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An Integrated Model for Optimization Oriented Decision Aiding and Rule Based Decision Making in Fuzzy Environment

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Abstract: In this paper a fuzzy decision aid system is developed base on new concepts that presented in the field of *fuzzy decision making in fuzzy environment (FDMFE)*. This framework aids decision makers to understand different circumstances of an uncertain problem that may occur in the future. Also, to keep decision maker from the optimization problem complexities, a decision support system, which can be replaced by optimization problem, is presented to make optimum or near optimum decisions without solving optimization problem directly. An application of developed decision aid model and decision support system is presented in the field of inventory models.

Key words: Fuzzy decision making in fuzzy environment · Optimum space · Decision support system · Fuzzy rule base

INTRODUCTION

Fuzzy decision makingcan be categorized into two subclasses as Deterministic Decision Making in Fuzzy Environment (DDMFE) andFuzzy Decision Making in Fuzzy Environment (FDMFE). Literature of the first subclass isricher than the otherone. The DDMFE models are completely reviewed by Rommelfanger [1], Inuiguchi and Ramik [2], Baykasogluand Gocken [3], Sahinidis [4] and Lai and Hwang [5]. Also, expected value, chance constrained and dependent-chance models are introduced by Liu [6, 7] as other approaches to deal with possibilistic mathematical programming. All of these models consider the coefficients of decision variables as fuzzy numbers whereas decision variables are crisp ones. This means that, in an uncertain environment, a crisp decision is made to meet some decision criteria.

Inuiguchi and Sakawa [8, 9] presented two different models to determine optimality degree of a feasible solution in single and multiple objective programming problems. Ramik and Vlach [10] presented a formulation of fuzzy mathematical programming problem (FMP) and then defined the concept of feasible solution and optimal solution of FMP. They showed that the feasible and optimal solutions of FMP areconvex fuzzy sets under some mild assumptions.Tanaka and Asai [11] proposed a possibilistic linear programming formulation where the coefficients of decision variables are crisp whereas decision variables are obtained as fuzzy numbers. Guo *et al*. [12] used linear programming (LP) and quadratic programming (QP) techniques to obtain fuzzy solutions. Tanaka *et al*. [13] dealt with the interactive case in which exponential distribution functions were used. In the other model, Tanaka *et al*. [14] took into consideration three kinds of possibility distribution for decision variables; interval possibility distribution, triangular possibility distribution and exponential possibility distribution. In their approach, possibility distribution of fuzzy parameters and each decision variable were considered as symmetric fuzzy numbers. Appadoo *et al*. [15] developed some concepts such as possibilistic mean, variance and covariance to a nonlinear type of fuzzy numbers called adaptive fuzzy numbers and used these concepts to develop expression for fuzzy net present value (FNPV) of

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future cash flows. Buckley *et al*. [16] developed a new approach to solve multi objective linear programming problems in which all of the parameters and the variables were depicted in the form of fuzzy numbers. Mehra *et al*. [17] proposed two new concepts of *(α, β)*-acceptable optimal solution and *(α, β)*-acceptable optimal value of a fuzzy linear fractional programming problem with fuzzy coefficients. Hashemi *et al*. [18] developed a two stage model to indicate possibility distribution of decision variables.Abiri and yousefli [19] implemented fuzzy decision making in fuzzy environment to location-allocation problem in which allocation decision variables are considered as fuzzy numbers.Ghazanfari *et al*. [20] presented a new approach to obtain non-symmetrical possibility distribution for decision variables in a possibilistic mathematical programming.

