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Numerical Solution for Mixed Convection Boundary Layer Flow About a Solid Sphere in a Micropolar Fluid with Convective Boundary Conditions

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Abstract: In this paper, the steady mixed convection boundary layer flow on a solid sphere with convective boundary conditions has been studied for cases of both assisting (heated sphere) and opposing flows (cooled sphere). The boundary layer equations are transformed into non-dimensional form and are reduced to a nonlinear system of partial differential equations, which are solved numerically using an implicit finite-difference scheme. Numerical solutions are obtained for the local skin friction coefficient, the local heat transfer coefficient, as well as the velocity and temperature profiles. The features of the flow and heat transfer characteristics for different values of the mixed convection parameter λ, the Prandtl number Pr, the micropolar parameter K, the conjugate parameter γ and the coordinate running along the surface of the sphere, x, are analyzed and discussed.

Key words: Boundary layer · Convective Boundary Conditions · Micropolar Fluid · Mixed convection · Solid sphere

INTRODUCTION

The mixed convective heat transfer has received much attention due to a large number of applications, which is frequently encountered in many industrial and technical processes including solar central receivers exposed to winds, electronic devices cooled by fans, nuclear reactors cooled during emergency shutdown, heat exchangers placed in a low-velocity environment from fixed or rotating bodies represents a problem that can be related to numerous engineering applications and industry. (Kafoussias and Williams [1]).

The theory of micropolar fluid was first proposed by Eringen [2]. This theory has generated much interest and many classical flows are being re-examined to determine the effects of the fluid microstructure. This theory is special class of the theory of microfluids, in which the elements are allowed to undergo only rigid rotations without stretch. The theory of micropolar fluid requires that one must add a transport equation representing the principle of conservation of local angular momentum to the usual transport equations for the conservation of mass and momentum and also additional local constitutive parameters are introduced. It should be mentioned that the mathematical background of the micropolar fluid flow theory is presented in the books by Eringen [3] and Lukaszewicz [4] and in the review papers by Ariman *et al*. [5, 6].

The Research that are related to this problem which studied by Nazar *et al*. [7, 8] and Salleh *et al*. [9] they studied the mixed convection boundary layer flow about a solid sphere in micropolar fluid with different conditions namely the constant surface temperature, constant heat flux and Newtonian heating, respectively.

In recent years, the researchers have been presented this problem, but with difference fluids and conditions, where Tham *et al*. [10] studied the mixed convection boundary layer flow about a solid sphere with constant

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surface temperature in nanofluid The laminar mixed convection boundary layer flow about an isothermal solid sphere embedded in a porous medium filled with a nanofluid for both cases of assisting and opposing flows have been studied by Tham and Nazar [11].

On the other hand, the convective boundary conditions in which the heat is supplied through a bounding surface of finite thickness and finite heat capacity recently used by Aziz [12] who obtained the similarity solution for laminar thermal boundary layer over a flat plate with a convective surface boundary condition. Subsequently, Ishak and Ishak *et al*. [13, 14] obtained the similarity solutions for flow and heat transfer over a permeable surface and the radiation effects on the thermal boundary layer flow over a moving plate with covective boundary conditions, respectively. The mixed convection boundary-layer flow past a horizontal circular cylinder embedded in a porous medium filled with a nanofluid with convective boundary condition presented by Rashad *et al*. [15] The numerical solutions of the steady magnetohydrodynamic two dimensional stagnation point flow of an incompressible nano fluid towards a stretching cylinder with convective boundary condition using fourth-order Runge-Kutta-Fehlberg method with a shooting technique. has been investigated by Akbar *et al*. [16]. Mohamed *et al*. [17] studied the numerical solutions of stagnation point flow over a stretching surface with convective boundary conditions using the shooting method. Recently Alkasasbeh *et al*. [18, 19] presented the numerical solutions of mixed and free convection boundary layer flow about a solid sphere with convective boundary conditions in a viscous fluid using the Keller-Box method, respectively.

Therefore, based on the above-mentioned studies, the aim of the present paper is to study the mixed convection boundary layer flow on a solid sphere placed in a micropolar fluid with convective boundary conditions. The governing dimensional boundary layer equations are first transformed into a system of non-dimensional equations via non-dimensional variables and then, into non-similar equations before they are solved numerically by the Keller box method as described in the book by Cebeci and Bradshaw [20] and Na [21] On other hand, we compared the present result with the previously published results which reported by Nazar *et al*. [7] for the case of constant surface temperature. However, three parameters are introduced in this paper, which are the conjugate parameter, material or micropolar parameter and the mixed convection parameter. To the best of our knowledge, this present problem (for the case of convective boundary condition) has not been presented before, so the reported results are new.

