were determined to see the estimates as they effect the construction activities. The expected time (mean) μ and the standard deviation σ for each activity was calculated using the following formulas $E(T)$ or $\mu = \frac{\alpha + 4m + \beta}{6}$ and $\sigma_i = \frac{\beta_i - \alpha_i}{6}$ respectively as shown in Table 3. The activities on the critical path were obtained from the actual time spent on each job as shown in Table 1 by calculating the earliest start and latest start of each job as show in Figure 2 and those nodes in which the two values are the same are called critical activities and from this we obtain the length of time the project takes which is 196 days. This reduced the number of days for the construction work from 293 days to 196 days. This implies that the building construction work was reduced by 97 days which has a serious implication on the profit margin for the company. This is demonstrated in the project diagram on Figure 2.

A classical task of the project scheduling is to create a network flow diagram (Project Network Diagram) and using exact activity duration or their estimates, to determine the completion time of the project and specify the critical activities. Furthermore at this scheduling stage, PERT-CPM provides a realistic, disciplined method for determining how to attain the project objectives,communicating,documenting project plans clearly and concisely using project Network Diagram. This Network diagram or time chart shows the start and finish times for each activity and the amount of leeway corresponding to each activity in relation to other activities in the project.

The monitoring stage helps the project manager to focus attention on the critical activities that may change the schedule of the entire project if they are not handled at the proper time... The application of PERT-CPM uses the expected value and variance of each activity which is obtained by the usual statistical method as shown in [12].

The Forward Pass and Backward Pass Algorithm: The mathematical problem which forms the subject matter of this section is that of determining a maximal steady state flow from one point to another in the network subject to capacity limitations on arcs, will be critically looked into.

Forward Pass Method (For Earliest Event Time): The computations starts at event (node) I and advance recursively to the final event (node), say N. At each event (node) we calculate it's earliest occurrence time (E) and earliest start and finish time for each activity that begins