All of the aforesaid models, just considered decision variables as fuzzy number and obtained a possibility distribution for each of them, but they didn't introduce a framework to interpret fuzzy decision making and practical use of obtained uncertain decisions. This issue is investigated through this paper. In this paper decision variables of a mathematical programming problem are considered as fuzzy variables and their possibility distributions are obtained using simulation approach. Some basic concepts are presented and fuzzy decision making in fuzzy environment is defined using these concepts. In the next step, optimum fuzzy decision variables are used to architect a rule based decision support system. Established fuzzy rule base helps decision maker by representing optimum or near optimum solutions, based on the realized values of uncertain parameters in the real time. In other word, developed decision support system can be seen as a new approach to infer (not calculate) optimum or near optimum values of decision variables without solving mathematical programming problem directly.One of the most appropriate applications of the developed model is in the Np-hard or nonlinear decision making problems, so that when uncertain parameters will be realized, optimum or near optimum solutions can be instantly calculated. The organization of this paper is as follows: In the next section, an integrated system for decision aiding and decision making is presented in two phases. To illustrate the effectiveness of the developed system a numerical example is presented in section 3. Presented example is a practical application in the area of inventory problems. The last section remarks conclusion and future researches.

A Framework to Decision Aid and Decision Making in Fuzzy Environment: Herein, we introduce a framework which consists of two phases. In the first phase, based on simulation, an approach is presented to obtain possibility distribution of decision variables through a possibilistic mathematical programming model and some definitions are presented to explain fuzzy decision making in fuzzy environment concept. Second phase uses the obtained possibility distributions to establish the rule based decision support system.

Phase 1: Providing Decision Aid System: In this phase we use conventional optimization and simulation methods to derive possibility distributions of decision variables through a possibilistic mathematical programming.Based on uncertain decisions, a new concept called*possibilisticoptimum space*is introduced as decision aid system. Possibilistic optimum space illustrates all optimum possible decisions which may be madein the future. Consider the following possibilistic programming problem:

$$\begin{aligned} &\max \ f(x,\xi) \\ &S.t. \\ &\quad g(x,\xi) \end{aligned} \tag{1}$$

Where, ξ is the vector of fuzzy parameters with known possibility distributions.It must be noticed that $f(.)$ and $g(.)$ could be linear or nonlinear whereas decision variable x is assumed to be just continuous and couldn't take discreet values.

Definition 2.1: One*state*of the uncertainmodel(1) is a crisp vector ζ of parameters that indicates a realized scenario of ξ.

Let $\xi = (\xi_1,\xi_2,....,\xi_n)$ be n-dimensionalfuzzy vector and $\zeta = (\zeta_1,\zeta_2,....,\zeta_n)$ be a state that may occur. ζ hasdegree of possibility $\pi(\zeta)$ for occurrence.

Using definition 1, a proposition can be proofed that shows decision variables of a possibilistic mathematical programming are fuzzy sets and have possibility distribution.

Proposition 2.1: Suppose that ξ_i; i = 1,2,...,n has a possibility distribution $\mu_{\xi i}$, then each decision variable x_j ; j = 1,2,...,n has a possibility distribution.

Proof: Let $\zeta = (\zeta_1,\zeta_2,....,\zeta_n)$ be a state of model (1). Occurrence possibility degree of $\zeta = (\zeta_1,\zeta_2,....,\zeta_n)$ is named α which can be calculated as follows:

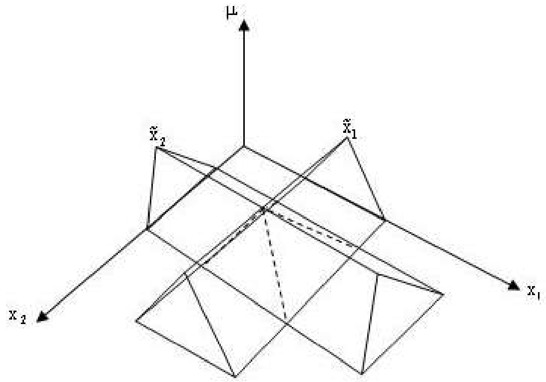

Fig. 1: Possibilistic optimum space

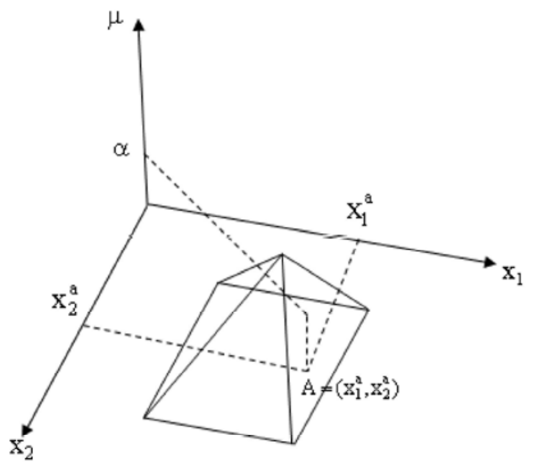

Fig. 2: Three dimensional depiction of POS

$$\alpha = \min_{1\le i\le n}\{\mu_{\xi_i}(\zeta_i)\,|\,\zeta_i \in \sup p(\xi_i)\}$$

Assume that optimum decision vector regarding to the state ζ is obtained as X^*. Since X^* is calculated based on the realized parameter vector ζ, we have $\pi(X^*) = \alpha$. On the other hand $X^* = \{x_1^*, x_2^*, ..., x_n^*\}$, so $\pi(X^*) = \min_{i=1,2,...n}\{\pi(x_i^*)\}$. Therefore it can be inferred $\pi(x_i^*) \ge \alpha\,;\, i = 1,2,...,n$. Considering the fact that ζ has a possibility distribution, so there is at least one state with possibility degree a = 1. Consequently there is at least one optimumdecision vector $X^* = \{x_1^*, x_2^*, ..., x_n^*\}$ regarding to this state with possibility degree $\pi(X^*) = 1$; i = 1,2,...,n. *So* This means that $\pi(x_i^*) = 1\,;\, i = 1,2,...,n$. So based on the definition of possibility distribution, X^* and $x_i^*\,;\, i = 1,2,...,n$ both have possibility distribution ■.

Definition 2.2: Embedded space through optimal decision variables is named *possibilisticoptimum space (POS)*. Consider $\tilde{x}_1$ and $\tilde{x}_2$ be two optimum fuzzy decision variables. Figure1 and Figure 2 illustrate possibilistic optimum space concept.

Each point of the pyramid base such as $A = (x_1^a, x_2^a)$ is optimal solution with possibility degree $\alpha \in [0,1]$. Using possibilistic optimum space, Decision maker is equipped by a useful decision aid system so that gives a wide vision to him/her to understand different circumstances that may beoccurred in the future. Possibilistic optimum solution doesn't make a decision for decision maker but improves his/her knowledge to recognize several aspects of situationsto make the best decision. But if decision maker likes to have a deterministic decision, most possible solution (MOS) which is introduced in the definition 3 can be proposed to him/her.

Definition 2.3: *Most Possible Solution (MPS)* is a solution (or set of solutions) of possibilistic optimum space that has possibility degree α = 1 for occurrence as optimum solution.

In the model (1) the possibilistic optimum space is the support set of joint possibility distribution of the optimum decision vector X^* and MPS is the $(x_1^*, x_2^*, ..., x_n^*)$ so that $\pi(x_i^*) = 1\,;\, i = 1,2,...,n$. MPS is the optimum solution with the highest possibility degree for occurrence in the future, so in the case that decision maker prefers to get a singleton decision; MPS is the best decision that can be presented to him/her.

To obtain POS and MPS, possibility distributions of optimum decision variables $x_i^*\,;\, i = 1,2,...,n$ and joint possibility distribution of optimum decision vector X^*, must be extracted from the model (1). To this end following algorithm is developed based on simulation:

Step1: For $\alpha \in [0,1]$, determine *a – level* set for all ζ_i; i = 1,2,...n. Using interval parameters, convert the possibilistic mathematical programming (1) into an interval mathematical programming at the level α.

Step 2: Randomly generate $\zeta_j = (\zeta_{j1}, \zeta_{j2}, ..., \zeta_{jn})$: j = 1,2,...,N from the α = *level* intervals and solve the model (1) in the deterministic space.