Mathematical Analyses: Consider a heated sphere of radius a, which is placed in a flow field with the undisturbed free stream velocity U_∞ and temperature T_∞ The surface of the sphere is subjected to a convective boundary conditions (CBC). The convective forced flow is assumed to be moving upward, while the gravity vector g acts downward in the opposite direction, where the coordinates $\bar{x}$ and $\bar{y}$ are chosen such that $\bar{x}$ measures the distance along the surface of the sphere from the lower stagnation point and $\bar{y}$ measures the distance normal to the surface of the sphere.

We assume that the equations are subjected to a convective boundary conditions of the form proposed by Aziz [12]. Under the Boussinesq and boundary layer approximations, the equations which govern the boundary layer flow are see (Salleh *et al*. [9] and Nazar *et al*. [7, 8]).

$$\frac{\partial}{\partial \bar{x}}(\bar{r}\,\bar{u}) + \frac{\partial}{\partial \bar{y}}(\bar{r}\,\bar{v}) = 0, \tag{1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \bar{u}_e\frac{d\bar{u}_e}{d\bar{x}} + \left(\frac{\mu+\kappa}{\rho}\right)\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + g\beta(T-T_\infty)\sin\left(\frac{\bar{x}}{a}\right) + \frac{\kappa}{\rho}\frac{\partial \bar{H}}{\partial \bar{y}}, \tag{2}$$

$$\rho j\left(\bar{u}\frac{\partial \bar{H}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{H}}{\partial \bar{y}}\right) = -\kappa\left(2\bar{H} + \frac{\partial \bar{u}}{\partial \bar{y}}\right) + \varphi\frac{\partial^2 \bar{H}}{\partial \bar{y}^2}, \tag{3}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \alpha\frac{\partial^2 T}{\partial \bar{y}^2} \tag{4}$$

Subject to the boundary conditions (Salleh *et al*. [9]; Aziz [12]).

$$\bar{u} = \bar{v} = 0, \text{ or } -k\frac{\partial T}{\partial \bar{y}} = h_f(T_f - T)\bar{H} = -n\frac{\partial \bar{u}}{\partial \bar{y}} \text{ as } \bar{y} = 0,$$

$$\bar{u} \to \bar{u}_e(\bar{x}), T \to T_\infty, H \to 0 \text{ as } \bar{y} \to \infty, \tag{5}$$

where $\bar{u}$ and $\bar{v}$ are the velocity components along the $\bar{x}$ and $\bar{y}$ directions, respectively, $\bar{H}$ is the angular velocity of micropolar fluid, T_f is the temperature of the hot fluid, g is the gravity acceleration, β is thermal expansion coefficient, $v = \mu/\rho$ is the kinematic viscosity and h_f is the heat transfer coefficient for the convective boundary conditions and Newtonian heating.

Let $\bar{r}(\bar{x})$ be the radial distance from the symmetrical axis to the surface of the sphere, φ is the spin gradient viscosity and $\bar{u}_e$ is the local free stream velocity which are given by.

$$\bar{r}(\bar{x}) = a\sin(\bar{x}/a), \varphi = (\mu + (\kappa/2))j, \bar{u}_e(\bar{x}) = \frac{3}{2}U_\infty \sin\left(\frac{\bar{x}}{a}\right), \tag{6}$$

We introduce now the following non-dimensional variables (Salleh *et al*. [9], Nazar *et al*. [7, 8] and Aziz [12]:

$$x = \frac{\bar{x}}{a}, y = \mathrm{Re}^{1/2}\left(\frac{\bar{y}}{a}\right), r(x) = \frac{\bar{r}(\bar{x})}{a},$$

$$u = \frac{\bar{u}}{U_\infty}, v = \mathrm{Re}^{1/2}\left(\frac{\bar{v}}{U_\infty}\right), H = \left(\frac{a}{U_\infty}\right)\mathrm{Re}^{1/2}\,\bar{H}, \theta = \frac{T - T_\infty}{T_f - T_\infty} \tag{7}$$

where $\mathrm{Re} = U_\infty \frac{a}{v},$ is the Reynolds number. Substituting variables (7) into equations (1) to (4) then become.