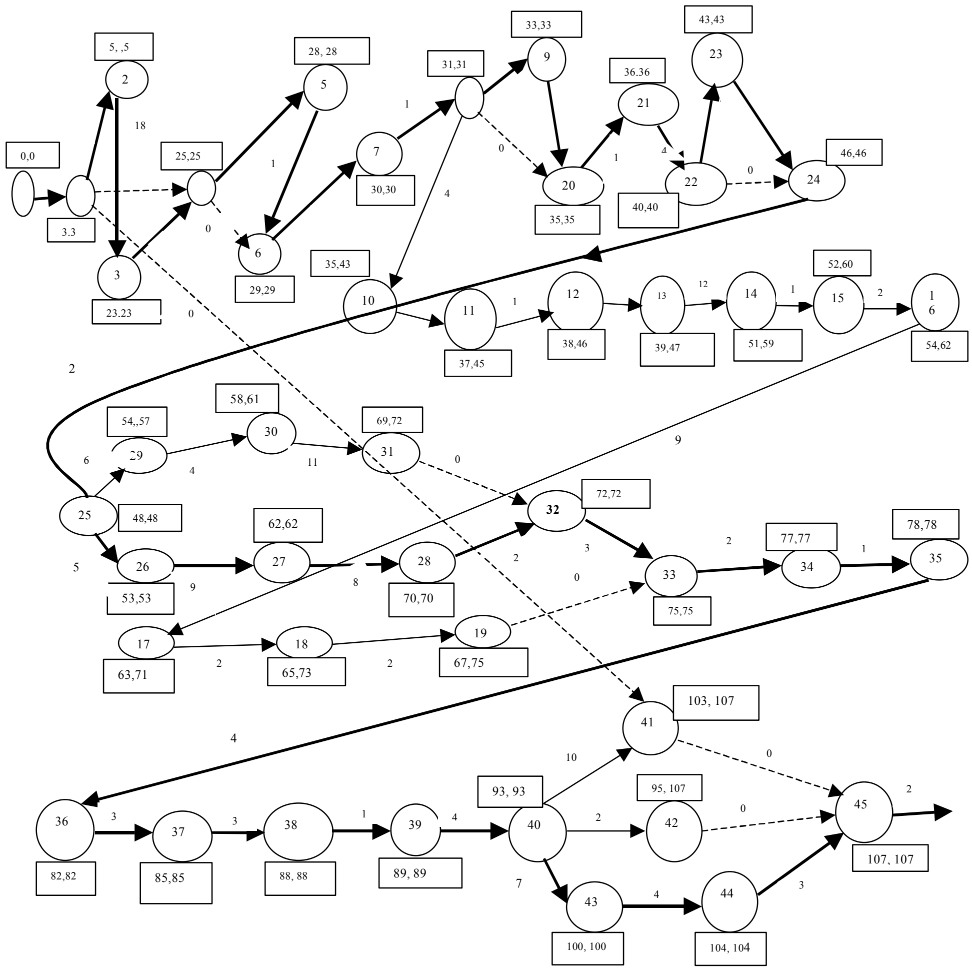

Fig. 2: Project Network Diagram showing Earliest Occurrence Time and Latest Occurrence Time for All the Activities of the Project.

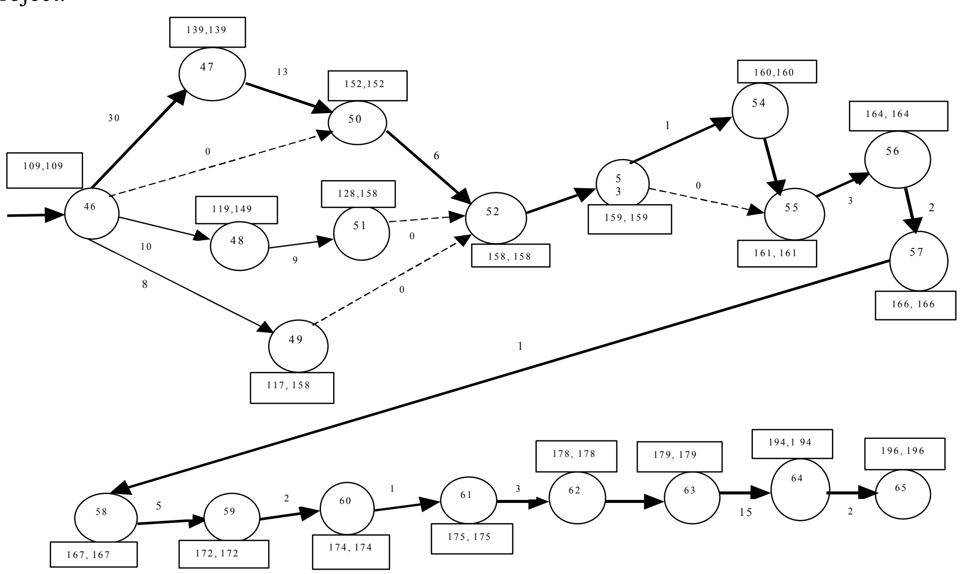

Fig. 2: Continues: Project Network Diagram showing Earliest Occurrence Time and Latest Occurrence Time for All the Activities of the Project.

Table 2: Earliest Occurrence Time and Latest Occurrence Time for All the Activities of the Project.

