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Fault Detection and Diagnosis in Nonlinear System Using Multi Model Adaptive Approach

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Abstract: Fault Detection and Diagnosis (FDD) using Linear Kalman Filter (LKF) is not sufficient for effective monitoring of nonlinear processes. Most of the chemical processes are nonlinear in nature while operating the process in wide range of process variables. In this paper we present an approach for designing of Multi Model Adaptive Linear Kalman Filter (MMALKF) for Fault Detection and Isolation (FDI) of a nonlinear system. The MMALKF uses a bank of Adaptive Linear Kalman Filters (ALKFs), with each ALKF based on different fault hypothesis. The effectiveness of the MMALKF has been demonstrated on Continuously Stirred Tank Reactor (CSTR) system. The proposed method is detecting and diagnosing the sensor and actuator soft faults which may occur either sequentially or simultaneously.

Key words: Multi Model Adaptive Linear Kalman Filter · Fault Detection and Isolation · Continuously Stirred Tank Reactor · State estimation · Residual generation

INTRODUCTION

Sensors and actuators are playing major role in generating controller output and implementing the control action. Malfunction may occur either in plant, sensors or in the actuators. The controllers are developed by assuming sensors give exact view of the process and the actuators are implementing the controller output exactly. If bias is present either in the actuator or in the sensor even though control algorithm is advanced one this may lead to improper control of the process loops. It will affect the product quality,economy, safety of the plant and also affects the atmosphere. So, detecting and diagnosing the soft failure is essential. The Fault Detection and Diagnosis (FDD) algorithm consists of making binary decision whether a fault has occurred or not, if fault has occurred isolating the faulty component and estimating the magnitude and time of occurance of fault.

Most of the FDD approaches use analytical redundancy. Faults are detected and diagnosed by comparing the noisy sensor output and expected output based on plant model [2-4]. The system considered here is a stochastic time invarient process and the expected output is generated by statistical filter. The difference between the process and the estimator output is error and called residuals, which are used to detect and diagnose different kinds of faults. This residual is also used to find the time of occurrence of faults.

This paper uses Linear Kalman Filter to estimate nonlinear system states. Most of the chemical processes are highly nonlinear in nature while operating the process in wide range of process variables. Accurate estimation of states is important for fault detection and control purposes. The widely used estimation technique for nonlinear system is Extended Kalman Filter (EKF). EKF linearizes all nonlinear transformations and substitutes Jacobian matrices in the KF equations. Linearization is reliable only if the error propagation is well approximated by linear transformation and for some nonlinear systems Jacobian matrix may not exists. Nonlinear estimation methods are computationally complex and most of the existing fault detection algorithms are designed for sequential faults not for simultaneous faults.

The aim of the present work is to develop a MMALKF, which uses multiple ALKFs each with different hypothesis [1]. First the nonlinear model is linearized around different operating points, then the LKFs (state estimators) are designed for each local linear model and the LKFs are fused using gain scheduling technique to get the Adaptive Linear Kalman Filter[8-10].

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The ALKFs has multiple models because each of which is designed for detecting specific sensor or actuator faults. The proposed technique will detect the faults which may occur sequentially as well as simultaneously and the time of occurrence of fault. The paper is organized in seven sections. The following sections deal with the design of fused linear model and ALKF respectively. Design of MMALKF is presented in Section 4. The process used for simulation is presented in section 5. The simulation results are discussed in section 6. The conclusions reached from the results are given in section 7.

Fused Linear Model: Let us consider a nonlinear stochastic system represented by the following state and output equations:

$$x_k = f(x_{k-1}, u_k, w_{k-1}) \tag{1}$$

$$y_k = h(x_k, \upsilon_k) \tag{2}$$

The nonlinear system is linearized around different operating points using Taylor series expansion. The linear system around operating points $(\bar{x}_i, \bar{u}_i)$ s given as follows,

$$x_i(k) = \Phi_i(x(k-1) - \bar{x}_i) + \Gamma_{ui}(u(k-1) - \bar{u}_i + \Gamma_n w(k) \tag{3}$$