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial y}(rv) = 0 \tag{8}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{du_e}{dx} + (1+K)\frac{\partial^2 u}{\partial y^2} + \lambda\theta\sin x + K\frac{\partial H}{\partial y} \tag{9}$$

$$u\frac{\partial H}{\partial x} + v\frac{\partial H}{\partial y} = -K\left(2H + \frac{\partial u}{\partial y}\right) + \left(1 + \frac{K}{2}\right)\frac{\partial^2 H}{\partial y^2}, \tag{10}$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{\mathrm{Pr}}\frac{\partial^2 \theta}{\partial y^2} \tag{11}$$

The boundary conditions (5) become.

$$u = v = 0, \text{ or } \frac{\partial \theta}{\partial y} = -\gamma(1-\theta)\ H = -\frac{1}{2}\frac{\partial u}{\partial y} \text{ at } y = 0$$

$$u_e(x) \to \frac{3}{2}\sin x, \theta \to 0, H \to 0 \text{ as } y \to \infty \tag{12}$$

where Pr is the Prandtl number and $\gamma = ah_f Gr^{-1/4}/k$ is the conjugate parameter for the convective boundary conditions, $K = \kappa/\mu$ is the material or micropolar parameter and λ is the mixed convection parameter which is given by:

$$\lambda = \frac{Gr}{\mathrm{Re}^2}, \tag{13}$$

with $Gr = g\beta(T_f - T_\infty)\ a^3\ /\ v^2$ is the Grashof number for convective boundary conditions. It is noticed that, if $\gamma \to \infty$ then we have $\theta(0) = 1$, which is the constant wall temperature and this case has been studied by Nazar *et al*. [7]. Also worth mentioning that $\lambda > 0$ corresponds to the assisting flow (heated sphere), $\lambda < 0$ corresponds to the opposing flow (cooled sphere) and $\lambda = 0$ corresponds to the forced convection flow.

To solve the system of equations (8) to (11), subjected to the boundary conditions (12), we assume the following variables:

$$\psi = xr(x)f(x, y),\ \theta = \theta(x, y),\ H - xh(x, y), \tag{14}$$

where ψ is the stream function defined as.

$$u = \frac{1}{r}\frac{\partial \psi}{\partial y} \text{ and } v = -\frac{1}{r}\frac{\partial \psi}{\partial x}, \tag{15}$$

which satisfies the continuity equation (8). Thus, equations (9) to (11) become.

$$(1+K)\frac{\partial^3 f}{\partial y^3} + (1 + x\cot x)f\frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial f}{\partial y}\right)^2 + \lambda\frac{\sin x}{x}\theta + \frac{9}{4}\frac{\sin x\cos x}{x} + K\frac{\partial h}{\partial y} = x\left(\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x \partial y} - \frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}\right) \tag{16}$$

$$\left(1 + \frac{K}{2}\right)\frac{\partial^2 h}{\partial y^2} + (1 + x\cot x)f\frac{\partial h}{\partial y} - \frac{\partial f}{\partial y}h - K\left(2h + \frac{\partial^2 f}{\partial y^2}\right) = x\left(\frac{\partial f}{\partial y}\frac{\partial h}{\partial x} - \frac{\partial f}{\partial x}\frac{\partial h}{\partial y}\right) \tag{17}$$

$$\frac{1}{\mathrm{Pr}}\frac{\partial^2 \theta}{\partial y^2} + (1 + x\cot x)f\frac{\partial \theta}{\partial y} = x\left(\frac{\partial f}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial f}{\partial x}\frac{\partial \theta}{\partial y}\right) \tag{18}$$

Subject to the boundary conditions.

$$f = \frac{\partial f}{\partial y} = 0, \frac{\partial \theta}{\partial y} = -\gamma(1-\theta) h = -\frac{1}{2}\frac{\partial^2 f}{\partial y^2} \text{ at } y = 0$$
$$\frac{\partial f}{\partial y} \to \frac{3}{2}\frac{\sin x}{x}, \theta \to 0, h \to 0 \text{ as } y \to \infty \tag{19}$$

It can be seen that at the lower stagnation point of the sphere ($x \approx 0$), equations (16) and (18) reduce to the following ordinary differential equations:

$$f''' + 2ff'' - f'^2 + \lambda\theta + \frac{9}{4} = 0 \tag{20}$$

$$\left(1 + \frac{K}{2}\right)h'' + 2fh' - f'h - K(2h + f'') = 0 \tag{21}$$

$$\frac{1}{\mathrm{Pr}}\theta'' + 2f\theta' = 0 \tag{22}$$

and the boundary conditions (19) become.

$$f(0) = f'(0) = 0, \text{ or } \theta'(0) = -\gamma(1-\theta(0))h(0) = -\frac{1}{2}f''(0),$$

$$f' \to \frac{3}{2}, \theta \to 0, h \to 0 \text{ as } y \to \infty \qquad (23)$$

where primes denote differentiation with respect to y.