S/NO	Events	Earliest Occurrence time (E_N)	Latest Occurrence Time (L_N)	{(E_N), (L_N)}	S/NO	Events	Earliest Occurrence time (E_N)	Latest Occurrence Time (L_N)	{(E_N), (L_N)}
1	0 – 1	3	3	{3, 3}	40	32 – 33	75	75	{75,75 }
2	1– 2	5	5	{5, 5}	41	33 - 34	77	77	{77, 77}
3	1– 4	25	25	{25, 25}	42	34 – 35	78	78	{78, 78}
4	1 – 41	103	107	{103,107 }	43	35 – 36	82	82	{82,82 }
5	2 – 3	23	23	{23, 23}	44	36 – 37	85	85	{85,85 }
6	3 – 4	25	25	{25,25 }	45	37 - 38	88	88	{88, 88}
7	4 – 5	28	28	{28, 28}	46	38 - 39	89	89	{89, 89}
8	4 – 6	29	29	{ 29, 29}	47	39 - 40	93	93	{93,93 }
9	5 – 6	29	29	{29, 29}	48	40 - 41	103	107	{103, 107}
10	6 – 7	30	30	{30, 30}	49	40 -- 42	95	107	{95,,107 }
11	7 – 8	31	31	{31, 31}	50	40 - 43	100	100	{100,100 }
12	8 – 9	33	33	{33, 33}	51	41 - 45	107	107	{107, 107}
13	8 – 10	35	43	{35, 43}	52	42- 45	107	107	{107,107 }
14	8 – 20	35	35	{35, 35}	53	43 - 44	104	104	{104,104 }
15	9 – 20	35	35	{35,35 }	54	44 - 45	107	107	{107,107 }
16	10 – 11	37	45	{37,45 }	55	45 - 46	109	109	{109, 109}
17	11 – 12	38	46	{38, 46}	56	46 - 47	139	139	{139, 139}
18	12 – 13	39	47	{39, 47}	57	46 -48	119	149	{119,149 }
19	13 – 14	51	59	{51, 59}	58	46 -49	117	158	{ 117,158 }
20	14 – 15	52	60	{ 52, 60}	59	46 -50	152	152	{152,152 }
21	15 – 16	54	62	{54, 62}	60	47 -50	152	152	{152,152 }
22	16 – 17	63	71	{63, 71}	61	48 -51	128	158	{128, 158}
23	17 – 18	65	73	{65,73 }	62	49 -52	158	158	{158, 158}
24	18 – 19	67	75	{67,75 }	63	50 -52	158	158	{158, 158}
25	19 – 33	75	75	{75,75 }	64	51 -52	158	158	{158, 158 }
26	20 – 21	36	36	{36, 36}	65	52 -53	159	159	{159, 159}
27	21 – 22	40	40	{40, 40}	66	53 -54	160	160	{160,160 }
28	22 – 23	43	43	{43, 43}	67	53 -55	161	161	{161, 161 }
29	22 – 24	46	46	{46,46 }	68	54 -55	161	161	{161, 161 }
30	23 – 24	46	46	{46,46 }	69	55 - 56	164	164	{164, 164}
31	24 – 25	48	48	{48, 48}	70	56 -57	166	166	{166, 166}
32	25 – 26	53	53	{53, 53}	71	57 -58	167	167	{167, 167}
33	25 – 29	54	57	{54,57 }	72	58 -59	172	172	{172, 172}
34	26 – 27	62	62	{62, 62}	73	59 -60	174	174	{174, 174}
35	27 – 28	70	70	{70, 70}	74	60 -61	175	175	{175, 175}
36	28 – 32	72	72	{72,72 }	75	61 -62	178	178	{178,178 }
37	29 – 30	58	61	{58,61 }	76	62 -63	179	179	{179, 179}
38	30 – 31	69	72	{69, 72}	77	63 -64	194	194	{194, 194}
39	31 – 32	72	72	{72, 72}	78	64 -65	196	196	{196, 196 }

at the event. When calculation ends at the final event N, its earliest occurrence time gives the earliest possible completion time of the entire project. The steps are as follows:

- Set the earliest occurrence time of initial event 1 to zero. That is E_1=0, for i=1
- Calculate earliest start time for each activity that begin at event i (=1). This is equal to earliest occurrence time of event 1, (tail event). That is, $ES_{ij} = E_i$, for all activities (I,j) starting at event i.
- Calculate the earliest finish time of each activity that begins at events i. this is equal to the earliest start time of the activity plus the duration of the activity. That is $EF_{ij} = ES_{ij} + t_{ij} = E_i + t_{ij}$, for all activities (i,j) beginning at event i.
- Proceed to the next event, say j;j>i
- Calculate the earliest occurrence time for the event j. This is the maximum of the earliest finish times of all activities ending into that event, that is E_j = Max $\{EF_{ij}\}$ = Max $\{E_i + t_{ij}\}$, for all immediate predecessor activities.
- If j =N (final event), the earliest finish time for the project, that is, the earliest occurrence time E_N for the final event is given by E_N = Max $\{EF_{ij}\}$ = Max $\{ E_{N-1} + t_{ij}\}$, for all terminal activities.