Step 3: Set $x_i^{\alpha L} = \min_j\{x_{ij}^{\alpha}\}$, $x_i^{\alpha R} = \max_j\{x_{ij}^{\alpha}\}$, $f_{\alpha}^{L} = \min_j\{f(X, \zeta_j)\}$, $f_{\alpha}^{R} = \max_j\{f(X, \zeta_j)\}$.

Step 4: Repeat steps 1 to 3 for α from 0 to 1.

Using the obtained lower and upper bonds in the several $\alpha - level$, the possibility distributions of decision variables and objective function can be derived. Extracted possibility distribution of decision variables will be used to establish POS and calculate MPS.

As mentioned before, the possibilistic optimum space gathers all of the possible optimum values for the decision variables regarding the future realization of the uncertain parameters. Hence, decision maker can have a wide vision of the possible situations and solutions that improves his knowledge over the problem. On the other side, if he wants to make a decision before the realization of the uncertain parameters, MPS would be the answer with the minimum risk.Otherwise, a decision support system (DSS) which is introduced in the next phase can help him/her to make decisions in the real time after parameter realization without engaging in the complexities of the mathematical programming. Mentioned DSS is architected based on the generated optimum knowledge in this phase.

Phase 2: Providing Rule Based Decision Support System: Special types of Decision Support Systems are Fuzzy Logic Controllers (FLC) which use rules to model processes in a simple way [21]. Using experts' knowledge and historical data of system's behavior are two common approaches that are implemented for fuzzy modeling. The first one is named direct approach and the latter is called indirect approach. In the direct approach, knowledge is obtained as If-Then rules with fuzzy predictions that establish relations between relevant system variables [22]. The most important issue in this approach isto make a proper prediction of relations between antecedent and consequents.Usuallyit is presumed a decision maker or some experts exist to determine these relations, but this assumption is not true forever because they are not always available and when they are, their knowledge is not always reliable, constant and compatible.Presented method in this section not only overcomes this problem, but also provides a novel approach to solve optimization problems based on fuzzy rule base. Mamdani'sinference mechanism is used to determine decision variables' values. The design stages of a FLC are as follows:

- Describe each input with linguistic variables. Herein, inputs are uncertain parameters of possibilistic mathematical programming (1). For example, each parameter can be expressed with five linguistic variables as shown in Figure 3.

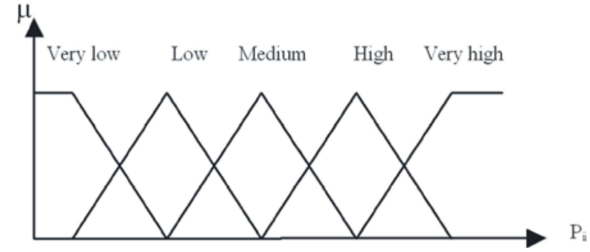

Fig. 3: linguistic variables of parameter P_i

- Define the outputs as fuzzy numbers. As mentioned above,in the conventional fuzzy controllers, the outputs are described as the linguistic terms similar to the inputs by decision maker. However, in this work, for each set of fuzzy parameters, a possibilistic linear programming, discussed in the previous phase, is solved and the optimum decision variables are obtained in the form of fuzzy numbers. Possibility distribution of the optimum decision variables are used as outputs.
- Establish fuzzy rule base. Using fuzzy parameters as antecedent and fuzzy decision variables as consequent, design the fuzzy rule base.
- Defuzzification. Based on the fuzzy rule base of possibilistic linear programming structured in the previous step, defuzzification is used to obtain a deterministic decision.

Presented algorithm, in addition to describesthe relationship between antecedent and consequents systematically, equips decision makers by a dynamic decision making toolwhichcan be usedin each stateto obtain optimum or near optimum decision variables' values without solving the original problem.To clarify the proposed algorithm, consider the following example.