$$y_i(k) = C_i x_i(k) + \upsilon_i(k) \tag{4}$$

Where $x \in R^n$ represents state variables, $u \in R^m$ represents inputs, $y \in R^r$ represents measured output and $w \in R^q and \upsilon \in R^r$ represents state and measurement noise respectively. $w(k)$and $u(k)$ are assumed to be Gaussian noises with covariance matrices Q and R respectively. Φ_i, $\Gamma_{ui}$and Γ_n and C_i are known time invariant matrices of appropriate size. The nonlinear system is represented by a fused linear model using gain scheduling technique at a given operating point. For a given input vector , $u(k)$ the state and output of fused linear model is represented as follows:

$$x(k) = \sum_{i=1}^{N} g_i \left\{ \begin{array}{l} \Phi_i(x(k-1) - \bar{x}i) \\ \quad + \Gamma_i(u(k-1) - \bar{u}i) + \bar{x}i \end{array} \right\} \tag{5}$$

$$y(k) = Cx(k) \tag{6}$$

To cover the entire operating horizon, five operating points has been selected (i=1 to 5). Let , y_m is the actual value of the measured process variable at current sampling instant and gi is the weighting factor [8-10].

$$\mathit{If}\ \ (y_m \geq y_5\ ), then\ g_1 = g_2 = g_3 = g_4 = 0\ , g_5 = 1 \tag{7}$$

$$\mathit{If}\ \ (y_4 < y_m \leq y_5\ ), then$$
$$g_1 = g_2 = g3 = 0, g_4 = \frac{y_m - y_4}{y_5 - y_4}, g_5 = 1 - g_4 \tag{8}$$

$$\mathit{If}\ \ (y_3 < y_m \leq y_4\ ), then$$
$$g_1 = g_2 = 0, g_3 = \frac{y_m - y_3}{y_4 - y_3}, g_4 = 1 - g_3, g_5 = 0 \tag{9}$$

$$\mathit{If}\ \ (y_2 < y_m \leq y_3\ ), then$$
$$g_1 = 0, g_2 = \frac{y_m - y_2}{y_3 - y_2}, g_3 = 1 - g_2, g_4 = g_5 = 0 \tag{10}$$

$$\mathit{If}\ \ (y_1 < y_m \leq y_2\ ), then$$
$$g_1 = \frac{y_m - y_1}{y_2 - y_1}, g_2 = 1 - g_1, g_3 = g_4 = g_5 = 0 \tag{11}$$

$$\mathit{If}\ (y_m \leq y_1), then\ g_1 = 1, g_2 = g_3 = g_4 = g_5 = 0 \tag{12}$$

The weighting factors are in the range of [0 1].

Adaptive Linear Kalman Filter: For the nonlinear model a ALKF can be designed to estimate the system states. This approach consists of family of local linear estimators and a scheduler. At each sampling instant the scheduler will assign weights (gain scheduling) for each linear local estimator and the weighted sum of the outputs will be the estimate of the current state. The scheduler assigns weight based on scheduling variable. The scheduling variable may be input variable or state variable or some auxiliary variable, the scheduling variable considered here is coolant flow rate of the process.

The LKF is designed for each local linear model using kalman filter theory as follows:

$$\hat{x}(k|k-1) = \Phi_i(x(k-1|k-1) - \bar{x}i) + \Gamma_i(u(k-1) - \bar{u}i) \tag{13}$$

$$\hat{y}_i(k|k-1) = C_i \hat{x}_i\ (k-1) \tag{14}$$

$$\hat{x}_i(k|k) = \hat{x}_i(k|k-1) + K_i\ (k)[(y(k) - \bar{y}_i) - \hat{y}_i\ (k|k-1)] \tag{15}$$

Where $k_i(k)$ represents Kalman gain matrix, $\hat{x}_i(k|k-1)$ represents predicted state estimates and $\hat{x}_i(k|k)$ represents corrected state estimates. The Kalman gain matrix can be calculated from the following equations.

$$P_i(k|k-1) = F_i P_i(k-1|k-1) + F_i^T + G_{ni} Q G_{ni}^T \quad (16)$$

$$V_i(k) = C_i(k) P_i(k|k-1) C_i^T + R \quad (17)$$

$$K_i(k) = P_i(k|k-1) C_i^T V_i^{-1}(k) \quad (18)$$

$$P_i(k|k) = (I - K_i(k) C_i) P_i(k|k-1) \quad (19)$$

Where $P_i(k|k-1)$ and $P_i(k|k)$ are the covariance matrices of errors in predicted and corrected state estimates of ith local estimator, respectively.