Table 3: Project activity, Optimistic estimate (α), Most likely estimate (m), Pessimistic estimate (β), Expected time E(T) = μ= $\frac{\alpha+4m+\beta}{6}$ and Standard Deviation $\sigma=\frac{(\beta-\alpha)}{6}$

S/N	Activity (I-j)	Optimistic time (α)	Most Likely time (m)	Pessimistic Time (β)	Expected time E(T) =$\mu\ \frac{\alpha+4m+\beta}{6}$	Standard Deviation $\sigma=\frac{(\beta-\alpha)}{6}$
1	0 - 1	2	3	4	3.00	0.33
2	1 - 2	1	2	4	2.17	0.50
3	1 - 4	0	0	0	0.00	0.00
4	1 – 41	0	0	0	0.00	0.00
5	2 – 3	16	18	20	18.00	0.67
6	3 – 4	1	2	3.5	2.08	0.42
7	4 – 5	1.5	3	4.5	3.00	0.50
8	4 – 6	0	0	0	0.00	0.00
9	5 – 6	0.2	1	1.8	1.00	0.27
10	6 – 7	0.5	1	2	1.08	0.25
11	7 – 8	0.5	1	2.5	1.17	0.33
12	8 – 9	1.5	2	3.5	2.17	0.33
13	8 – 10	2	4	5	3.83	0.50
14	8 – 20	0	0	0	0.00	0.00
15	9 – 20	1	2	3.5	2.08	0.42
16	10 – 11	1	2	4	2.17	0.50
17	11 – 12	0.8	1	1.2	1.00	0.07
18	12 – 13	0.2	1	1.8	1.00	0.27
19	13 – 14	10	12	14	12.00	0.67
20	14 – 15	.8	1	1.5	1.05	0.12
21	15 – 16	1	2	3	2.00	0.33
22	16 – 17	4	9	11	8.50	1.17
23	17 – 18	1.4	2	2.6	2.00	0.20
24	18 – 19	1	2	3	2.00	0.33
25	19 – 33	0	0	0	0.00	0.00
26	20 – 21	0.5	1	2.5	1.17	0.33
27	21 – 22	3	4	5	4.00	0.33
28	22 – 23	2	3	4	3.00	0.33
29	22 – 24	0	0	0	0.00	0.00
30	23 – 24	2.5	3	4	3.08	0.25
31	24 – 25	1.5	2	4	2.25	0.42
32	25 – 26	4	5	7	5.17	0.50
33	25 – 29	3	6	8	5.83	0.83
34	26 – 27	7	9	11	9.00	0.67
35	27 – 28	6.5	8	10	8.08	0.58
36	28 – 32	1	2	3	2.00	0.33
37	29 – 30	3	4	5	4.00	0.33
38	30 – 31	9	11	13	11.00	0.67
39	31 – 32	0	0	0	0.00	0.00
40	32 – 33	1.5	3	4.5	3.00	0.50
41	33 - 34	1.5	2	3.5	2.17	0.33
42	34 – 35	0.5	1	1.8	1.05	0.22
43	35 – 36	5	4	7	4.67	0.33
44	36 – 37	1	3	5	3.00	0.67
45	37 – 38	2	3	4	3.00	0.33
46	38 – 39	0.5	1	2.5	1.17	0.33
47	39 – 40	2.5	4	5.5	4.00	0.50
48	40 – 41	9	10	11	10.00	0.33
49	40 – 42	1	2	3	2.00	0.33