Example: In this example, Possibilistic Economic Order Quantity (PEOQ) model with demand-dependent unit cost under limited storage capacity is investigated. Fuzziness is considered in the storage space, set up cost and inventory holding cost. Notations and assumptions that used in this section are as follows:

Notations:

Q : Order quantity
D : Demand per unit time
$\tilde{C}$: Inventory holding cost
$\tilde{A}$: Set up cost
P : Unit production cost
S : Storage space for one item

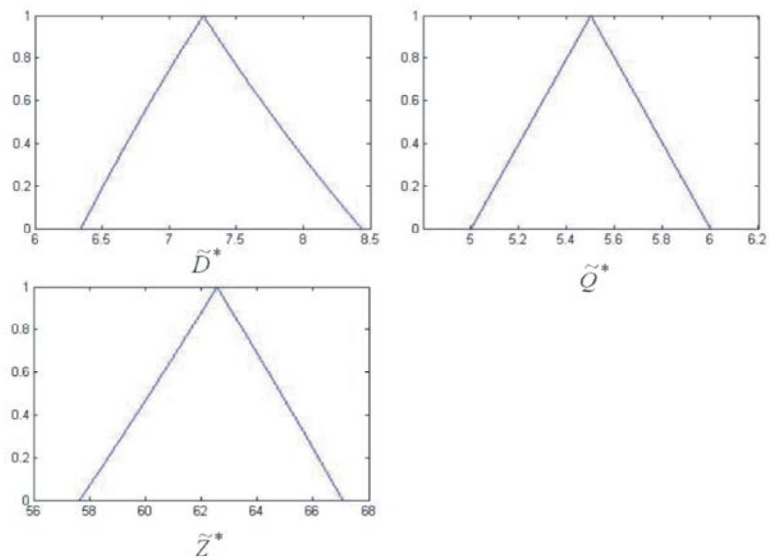

Fig. 4: Possibility distributions of decision variables $\tilde{D},\tilde{Q}$ and objective function $\tilde{Z}$

$\tilde{I}$: Inventory space constraint
K : Predetermined Constant

Assumption:

- Set up cost is a function of order quantity as follows:

Setup cost= $\tilde{A} \times Q^{\gamma}$; $0 < \gamma < 1$

- Production cost is a function of demand as follows:

$P = KD^{-\beta}$; $\beta > 1$

Based on the mentioned notations and assumptions, fuzzy economic order quantity model under storage space constraint is developed as follows:

$$\begin{aligned} &\min \ Z = \tilde{A}Q^{\gamma-1}D + KD^{1-\beta} + \frac{Q}{2}\tilde{C} \\ &S.t. \\ &\qquad SQ \le \tilde{I} \\ &\qquad D,Q \ge 0 \end{aligned} \tag{2}$$

Purpose of solving the above model is deriving optimum possibility distribution of decision variables Q and D. Consider several parameters as follows:

K=100; $\gamma = 0.5$; $\beta = 1{,}5$; $S = 10$; $\tilde{A} = (5,6,7)$;
$\tilde{C} = (2,2.5,3)$; $\tilde{I} = (50,55,60)$

To solve this problem, upper and the lower bounds of decision variables and objective function are obtained at several α level values from 0 to 1. In this way optimum values of the decision variables $\tilde{D}^*,\tilde{Q}^*$ and objective function $\tilde{Z}^*$ are obtained as Figure 4.

When the optimum values of the decision variables are plotted on the coordinate axes, a possibilistic optimum space is obtained in which each point potentially could be optimal solution with the degree of possibility $\alpha \in [0,1]$. POS of is depicted in Figure 5.

Also, considering definition 3, obtained MPS is shown in Figure6.

Using fuzzy parameters and decision variables, following If-Then rule could be structured.

$$\begin{aligned} &If \ \ \tilde{A} = (5,6,7) \, and \, \tilde{C} = (2,2.5,3) \, and \, \tilde{I} = (50,55,60) \\ &\qquad Then \ \ \tilde{D} = (6.342,7.255,8.434) \, and \, \tilde{Q} = (5,5.5,6) \end{aligned}$$

To design fuzzy rule base as a decision support system for model (2), uncertain parameters must be described in the terms of linguistic variables. For this purpose, $\tilde{A},\tilde{C}$ and $\tilde{I}$ are described with three linguistic variables Low, Medium and High. Figures7-9 show fuzzy values of the uncertain parameters.