The ALKF (global estimator) dynamics will be weighted sum of individual LKF and it is given below,

$$\hat{x}(k|k) = \sum_{i=1}^{N} g_i \begin{Bmatrix} \hat{x}_i(k|k-1) + \\ k_i(k)[(y(k) - \bar{y}_i) - \hat{y}_i(k|k-1)] \end{Bmatrix} \quad (20)$$

Multi Model Adaptive Linear Kalman Filter: This approach uses multiple ALKF. Each ALKF is designed based on specific hypothesis to detect a specific fault. The fault considered here is soft fault of fixed bias. The same approach can be used to detect dritf like (time varying) faults. This approach is capable of detecting multiple sequential as well as multiple simultaneous faults which occur either in sensors or in actuators.

If a bias of magnitude $B_{s,j}$ occurs at time t in the j[th] sensor, then the measurement equation is given by,

$$y(k) = Cx(k) + \upsilon(k) + B_{s,j} F_{y,j} \varphi(k-t) \quad (21)$$

Where $F_{y,j}$ is a sensor fault vector with j[th] element equal to unity and other elements equal to zero.

$$\varphi(k-t) = \begin{Bmatrix} 0 & if\ k < t \\ 1 & if\ k > t \end{Bmatrix} \quad (22)$$

If a bias of magnitude $B_{a,j}$ occurs in the j[th] actuator at time t then the state equation is given by,

$$\begin{aligned} x(k+1) &= \Phi x(k) + \Gamma_{ui}(u(k) \\ &+ B_{a,j} F_{n,j} \varphi(k-t)) + \Gamma_n w(k) \end{aligned} \quad (23)$$

Where $F_{u,j}$ is an actuator fault vactor with jth element equal to one and other elements equal to zero [5-7].

All the ALKF except the one using correct hypothesis will produce large estimation error. By monitoring the residuals of each ALKF, the faulty element(sensor or actuator) can be detected and isolated. Similarly we can model faults due to unmeasured disturbances and parameter changes. We can model these because the process dynamics are derived using first principles.

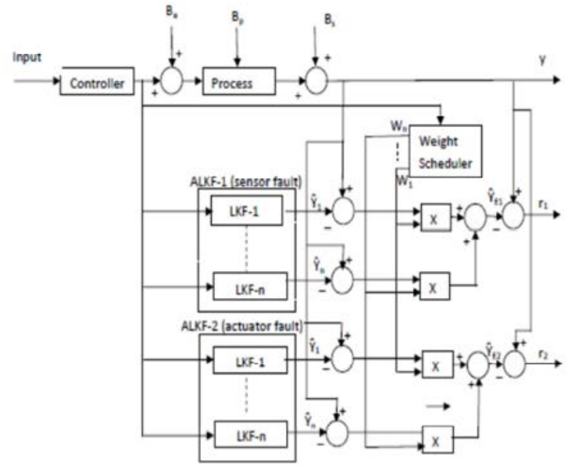

Fig. 1: Structure of the proposed MMALKF

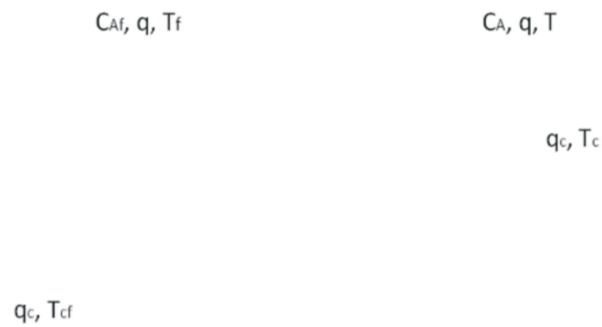

Fig. 2: (Schematic of CSTR)