The physical quantities of interest in this problem are the local skin friction coefficient, C_f and the local heat transfer coefficient, Q_w (x) which are defend by.

$$C_f = \frac{a}{U_\infty}\mathrm{Re}^{-1/2}\left[(\mu+\kappa)\frac{\partial \bar{u}}{\partial \bar{y}} + \kappa H\right]_{\bar{y}=0} \quad Q_w(x) = \frac{a}{k(T_f - T_\infty)}\mathrm{Re}^{-1/2}\left(\frac{\partial \bar{T}}{\partial \bar{y}}\right)_{\bar{y}=0} \qquad (24)$$

Using the non-dimensional variables (7), we have.

$$C_f = \left(1+\frac{K}{2}\right)x\frac{\partial^2 f}{\partial y^2}(x,0), \text{ and } Q_w(x) = \gamma(1-\theta(x,0)) \text{ (CBC)} \qquad (25)$$

RESULTS AND DISCUSSION

The nonlinear partial differential equations (16) to (18) subject to the boundary conditions (19) were solved numerically using an efficient, implicit finite-difference method known as the Keller-box scheme for both cases of Newtonian heating and convective boundary conditions with several parameters considered, namely the mixed convection parameter λ, the material or micropolar parameter K, the Prandtl number Pr, the conjugate parameter γ and the coordinate running along the surface of the sphere, x. The numerical solutions start at the lower stagnation point of the sphere, ($x \approx 0$), with initial profiles as given by the nonlinear ordinary differential equations (20) to (22) subject to the boundary conditions (23) and proceed round the sphere up to 120° (Nazar *et al*. [7, 8]). The values of the skin friction coefficient C_f and the local heat transfer coefficient $Q_w(x)$ have been obtained at different positions x with various of the conjugate parameter γ mixed convection parameter λ which aiding flow (heated sphere) and opposing flow (cooled sphere) and for the values of Prandtl number considered are Pr = 0.7 and 7 which correspond to air and water, respectively.

The values of the heat transfer coefficient $-(\partial\theta/\partial_y)$ and the skin friction coefficient ($\partial^2 f/\partial y^2$ for various values of λ when Pr = 0.7, K = 1 and $\gamma \to \infty$ are presented in Table 1. Some numerical results obtained by an implicit finite-difference scheme as reported by Nazar *et al*. [7] for the case of constant surface temperature, are also included in these table for comparison purposes. It is found that the agreement between the previously published results with the present ones is excellent. We can conclude that this numerical method works efficiently for the present problem and we are also confident that the results presented here are accurate.

Tables 2 to 5 shown the values of the local heat transfer coefficient $Q_w(x)$ and the local skin friction coefficient C_f at the different positions x for various values of λ when Pr = 0.7, K = 1, 3 and γ = 0.5, respectively. It is found that the local heat transfer coefficient the local skin friction coefficient increases as the mixed convection λ increase. Also for fixed x and λ as the values of micropolar parameter K increases from 1 to 2, this results in an increases of the value of the local skin friction coefficient, as well as an decrease in the value of values of the local heat transfer coefficient. Moreover, the numerical solutions indicate that the value of λ which first gives no separation lies between 2.31 and 2.32 for K = 1 and between 2.55 and 2.54 for K = 3.

Figures 1 to 2 present the variation of the local heat transfer coefficient $Q_w(x)$ and local skin friction coefficient C_f with x when Pr = 0.7, K = 1, 3, λ = *1* and various values

Table 1: Values of the heat transfer coefficient $-(\partial\theta/\partial y)$ and the skin friction coefficient ($\partial^2 f/\partial y^2$) for various values of λ when Pr = 7, K = 1 and $\gamma \to \infty$

	$-(\partial\theta/\partial y)$		$(\partial^2 f/\partial y^2)$	
λ	Nazar *et al*. [7]	Present	Nazar *et al*. [7]	Present
-3	0.6770	0.677045	0.9383	0.938298
-2	0.7064	0.706388	1.2113	1.211342
-1	0.7312	0.731223	1.4617	1.461763
-0.5	0.7427	0.742699	1.5840	1.584003
1	0.7745	0.774456	1.9444	1.944369
2	0.7935	0.793543	2.1750	2.174998
3	0.8109	0.810897	2.3976	2.397570
4	0.8271	0.827102	2.6134	2.613378
5	0.8425	0.842549	2.8271	2.827117