Based on the several values of uncertain parameters and related values of decision variables, a DSS in the form of fuzzy rule base is designed. Table 1 shows features of designed fuzzy rule base using fuzzy logic toolbox of MATLAB software.

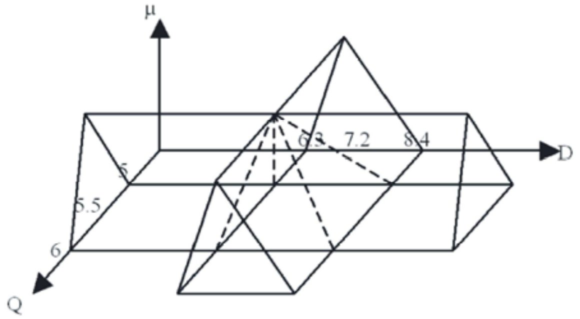

Fig. 5: Possibilistic optimum space of PEOQ problem

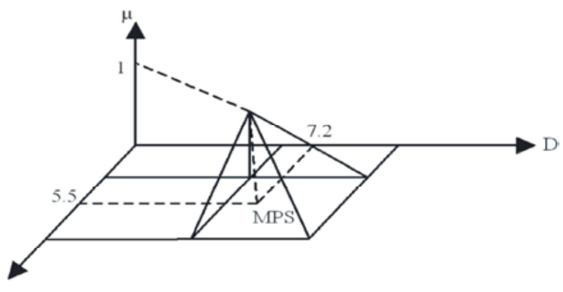

Fig. 6: Most Possible Solution (MPS) of PEOQ problem

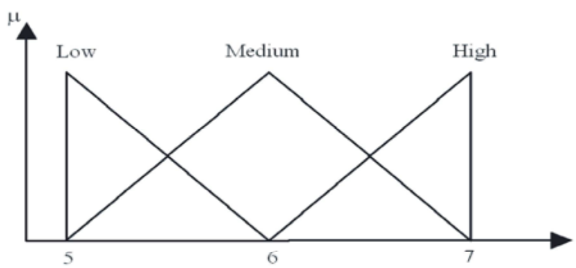

Fig. 7: Fuzzy values of fuzzy parameter $\tilde{A}$

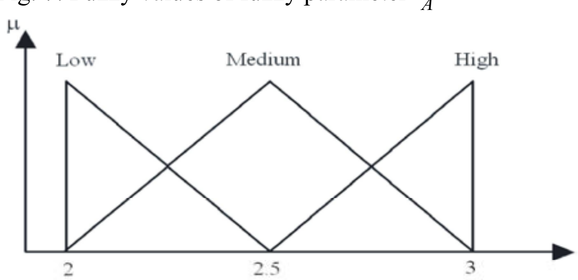

Fig. 8: Fuzzy values of fuzzy parameter $\tilde{C}$

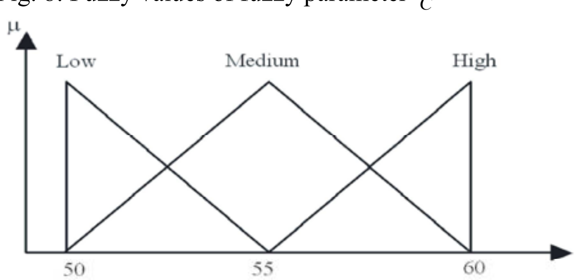

Fig. 9: Fuzzy values of fuzzy parameter $\tilde{I}$

Table 1 Features of fuzzy rule base

Rule base type	Mamdani
And Method	Min
Or Method	Max
Defuzzification Method	Centroid
Implication Method	Product
Aggregation Method	Max
Input	[1x3 struct]
Output	[1x2 struct]
Number of rule	[1x27 struct]

Table 2: result of comparison between FLC and optimum outputs

Deviations from optimum solution	*D*	*Q*
Minimum deviation (%)	0.00267	0.0221
Maximum deviation (%)	7.32	13.31
Mean of deviations (%)	3.03	2.92

Designed fuzzy rule base can solve each state of the model (2) effectively. 1000 randomly samples are solved by developed fuzzy rule base and outputs are compared with optimum solutions. Results of this comparison are shown in Table 2.