The proposed MMALKF scheme is given in Fig. 1. Each ALKF consists of five LKFs developed at 5 different operating points. The weights are calculated by using coolant flow rate of the process as scheduling variable. The LKF outputs are weighted and added to get the global output estimate $(\hat{y})$. The process output is compared with the ALKF output to generate residuals. Under fault free condition the magnitude of the residuals are maximum. If fault occurs in any of the sensor or actuator, all the estimators except the one using the correct hypothesis will produce large estimation error.

If the ALKF is designed for 1% error and the error occurred is less than or above 1%, then the residual generated will be different from the one during the normal operating condition. By closely observing the innovations, the faults which occurs either sequentially or simultaneously can be isolated and the time of occurance can also be detected.

Table 1: Nominal operating condition for CSTR

Process variable	Normal Value
Tank volume (V)	100 L
Feed flow rate (q)	100.0 L/ min
Feed concentration (C_{Af})	1 mol/ L
Feed temperature (T_f)	350.0 K
Coolant flow rate (q_c)	103 L/ min
Inlet coolant temperature (T_{cf})	350.0 K
Liquid density (ρ, ρc)	1 * 10^3 g/L
Specific heats(Cp, C_{pc})	1 cal/(g k)
Reaction rate constant (k_0)	7.2 * 10^{10} min^{-1}
Activation energy term (E/R)	1 * 104 K
Heat of reaction (-ΔH)	-2 * 10^5 cal/ mol
Heat transfer term (hA)	7 * 10^5 cal/ (min k)
product concentration (C_A)	0.0989 mol/ L
Reactor temperature (T)	438.7763 K

Continuously Stirred Tank Reactor: Simulated CSTR process was considered to test the efficacy of the proposed method. The schematic of the system is shown in Fig 2. An irreversible exothermic reaction A → B occurs in a constant-volume reactor that is cooled by a single coolant stream. The two state variables of the process are concentration and temperature. The first principle model of the system is given by the following equations.

$$\frac{dC_A(t)}{dt}=\frac{q(t)}{V}(C_{A0}(t)-C_A(t))-k_0C_A(t)\exp\left(\frac{-E}{RT(t)}\right) \quad (24)$$

$$\frac{dT(t)}{dt}=\frac{q(t)}{V}(T_0(t)-T(t))-\frac{(-\Delta H)k_0C_A(t)}{\rho C_p}\exp\left(\frac{-E}{RT(t)}\right)+\frac{\rho_c C_{pc}}{\rho C_p}q_c(t)\left\{1-\exp\left(\frac{-hA}{q_c(t)\rho C_p}\right)\right\}T_{c0}(t)-T(t) \quad (25)$$

The steady state operating point data used in the simulation studies is given in Table 1 [12].

The continuous linear state space model is obtained by linearizing the differential equations (24) and (25) around nominal operating point and. The state vector is and the input vector is.

RESULTS

The CSTR process is simulated using first principles model as given in (24) and (25) and the true state variables are computed by solving the nonlinear differential equations using Matlab 7.1. The dynamic behavior of the CSTR process is not same at different operating points and the process is nonlinear. This can be verified from damping factor and natural frequency obtained at different operating points given in Table 2.

Table 2: Damping factor and Natural frequency at different operating points

Operating Points			Damping Factor	Natural Frequency (rad/sec)
q_c (l/min)	$\bar{C}_A$ (mol/l)	$\bar{T}$ (K)		
97	0.0795	443.4566	0.661	3.93
100	0.0885	441.1475	0.540	3.64
103	0.0989	438.7763	0.416	3.34
106	0.1110	436.3091	0.285	3.03
109	0.1254	433.6921	0.141	2.71

Table 3: Estimation Error

State Variable	RMSE
CA	2.0920*10^{-5}
T	0.0419

Fused Linear Model: To validate the performance of ALKF the process states are estimated and compared with the rigorous non-linear model. The process and measurement noise covariance are assumed to be 0.25% of coolant flow rate and 0.5% of state variables respectively. Fig.3 shows the variation in coolant flow rate introduced. Fig.4 and Fig. 5 shows the estimation of system states when the noises are uncorrelated It has been observed that the ALKF exactly estimates the system states without dynamic and steady state error. Fig.6 and Fig.7 shows the estimation error. Table. 3 shows the estimated RMSE of product concentration and reactor temperature.