Table 2: Values of the local heat transfer coefficient $Q^w(x)$ at the different positions x for various values of λ when Pr = 0.7, K = 1 and γ = 0.5

λ								
x	-5.4	-3	-1	1	2.31	2.32	3	5
0°	0.458238	0.460309	0.469099	0.473775	0.477636	0.478117	0.480286	0.489619
10°	0.453009	0.459147	0.465056	0.471706	0.475302	0.476477	0.479015	0.486633
20°		0.457187	0.462258	0.468964	0.472645	0.473865	0.477146	0.485845
30°		0.453200	0.456607	0.462923	0.466412	0.467575	0.472974	0.481894
40°			0.448586	0.454249	0.457405	0.458462	0.466287	0.475549
50°			0.438349	0.443087	0.445760	0.446662	0.456310	0.466969
60°			0.426961	0.429974	0.432005	0.432697	0.443177	0.456490
70°				0.414318	0.415470	0.415868	0.425321	0.443429
80°				0.396785	0.396816	0.396823	0.404084	0.427992
90°				0.375575	0.376045	0.376159	0.384742	0.410196
100°					0.354905	0.355072	0.369892	0.392368
110°					0.335943	0.336100	0.356285	0.378270
120°						0.319876	0.349935	0.364816

Table 3: Values of the local skin friction coefficient, C_f at the different positions x for various values of λ when Pr = 0.7, K = 1 and γ = 0.5

λ								
x	-5.4	-3	-1	1	2.31	2.32	3	5
0°	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
10°	0.434092	0.452117	0.471650	0.496164	0.510519	0.515420	0.526340	0.562336
20°		0.879233	0.916987	0.963392	0.990862	1.000299	1.022354	1.094929
30°		1.260321	1.312772	1.374291	1.411092	1.423795	1.458550	1.562350
40°			1.639467	1.706404	1.747112	1.761265	1.808742	1.937025
50°			1.883386	1.943530	1.981198	1.994455	2.050702	2.197661
60°			2.034285	2.075047	2.101905	2.111604	2.169688	2.329338
70°				2.109682	2.116533	2.119463	2.164476	2.335975
80°				2.023588	2.031492	2.044413	2.053081	2.218346
90°				1.830014	1.851889	1.872065	1.934970	1.996813
100°					1.538420	1.555003	1.678325	1.708946
110°					1.389932	1.416533	1.461896	1.507554
120°						1.224859	1.271793	1.360478

Table 4: Values of the local heat transfer coefficient $Q_w(x)$ at the different positions x for various values of λ when Pr = 0.7, K = 2 and γ = 0.5

λ								
x	-5.9	-3	-1	1	2.54	2.55	3	5
0°	0.450823	0.455644	0.456113	0.460606	0.466133	0.466878	0.476956	0.477247
10°	0.445287	0.450599	0.454811	0.459484	0.462772	0.463417	0.464603	0.470124
20°		0.448208	0.452363	0.456998	0.460291	0.460947	0.462152	0.468877
30°		0.443868	0.447490	0.451775	0.454830	0.455442	0.456740	0.465119
40°		0.438636	0.440649	0.444392	0.447071	0.447609	0.448794	0.459347
50°			0.432032	0.435053	0.437222	0.437659	0.438615	0.451744
60°			0.422164	0.424313	0.425857	0.426168	0.426854	0.442700
70°				0.411833	0.412604	0.412759	0.413126	0.431795
80°				0.387110	0.398013	0.398188	0.398201	0.419466
90°				0.372967	0.383083	0.381073	0.382724	0.406045
100°					0.365929	0.366155	0.367487	0.391979
110°					0.355189	0.352506	0.353630	0.377309
120°						0.340081	0.340332	0.360752

Table 5: Values of the local skin friction coefficient, C_f at the different positions x for various values of λ when Pr = 0.7, K = 2 and γ = 0.5

λ								
x	-5.9	-3	-1	1	2.3	2.55	3	5
0°	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
10°	0.515240	0.533663	0.549936	0.569725	0.584709	0.587790	0.593512	0.621698
20°		1.043496	1.073712	1.110818	1.139137	1.144990	1.155878	1.212544
30°		1.508721	1.548308	1.596747	1.633995	1.641723	1.656520	1.739450
40°		1.909917	1.954886	2.006314	2.046340	2.054689	2.070775	2.176424
50°			2.280983	2.325340	2.360633	2.368063	2.382437	2.505358
60°			2.516202	2.543347	2.566189	2.571101	2.580767	2.713207
70°				2.677421	2.679633	2.680316	2.682070	2.814623
80°				2.704267	2.710539	2.714652	2.735972	2.816703
90°				2.700097	2.703001	2.707898	2.721561	2.747396
100°					2.634895	2.617974	2.586765	2.645973
110°					2.601425	2.580291	2.541319	2.549289
120°						2.584262	2.541351	2.486423