As Table 2 shows, maximum deviation for D and Q are 7.32% and 13.31% respectively while mean of deviations are 3.03% and 2.92%. Figures 10 and 11respectivly show cumulative proportions of the D and Q deviations.

As it depicted in figure 10, over than 80% of deviations between FLC output and optimum solution for decision variable D are less than 5%. For decision variable Q this proportion is over than 96%. Based on Table 2 and Figures 10 and 11, it can be claimed that the proposed decision support system can be reliable for decision makers and departs them from complex mathematical programming computations.

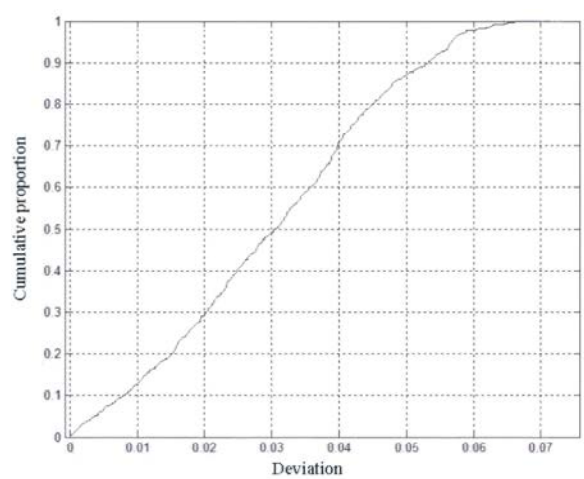

Fig. 10: Cumulative proportion of the D deviations

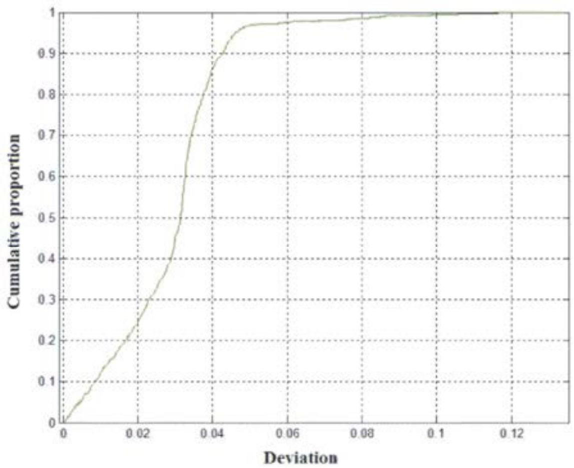

Fig. 11: Cumulative proportion of the Q deviations

CONCLUSION

Most of the researchers presented some methods to tackle decision making in uncertain environment problems. Also, some of them developed their methods in fully uncertain environment so that decision variables are obtained in the form of fuzzy numbers, but they did not explain how fuzzy decisions can be implemented in the real situations. Developed decision support system is an appropriate application of uncertain decisions that equips decision maker with a powerful tool that make him/her needless to engaging in mathematical programming complexity, while that results of presented example shows developed DSS is an efficient alternative for complex mathematical programming problems.

Additional advantage of the presented DSS is use of If-Then rule base to inference decisions, which is more understandable for decision makers in compare to optimization problems. Also, decision aiding system which presented in the first phase of the developed system, gives a wide vision of the possible situations and solutions to decision maker that improves his/her knowledge over the problem.

For the future research, the presented model can be extended in the other uncertain environments such as probabilistic environment. For this end, at first a probabilistic rule based inference system as same as Mamdani's inference mechanism should be developed in which independent variables are appeared with their probability density function in the antecedent and similarly dependent variables are appeared in the consequence. Moreover a new implication function must be developed to inference using this type of rule based DSS in the probabilistic environment.

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