World Applied Sciences Journal 28 (12): 2120-2121, 2013
ISSN 1818-4952

DOI: 10.5829/idosi.wasj.2013.28.12.1207

Relation Between a Planar and Outerplanar of Graph with Sparse of Graph

B. Nikfarjam and M. Yunusi

Department of mathematics, Tajik National University

Abstract: A graph is a sparse if $\Delta_G(x) \geq 0$ where $\Delta_G(x) = k(|x|-1) - e_x$ for every set x of vertices of a graph and e_x is the number of edge joining pairs of element of x. in this paper we study relation between a planar and outerplanar of a graph with sparse of a graph and we show that G is decomposable into K forest.

Key words: Sparse · Outer planar

INTRODUCTION

Outer planar graph are a widely studied graph class with application in graph drawing [1,2] and with intersecting theoretical properties [3-5]. The maximum outer planar sub graph problem (MOPS) is NP-hard. The problem of finding a MPS has important application in circuit layout,automated graph drawing,and facility layout.the maximum planar sub graph problem (MPS) is too NP-hard. The problem of finding a MOPS are a widely studied graph with application in graph drawing and with intersecting theorectical properties.

A graph is planar if it admits a plane drawing, otherwise the graph is nonplanar. A graph is outerplanar if it admits a plane embedding where all its vertices surround the same region and no two distinct edge intersect, otherwise the graph is nonouterplanar [6].

Characterization of Outer Planar Graphs:

Def (2.1): If a graph G' = *(V, E') is* a outer planar sub graph of G such that every graph G'' obtained from G' by adding on edge from $E \setminus E'$ is non- outer planar, then G' is called a maximal outer planar sub graph of G.

Def (2.2): Let G' = (V, E') be a maximal outer planar sub graph of G. if there is no outer planar sub graph $G'' = (V, E'')$ *of G* with $\backslash E''\backslash > \backslash E'\backslash$, then G' is a maximum Outer planar sub graph.

Def (2.3): A graph H is said to be homeomorphic from G if either H $\cong$G or H is Isomorphic to a subdivision of G.

Theorem (2.1): A graph is outer planar if and only if it has no sub graph home orphic to K_4or $K_{2,\,3}$

Theorem (2.2): A graph is outer planar if and only if G + $K_1$is planar.

Theorem (2.3): [7] Let G' = (V, E") be a maximum outer planar sub graph of a Graph G = *(V, E)* then $\backslash E'\backslash = 2\backslash V\backslash - 3$.

Theorem (2.4): [7] Let G' = (V, E') be a maximum outer planar sub graph of a graph G = (V, E) which does not contain any triangle. Then \ E' \ = 3\V\/ $_2$ - 2

Theorem (2.5): [8] The maximum outer planar sub graph of $Q_n$contains 3× 2^{n-1} - 2 edges.

Theorectical Bounds:

Theorem (3.1): Let $G' = (V, E')$ be a maximum planar subgraph of a graph G=(V,E) which does not contain any triangle. Then $|E| \leq 2|V| - 4$

Proof. See [9]

Claim (3.1): Every planar graph G satisfies $|E(G)| < 2(|V|-1)$ if G has no triangle.

Proof: By according theorem(3.1),since $|E| \leq 2|V| - 4$ i.e

$|E(G)| \leq 2\,|V|/2-2$

Theorem (3.2): [9] let $|E(G)| < \frac{3}{2}(|V|-1)$ be a maximum outerplanar subgraph of a graph G=(V,E) which dies not contain any triangle.

Then $|E| \leq 2|V|/2 - 2$

Corresponding Author: B. Nikfarjam, Department of mathematics Tajik National University.

Claim (2.2): Every outerplanar graph G satisfies $|E(G)| < \frac{3}{2}(|V|-1)$ if G has no triangle.

Proof: By according theorem (3.2) since $|E(G)| < {}^{3|3|}\!/_{2} - 2)$ i.e

$$|E(G)| \leq {}^{3|V|}\!/_{2} - 2 = \frac{3|V|-4}{2} < \frac{3|V|-3}{2} < {}^{3}\!/_{2}(|V|-1) \cdot$$

Theorem 2.3: [10]a graph is decomposable into K forest if and only if it is sparse.

New Results for Sparse:

Claim (4.1): Let G be a planar graph which does not contain any triangle. Then G is sparse.

Proof: By claim(3.1) since every planar graph G satisfies

$|E(G)| \leq 2(|V(G)|$-1 then 2(|v|-1) -|E(G)|$\geq$ 0
i.e Δ(G) $\geq$ 0 then G is sparse.

Hence G is decomposable into K forest. (by theorem (2.3)).

Claim (4.2): Let G be a outerplanar graph which does not contain any triangle. Then G is sparse.

Proof: By claim (2.2) since every outerplanar graph G satisfies

$|E(G)| \leq {}^{3}\!/_{2}(|V|(G)-1)$ then ${}^{3}\!/_{2}(|V|(G)-1) - |E(G)| \geq 0$
i.e. $\Delta(G) \geq 0$Then G is sparse.

Hence G is decomposable into K forest.

CONCLUSION

In this article we present have concerning outer thickness of a graph is given. Since the thickness and outer thickness is one of the classical and standard measure of non- outer planarity and non- planarity of graph.in particular in this paper we study relation between a planar and outerplanar of a graph with sparse of a graph and we show that G is decomposable into K forest.

REFERENCES

1. Biedl, T., Drawing outer planer graphs in o (n logn) area. In graph drawing: 10th international symposion, GD 2002, volume 2528 of lecture rotes in computer Science, pp: 54-65.
2. Kuat., G., 1996. Augmenting outer plant graphs. Journal of Algorithms, 21: 1-25.
3. Fesner, S. and G. Liotta, S.W. is math.straight –line dreary on restricted inteye grids in tow and three dimensions. In graph drawing: 9th international symposium GD 2001, Volume 2265 of lecture notes in computer science, pp: 328- 342
4. Maheshwari, A. and N. Zeh, 1999. External / memory algorithms for outer planer graphs. In proceeding of 10th international symposium and computations, volume 1741 of Lecture Notes in Computer Science, pp: 307-316.
5. Manning, J. and M.J. Atalleh, 1992. Fast detection and display of symmetry in outer planer graphs. Discrete Applied Mathematics, 39(1): 13-35.
6. Poranen, T., 2004. Approximation Algorithms for Some Topological Invariants of Graphs.
7. Kedlaya, K.S., 1996. Outerplanar partition of planar graphs, pp: 238-248.
8. Guy, R.K. and R.J. Nowakowski, 1990. The outer thickness and outer coarseness of graphs II. The complete biapartite graph. In R.Bodendiek editor, contemporary methods in graph theory, 313 - 322. B.I. Wissenehaftsvelage 1990.
9. Harary, F., 1971. Graph theory. Addision-wesley.
10. Nikfarjam, B. and M. Yunusi, The integer programming formulation of the coarseness and thickness problem,Global Engineers and Technology Review, 2(3): 16-19.