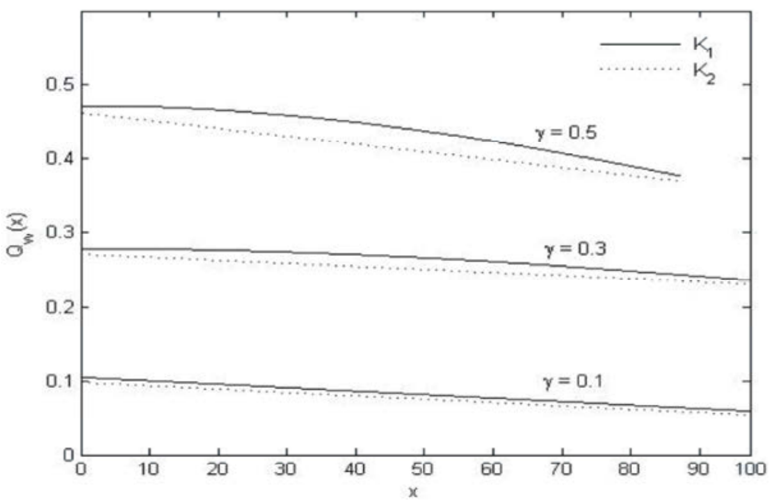

Fig. 1: Variation of the local heat transfer coefficient $Q_w(x)$ with x when Pr = 0.7, K = 1, 3, λ = 1 and various values of iγ

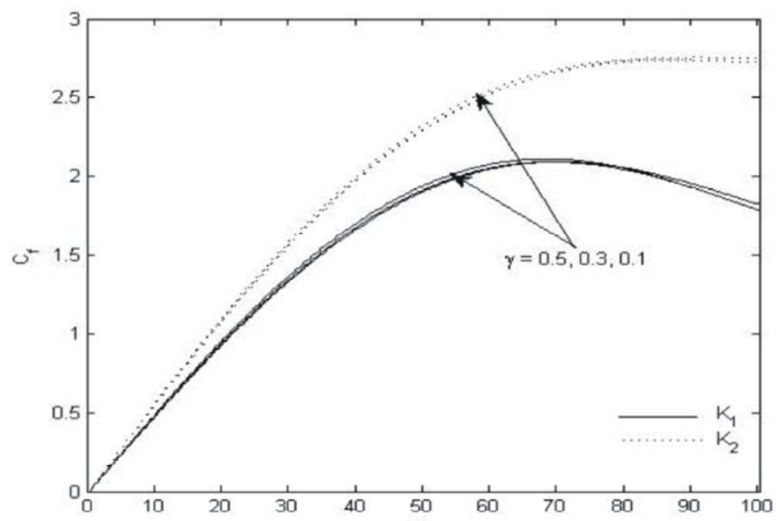

Fig. 2: Variation of the local skin friction coefficient, C_f with x when Pr = 0.7, K = 1, 3, λ = 1 and various values of γ

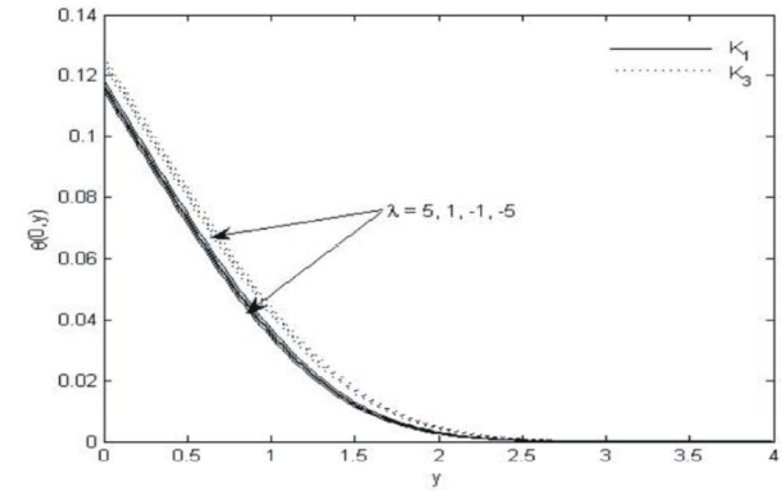

Fig. 3: Temperature profiles $\theta(0, y)$ for various values of λ when Pr = 0.7, K = 1, 3 and γ = 0.1