Sensor and Actuator Bias Detection: Two estimators were designed to detect the biases in the CA sensor, T sensor and an actuator which controls . To detect sequential or simultaneous sensor and actuator faults using multi model approach, the first estimator is designed for sensor faults detection with -5% hypothesis and the second estimator is designed for actuator fault detection with 0% hypothesis. The magnitude of fault occurred is estimated from the magnitude of residual generated and the time of occurance of fault is the time at which the residual changes its trend and the fault is confirmed by comparing the mean of the residual over a period of time with the threshold value.

Table 4 shows the residual generated for different % of biases either in the sensors/actuator or in both the sensors and actuator at the same time. The bias was introduced at 50^{th} sampling instant. Fig. 8 & Fig.9 shows the residuals generated by estimator1 and Fig. 10 temperature residuals generated by estimator2 when both sensor and actuator faults are introduced simultaneously. To estimate actuator fault either the temperature or the concentration residual generated by estimator 2 can be

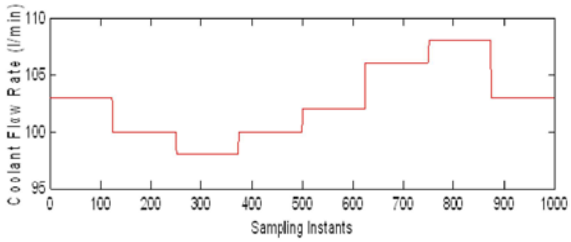

Fig. 3: Coolant flow rate (l/min)

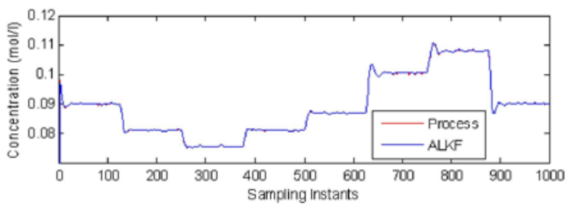

Fig. 4: Estimation of product concentration (mol/l)

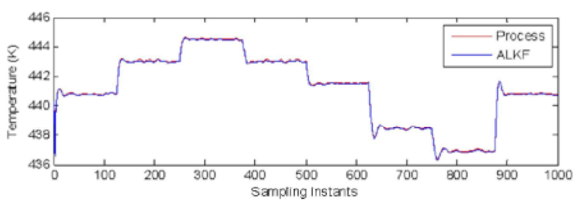

Fig. 5: Estimation of reactor temperature (K)

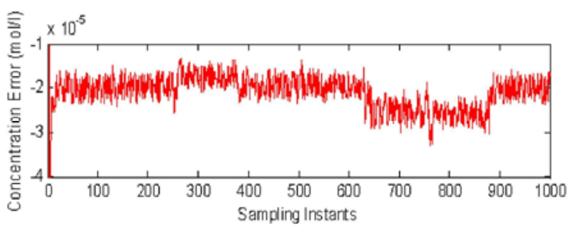

Fig 6: Product concentration error

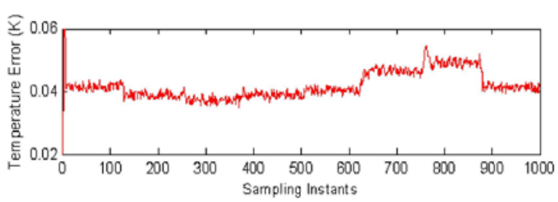

Fig. 7: Reactor temperature error

used. Here, temperature residual is considered for actuator fault diagnosis. Fig.11 shows the plot of Kalman gains, from this we can conclude it converges quickly.

Fig.12, Fig.13, Fig.14 shows the residual generated by estimator 1 and estimator 2 after introducing 2% bias in both sensor and actuator.