Table 3: Continued

S/N	Activity (I-j)	Optimistic time (α)	Most Likely time (m)	Pessimistic Time (β)	Expected time E(T) =μ $\frac{\alpha+4m+\beta}{6}$	Standard Deviation $\sigma = \frac{(\beta-\alpha)}{6}$
50	40 – 43	4	7	10	7.00	1.00
51	41 – 45	0	0	0	0.00	0.00
52	42 – 45	0	0	0	0.00	0.00
53	43 – 44	3	4	5	4.00	0.33
54	44 – 45	2	3	4	3.00	0.33
55	45 – 46	1.5	2	3.5	2.17	0.33
56	46 – 47	29	30	31	30.00	0.33
57	46 – 48	9	10	11	10.00	0.33
58	46 – 49	7	8	9	8.00	0.33
59	46 – 50	0	0	0	0.00	0.00
60	47 – 50	11.5	13	14.5	13.00	0.50
61	48 – 51	7.5	9	10.5	9.00	0.50
62	49 – 52	0	0	0	0.00	0.00
63	50 – 52	4	6	10	6.33	1.00
64	51 – 52	0	0	0	0.00	0.00
65	52 – 53	1.5	1	2	1.25	0.08
66	53 – 54	0.5	1	1.5	1.00	0.17
67	53 – 55	0	0	0	0.00	0.00
68	54 – 55	0.5	1	1.5	1.00	0.17
69	55 – 56	2.5	3	4	3.08	0.25
70	56 – 57	1.5	2	4	2.25	0.42
71	57-58	0.5	1	1.5	1.00	0.17
72	58-59	4	5	7	5.17	0.50
73	59-60	1.5	2	3	2.08	0.25
74	60-61	0.5	1	2	1.08	0.25
75	61-62	2	3	5	3.17	0.50
76	62-63	0.5	1	2	1.08	0.25
77	63-64	14	15	16	15.00	0.33
78	64-65	1	2	3	2.00	0.33

Backward Pass Method (For Least Allowable Time): Following the completion of the forward pass, the backward pass computations start at event (node) N and proceed through the events in the decreasing order of event numbers and end at the initial event (node) 1. At each event, we calculate its latest occurrence time (L) and latest finish and the latest start time for each activity that is terminating at the event and the procedure continues till the initial event 1.

The Steps Are as Follows:

- Set the latest occurrence of last event, N, equal to its earliest occurance time (known from forward pass method). That is, $L_N = E_N$, j = N
- Calculate latest finish time of each activity which ends at event j.This is equal to least occurrence time of final event. That is $LF_{ij} = L_{ij}$ for all activities (i,j) ending at event j.
- Calculate the latest start times of all activities ending at j. It is obtained by subtracting the duration of the activity from the Latest time of the activity. That is $LF_{ij} = L_j$ and $LS_{ij} = LF_{ij} - t_{ij}$, for all activity (i,j) ending at event j.
- Proceed backward to event in the sequence that decreases j by 1.
- Calculate the latest occurrence time of event i(i<j). This is the minimum of the start times of all activities from the event. That is L_i = Min $\{L_{i-1}\text{-}t_{ij}\}$, for all immediate successor activities.

7.If j = 1 (initial event), then the latest finish time for project, that is Latest occurrence time L_i for the initial event is given by L_i = Min $\{LS_{ij}\}$ = Min $\{L_j\text{-}1\text{-}t_{ij}\}$, for all immediate successor

Determination of the Critical Path: The critical path method is an algorithmic approach for finding critical paths. There are two phases in the calculation of the critical path; the first is called the Forward Pass Method while the second phase is called the Backward Pass Method.

Calculation of Lower Bound and Upper Bound Estimates in Days for Critical Activities (Jobs) of the Project: Applying the formula in theorem 2, i.e. Chebyshev's inequality

$$P(\mu - k\sigma < X < \mu + k\sigma) \geq 1 - \frac{1}{k^2}$$

where $K = 1.96$ that is at $\alpha = 0.05$,
We calculate the lower bound and upper bound estimates in days for the critical activities.

For Activity 0 - 1:

$$P(\mu - k\sigma < X < \mu + k\sigma) \geq 1 - \frac{1}{k^2}$$

$$P(3.00 - 1.96 \times 0.33 < X < 3.00 + 1.96 \times 0.33) \geq 1 - \frac{1}{1.96^2}$$

$$P(3.00 - 0.6468 < X < 3.00 + 0.6468) \geq 0.7397$$

$$P(2.35 < X < 3.65) \geq 0.7397$$

For Activity 1 - 2:

$$P(\mu - k\sigma < X < \mu + k\sigma) \geq 1 - \frac{1}{k^2}$$

$$P(2.17 - 1.96 \times 0.5 < X < 2.17 + 1.96 \times 0.5) \geq 1 - \frac{1}{1.96^2}$$