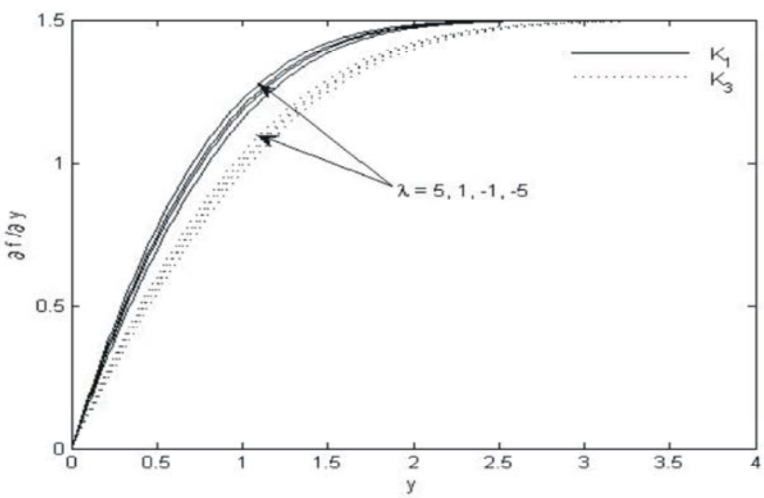

Fig. 4: Velocity profiles $(\partial f/\partial y)(0, y)$, for various values of λ when Pr = 0.7, K = 1, 3 and γ = 0.1

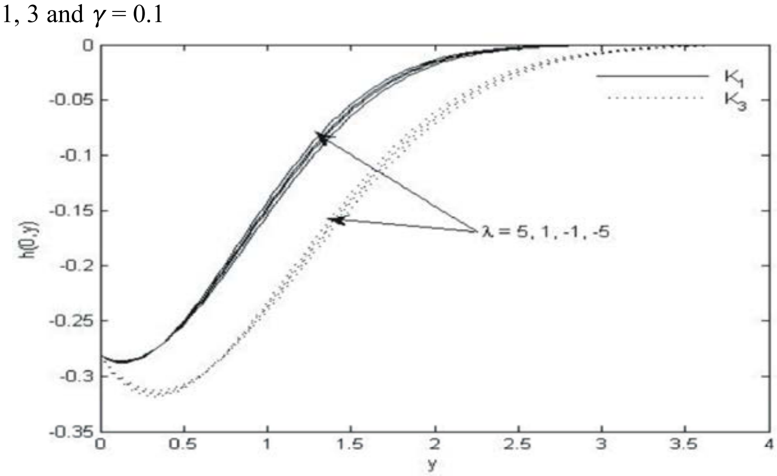

Fig. 5: Angular velocity profiles $h(0, y)$ for various values of λ when Pr = 0.7, K = 1, 3 and γ = 0.1

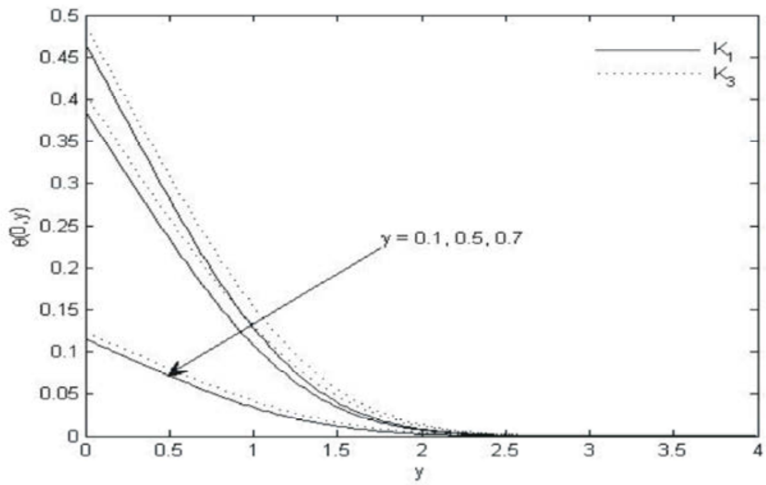

Fig. 6: Temperature profiles $\theta(0,y)$ for various values of γ when $\lambda = 5$ $K = 1$, 3and Pr = 0.7