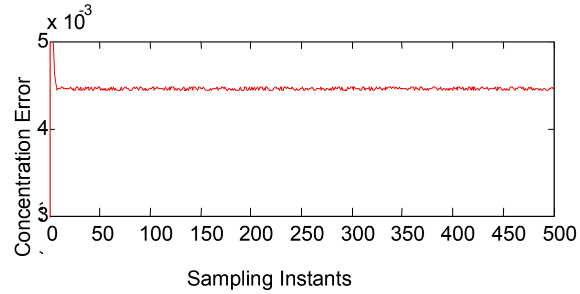

Fig. 8: Residual generated in concentration by Estimator1 when no bias is present

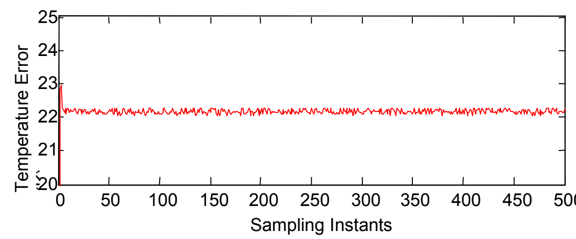

Fig. 9: Residual generated in temperature by Estimator1 when no bias is present

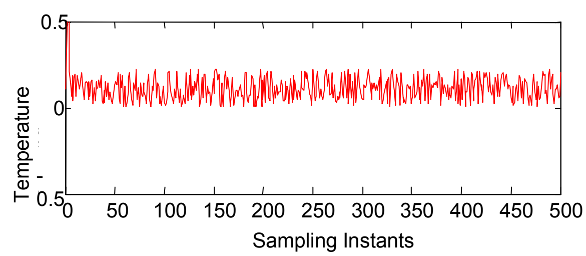

Fig. 11: Kalman Gains

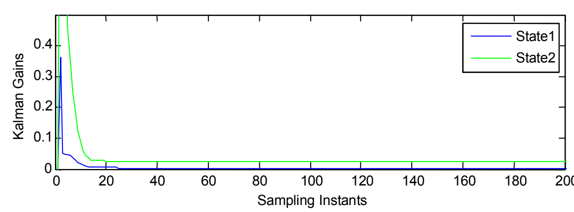

Fig. 12: Residual generated in concentration by Estimator1 When both sensor and actuator biases are present

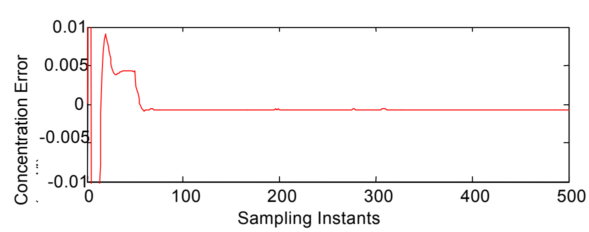

Fig. 13: Residual generated in temperature by Estimator1 When both sensor and actuator biases are present

Table 4: Sequential and simultaneous bias detection using MMALKF

% of bias introduced			Mean value of the residual generated: Estimator 1 for sensor bias detection (hypothesized with -5% bias)		Estimator 2 for actuator bias detection (hypothesized with 0% bias)
Sensor 1 (C_A in mol/l)	Sensor 2 (T in K)	Actuator (q_c in l/min)	State 1 - CA $(C_A - \hat{C}_A)$	State 2 - T $(T - \hat{T})$	State 2 - T $(T - \hat{T})$
0%	0%	0%	0.0045	22.1668	0.1094
1%	1%	0%	0.0031	17.8064	-4.2509
2%	2%	0%	0.0022	13.4075	-8.6498
3%	3%	0%	0.0013	8.9836	-13.0737
0%	0%	1%	0.0015	22.9369	0.8795
0%	0%	2%	0.0011	23.6430	1.5857
1%	1%	1%	$2.7356*10^{-4}$	18.5214	-3.5360
2%	2%	2%	-0.0032	14.7917	-7.2657