$$P(2.17 - 0.98 < X < 2.17 + 0.98) \geq 0.7397$$

$$P(1.19 < X < 3.15) \geq 0.7397$$

For Activity 2 - 3:

$$P(\mu - k\sigma < X < \mu + k\sigma) \geq 1 - \frac{1}{k^2}$$

$$P(18 - 1.96 \times 0.67 < X < 18 + 1.96 \times 0.67) \geq 1 - \frac{1}{1.96^2}$$

$$P(18 - 1.3132 < X < 18 + 1.3132) \geq 0.7397$$

$$P(16.69 < X < 19.31) \geq 0.7397$$

For Activity 3 - 4:

$$P(\mu - k\sigma < X < \mu + k\sigma) \geq 1 - \frac{1}{k^2}$$

$$P(2.08 - 1.96 \times 0.42 < X < 2.08 + 1.96 \times 0.42) \geq 1 - \frac{1}{1.96^2}$$

$$P(2.08 - 0.8232 < X < 2.08 + 0.8232) \geq 0.7397$$

$$P(1.2568 < X < 2.9032) \geq 0.7397$$

And so on for all the Critical activities.

DISCUSSION

The Critical Path : Figure 2, shows all the activities of the project represented in a diagram called Network Diagram. The critical path in the Network diagram has been shown by thick lines joining all those events where the Earliest Occurrence time (E_N) and the Latest Occurrence time (L_N) are equal. According to the data collected and from the calculations; the critical path of constructing the Building (five bed-room Duplex) is represented by 0-1 -2-3-4-5-6-7-8-9-20-21-22-23-24-25-26-27-28-32-33-34-35-36-37-38-39-40-43-44-45-46-47-50-52-53-55-56-57-58-59-60-61-62-63-64-65. Activities not included in the critical path are called non-critical activities. From the Network diagram, non-critical events are 8-10-11-12-13-14-15-16-17-18-19; 29-30-31; Dummy activities are 1- 4; 1- 41; 4 - 6; 8-20; 19-33; 22-24; 31-32; 41-45; 46-50; 49-52; 51-52 and 53-54. The time duration of the critical path, which represents the overall time for the completion of the Building (five bed room Duplex) is equal to 196 days which is 28 weeks

Calculating Time in Days: Using the three time durations in Table 4, the mean and standard deviation are calculated for each activity, which accounts for uncertainties in the time estimates. The means and standard deviations are used to calculate the lower bound and upper bound probabilities of each critical activity. The Weibull distribution used in this work can take only a variety of non symmetric shapes allowing the mode, which is the most likely time to fall anywhere between the two end points which represents the most optimistic (α) and most pessimistic (β) times.

The mean of the Weibull distribution represents the expected time E(T) of an activity given as a weighted average of the three time estimates. The calculations above show that the expected time for completion does not necessarily have to equal the most likely time. It is also important to note that the standard deviation increases as the difference between the most optimistic (α) and most pessimistic time (β) estimate increases. Therefore, the less certain a person is in estimating the actual time for an activity, the greater the associated deviation is from the expected time.

Amount of Slack Associated with Some Non-Critical Events: Table 5 lists some noncritical events with its associated earliest occurrence times, latest occurrence times and the slack which is the float. Slack which

Table 4: Result of Lower Bound and Upper Bound Estimates in Days for Critical Activities