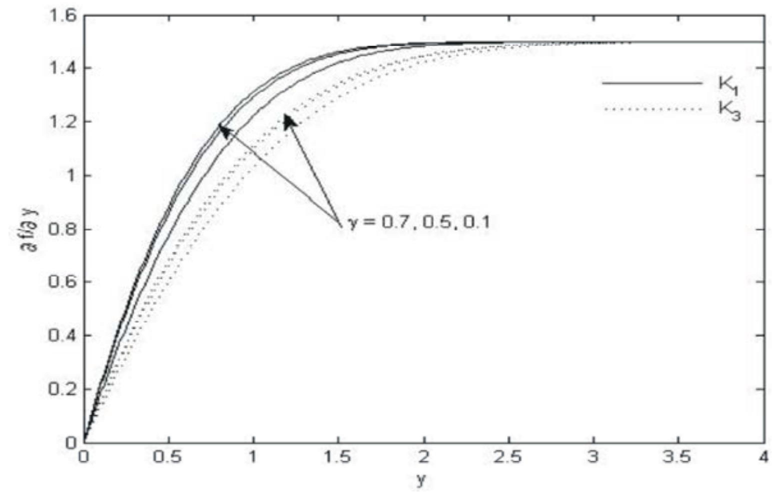

Fig. 7: Velocity profiles $(\partial f / \partial y)(0,y)$ for various values of γ when $\lambda = 5$ $K = 1$, 3 and Pr = 0.7

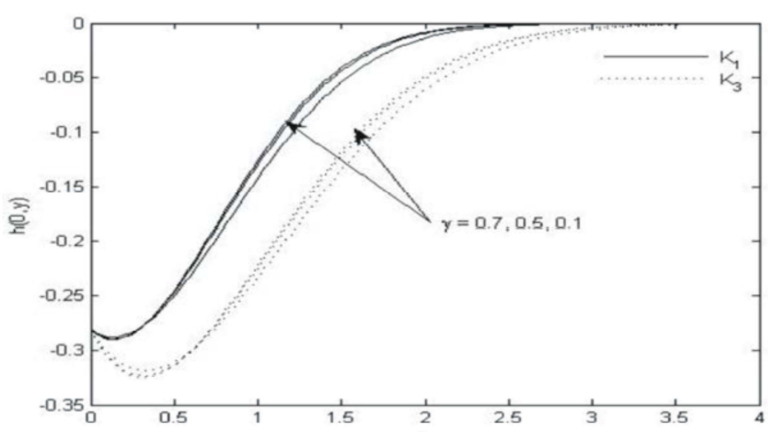

Fig. 8: Angular velocity profiles $h(0, y)$ for various values of γ when $\lambda = 5$ $K = 1$, 3 and Pr = 0.7

of the conjugate parameter γ respectively. It can be seen that, the values of local heat transfer coefficient and local skin friction coefficient increases with the increase of the conjugate parameter γ.

The temperature, velocity and angular velocity profiles are plotted in Figures 3 to 5 for some values of λ when Pr = 0.7, K = 1, 3 and γ = 0.5, respectively. We found that for fixed values of K, the velocity and angular velocity profiles increase, while the temperature profiles decrease when the mixed convection parameter λ increases and when fixed λ as micropolar parameters K increases the value of temperature profiles increase, but the velocity and angular velocity profiles decrease.

Figures 6 to 8 display the temperature, velocity and angular velocity profiles for some values of γ when λ = 5 K = 1, 3 and Pr = 0.7, respectively. It is found that for fixed values of K and λ the values of temperature, velocity and angular velocity profiles increases, when the conjugate parameter γ increase.

CONCLUSIONS

In this chapter, we have numerically studied the problem of mixed convection boundary layer flow over a sphere with convective boundary conditions in micropolar fluid, using the Keller-box method. It is shown how the mixed convection parameter λ, the micropolar parameter K and the conjugate parameter γ, affects on the local skin friction coefficient C_f local heat transfer coefficient $Q_w(x)$ the temperature $\theta(0, y)$ velocity $(\partial f/\partial y)(0,y)$ and angular velocity profiles $h(0, y)$ We can conclude that:

- When the conjugate parameter increases the values of local heat transfer coefficient and local skin friction coefficient increases. However, as the mixed convection λ increase the values of the local heat transfer coefficient and the local skin friction coefficient increases.
- The temperature profiles increase but the velocity and angular velocity profiles decrease when the mixed convection parameter λ decreases and also when K increases the value of temperature profiles increase, while the velocity and angular velocity profiles decrease.
- An increases in the values of the conjugate parameter γ leads to increases of the temperature, velocity and angular velocity profiles.

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