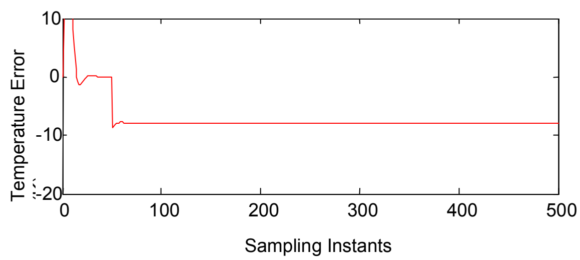

Fig. 14: Residual generated in temperature by Estimator 2

While testing the efficacy of the MMALKF the coolant flow rate is fixed at 100 L/min and corresponding steady state variables are [0.0885; 441.1475]. Estimator 1 is hypothesized with -5% sensor bias and estimator 2 is hypothesized with 0% actuator bias. Since, in the absence of bias, residual generated by estimator 1 is 5% of [0.0885; 441.1475] = [0.0044; 22.057].

When both sensor and actuator biases are present and estimator 2 is [0; 0]. From the Table 4 it is clear that, when only sensor bias is present the change in residual generated by estimator1 exactly shows the time of occurance and its magnitude. And the residual generated by estimator 2 is negative, this means that the Kalman filter relies less on measurement and more on system model. In the presence of actuator bias only the estimator 2 residuals exactly shows the magnitude and its time. When sensor and actuator bias are present the residuals generated by the estimator 1 and estimator 2 are indicative of its magnitude and time.

CONCLUSION

In this paper we have proposed MMALKF approach that includes adaptive gain scheduling algorithm along with the multiple linear kalman filters to detect and isolate multiple sensor and actuator faults which occurs sequentially and simultaneously. The efficiency of the proposed approach was tested through extensive simulation on CSTR process. The MMALKF can be used to develop a nonlinear model based FDI scheme for faults which occurs sequentially and simultaneously and fault tolerant control schemes. The proposed MMALKF performs better even in the presence of considerable amount of plant-model mismatch.

REFERENCES

1. Venkatasubramanian, V., R. Rengaswamy, K. Yin and S.N. Kavuri, 2003. A review of process fault detection and diagnosis, Part I: Quantitative model-based methods. Computers & Chemical Engineering, 27: 293-311.
2. Theilliol, D., H. Noura and J. Ch. Ponsart, 2002. Fault diagnosis and accommodation of a three-tank system based on analytical redundancy, ISA Transaction, 41: 365-382
3. Paul M. Frank, 1990 'Fault Diagnosis in Dynamic Systems Using Analytical and Knowledge-based Redundancy - A Survey and Some New Results', Automatica, 26(3): 459-474
4. Anjali P. Deshpande, Ujjwal Zamad and Sachin C. Patwardhan. Online Sensor/Actuator Failure Isolation and Reconfigurable Control Using the Generalized Likelihood Ratio Method. Ind. Eng. Chem. Res., 48(3): 1522-1535.
5. Senthil, R., K. Janarthanan and J. Prakash, Nonlinear State Estimation Using Fuzzy Kalman Filter. Ind. Eng. Chem. Res., 45(25): 8678 - 8688
6. Dan Simon, 2006. Optimal state estimation. John Wiley & sons, New Jersey.

7. Prakash, J., S.C. Patwardhan, S. Narasimhan, 2002. A supervisory approach to fault tolerant control of linear multivariable systems. Industrial Engineering Chemistry Research, 41: 2270-2281
8. Danielle, D. and D. Cooper, 2003. A Practical Multiple Model Adaptive Strategy for Multivariable Model Predictive Control. Control Engineering Practice, 11: 649-664.
9. Anjali P. Deshpande and Sachin C. Patwardhan, 2008. Online Fault Diagnosis in Nonlinear Systems Using the Multiple Operating Regime Approach. Industrial Engineering Chemistry Research, 47: 6711-6726
10. Vinodha, R., S. Abraham Lincoln and J. Prakash, 2010. Multiple Model and Neural based Adaptive Multi-loop PID Controller for a CSTR Process. World Academy of Science, Engineering and Technology, 68: 505-510