S/NO	Critical Activities	Lower Bound	Upper Bound	S/NO	Critical Activities	Lower Bound	Upper Bound
1	0 – 1	2.4	3.6	25	37 – 38	2.4	3.6
2	1 – 2	1.1	3.2	26	38 – 39	0.6	1.6
3	2 – 3	16.7	19.3	27	39 – 40	3.0	5.0
4	3 – 4	1.3	2.9	28	40 – 43	5.0	9.0
5	4 – 5	1.9	4.1	29	43 - 44	3.4	4.6
6	5 – 6	0.5	1.5	30	44 - 45	2.4	3.6
7	6 – 7	0.6	1.6	31	45 - 46	1.5	2.8
8	7 – 8	0.5	1.8	32	46 – 47	29.4	30.6
9	8 – 9	1.5	2.8	33	47 – 50	12.0	14.0
10	9 – 20	1.3	2.9	34	50 – 52	4.4	8.3
11	20 – 21	0.6	1.6	35	52 – 53	1.1	1.4
12	21 – 22	3.4	4.6	36	53 – 54	0.7	1.3
13	22 -23	2.4	3.6	37	54 – 55	0.7	1.3
14	23 - 24	2.6	3.6	38	55 – 56	2.6	3.6
15	24 – 25	1.4	3.1	39	56 – 57	1.4	3.1
16	25 – 26	4.2	6.2	40	57-58	0.7	1.3
17	26 – 27	7.7	10.3	41	58-59	4.2	6.2
18	27 – 28	6.9	9.2	42	59-60	1.6	2.6
19	28 – 32	1.4	2.6	43	60-61	0.6	1.6
20	32 – 33	2.0	4.0	44	61-62	2.2	4.2
21	33 - 34	1.5	2.8	45	62-63	0.6	1.6
22	34 – 35	0.6	1.5	46	63-64	14.4	15.6
23	35 – 36	4.0	5.3	47	64-65	1.4	2.6
24	36 - 37	1.7	4.3				

Table 5: Non-Critical Events and their Slacks

Events	Earliest Start Time (Ei)	Latest Start Time (Li)	Slack (Float)
8-10	35	43	8
10 - 11	37	42	5
11 - 12	38	46	8
12 - 13 and others	39	47	8
25 - 29	54	57	3
40 - 41	103	107	4
40 - 42	95	107	12

represents the amount of free time allowed among activities can absorb delays that may arise in the process. For example, event 40 - 42 which is completing of electrical and plumbing piping can be delayed for about 12 days without lengthening time of completion of the Building.

Estimation of Project Completion Time: Since we expect variation in the duration therefore, the probability of completing the project in a given time (26 weeks) can be calculated as shown below. The probability distribution of time for completing an event can be approximated by the normal distribution by the application of the central limit theorem. Hence, the probability of completing the project on schedule i.e. (desired) time, T_s is given by

$$Z = \frac{Ts - E(Ts)}{6}$$

where E(T) = expected completion time of the project

Z = the standardized normal

$\delta_c^2 = \delta_1^2 + \delta_2^2 +$ ---, $+ \delta_n^2$ is the sum of variances of critical activities.

Since the expected completion time of the project is obtained by summing the expected time of all the critical activities. Also, since it is assumed that activities are independent, therefore the variance of the critical path can be obtained by summing variances of the critical activities.

$$\delta_c^2 = 0.33^2 + 0.50^2 + 0.67^2 + 0.42^2 + 0.50^2 + 0.27^2 + 0.25^2 + \ldots + 0.33^2$$

$$\Sigma\delta_c^2 = 8.8294$$

$\therefore \quad \delta_c = \sqrt{8.8294} = 2.9714$

$\Longrightarrow T_s = 26; \quad E(T_s) = 28 \text{ and } \quad \delta_c = 2.9714$

$\therefore$ The Probability of meeting the scheduled time is given by

$Z = \frac{T_S - E(T_S)}{\delta}$ i.e $P\left[Z = \frac{26-28}{2.9714}\right] \Longrightarrow P(\leq -0.670)$

$P(Z \leq -0.670) = 1 - 0.7486 = 0.2514$

Thus, the probability that the project will be completed in less than or equal to 26 weeks is 0.2514 (from normal distribution table). From the above calculation, it was observed that the average time of doing the job does not deviate so much from the theoretical expected mean of the Weibull distribution which was applied in the study.

CONCLUSION

Obviously, there is a discrepancy in the actual time taken to execute the project and the duration of the project as calculated using PERT-CPM model in this work. The reason for this discrepancy is because we used Scientific method in this work in finding the completion time but the Company did not use any Scientific method. Also from the calculations, most of the activities were critical activities. This means that schedule of the activities of this building is highly sensitive and depends on completing these critical activities on time. These critical activities are most likely to be bottlenecks; therefore, attention should be focused on them in order to finish the project on schedule.

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