# New Structural Approach for Bond Graph to Digraph Conversion: a Sensor Placement Optimisation for Fault Detection and Isolation 

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#### Abstract

In this paper, we propose a new structural and qualitative approach to convert from Bond Graph to Digraph representation of a structural system which allows sensors placement optimization to solve faults detection and isolation problem. When the detection and isolation of faults of an existing system's sensors is impossible or uncertain, a reconfiguration sensor placement of this system should be considered. This paper proposes how this reconfiguration takes place by recovering all missing or redundant parts of the system. This novel approach is illustrated over a thermo-fluid tank application.


Key words: Structural Approach • Sensor • Optimisation • Detection • Isolation

## INTRODUCTION

The structural approach was introduced for systems with unknown parameters in absence of quantitative or incorrect values due to measurement errors, this last aspect being unavoidable in modelling physical processes. Thus, it is a matter of a maximum exploitation of the structural approach behind any computation. The method is based on the graph theory. While constructing the structural graph (or structure matrix) of the system, some information maybe lost. This is due to the fact that construction of the state equations does not clearly complete all the constitutive relations of the considered system. The Bond graph (BD) tool defined in [1] formalized in $[2,3]$ is a network type language which represents graphically, with a single language, many physical systems. It is placed like an intermediate stage between the physical description of a dynamic system and the construction of a mathematical model phase, by graphic representation of the powers exchanges between the various components of the system. The modelling of a physical system with BD does not require the writing of general laws of conservation. It is based, principally, on the characterization of the power exchanges phenomena within the system.

Bond Graph to Digraph Conversion: Bell and Perelson [4-6] established a comparison between BG and the linear graphs from the modelling point of view and mathematical formulation. In addition, they proposed the conversion of BG into a linear graph. The conversion from the Matroïdes to BG was proposed by Birkett [7]. Rahmani [8] proposed a new concept relating to BG , entitled family of causal cycles like equivalence with the concept of family of cycles in a digraph. These approaches are not largely exploited to solve various control problems due to their complexities and implementation difficulties.

In this paper we propose a new qualitative approach for the conversion from BG to Digraphs. This approach is based on structural and physical systems particularities such as interconnection between subsystems [9]. So, it is efficient to represent system's structure which allows drawing the digraph directly. It can be employed to associate a linear structured system with a digraph, referring to results presented in [10, 11].

To draw directed graph that represents system structure, we need to define the structural adjacency matrix, this matrix is a squared matrix composed by elements of the system [11]. If we assume a digraph $\mathrm{G}(\mathrm{V}, \mathrm{A})$ where V represents vertices and A represents arcs of the graph G, thus, the structural adjacency matrix of $G$ is therefore a squared Boolean matrix $\mathrm{M}=(\mathrm{mij})$, of size $\mathrm{n} \times \mathrm{n}$, defined as follows;

[^0]$\left\{\begin{array}{cl}m i j=1 & \text { if }(i, j) \in A \\ 0 & \text { else }\end{array}\right.$
To draw the structural adjacency matrix from the BG, we have to put the elements of the system in lines and columns [11]. Then we check all the connections between them and we fill the Boolean matrix by taking into account these following instructions:

- All the elements R, C and I are an essential components in the matrix because their interconnections are represented by the propagation of the flows on the physical system.
- By separating all the detectors De and Df from the other elements the sensor placement interpretation will be simpler. Their interconnections with the other elements are represented by the propagation of the information on the physical system.
- The Sources and the controllers are the secondary elements in the matrix, thus we can ignore them. We must put the essential elements on the matrix and separate them from the detectors because generally, there is an interaction between the detectors and the controllers.
- We must make the same number of elements and it is preferable to avoid changing their positions in line and column in the matrix.
- We consider the elements TF and GY as junctions and we cannot represent them in the matrix because they represent the energy exchange between the elements. Generally if the link between detectors exists which represent the exchange of information means that we have a redundancy.

Study Case: In this section, with respect to the considered instructions, we will construct the structural matrix and convert the BG to an associate digraph of an industrial application.

A simple example of an industrial water-heater shown in Figure 1, which represents a thermo-fluid system with localized parameters, is chosen to show the various steps in the conversion from the BG to the Digraph that can be eventually associated to a structured system. The system is constituted by a tank, a pump, a valve and a heater. Sensors are needed to monitor and control the thermo-fluid system. These sensors are temperature $(\mathrm{T})$, level (L), pressure ( P ) and flow ( F ). The control sensors (T) and (L) are used to control the behaviour of the system. The control circuit is completed by controllers for the level (LC) and the temperature (TC).


Fig. 1: Thermo-fluid System


Fig. 2: Bond Graph Model of the Thermo-fluid System

Based on the thermo-fluid system given in Figure 1, we have derived the corresponding BG model $[12,13]$ and presented in Figure 2.

Note that, the level controller (LC) acts upon a pump to maintain a constant level of water in the tank. The water inside the tank is heated by using the thermal energy provided by the heater and the temperature is kept constant at a desired set point by the temperature controller (TC) acting on the thermal source. The valve at the output of the tank is used to deliver a hot water to the consumer. The pump is considered as a flow source and thus, its output is known. The level (L) and temperature ( T ) sensors are respectively used to regulate the level and the temperature of the tank. The flow sensor $(\mathrm{F})$ is used to measure the amount of water leaving the tank. The pressure sensor $(\mathrm{P})$ at the bottom of the tank is a material redundancy to the level sensor $\left(L=\frac{P}{p . g}\right)$.
In addition, the output control signal of each controller is considered as a known value $[12,13]$.

However, we construct the structural adjacency matrix from the system model directly from the detailed

Table 1: Structural Adjacency Matrix of the Thermo-fluid System

|  | $\mathrm{P}_{\mathrm{De}}$ | Loe | T se | mot | Qor | Fr | Tank | Valve | Pump | Heater | LC | TC |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| P De | 0 | $1!$ | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| LDe | $1!$ | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| TDe | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| m Df | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| Ọ Df | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| F Df | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| Tank | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| Valve | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| Pump | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| Heater | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| LC | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| TC | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |

instrument plan given in Figure 1 without going through the BG. On the other hand, we construct this matrix starting from the BG model; the result of the obtained matrix is exactly similar. This approach can be considered as a verification and validation of the BG model.

Table 1 represents the structural adjacency matrix which gives the elements of the thermo-fluid system in lines and columns. Table 1 is constructed upon the BG model following the mentioned instructions where the matrix element 1 represents a link between components of lines and columns. Notice that, the symbol 1 means that there a connection between elements in the BG model.

In addition, we can decompose the structural adjacency matrix of the thermo-fluid system into 3 sub-matrices which are defined as followed:

- The Structural Interconnection Matrix represents the connections between the essential and secondary elements themselves (R, C, I, sources and controllers, in the BG model).
- The Structural Reachability Matrix represents the connections between the essential and secondary elements (R, C, I, sources and controllers, in the BG model) from one side and the detectors (De, Df in the BG model) from the other side.

This matrix represents the sensor placement on the system, if any component has a zero link with any sensor, the observability of its state is impossible. We must add a new sensor linking to this element, $[10,11]$ to be able to detect and isolate the behaviour of this sensor.

- The Structural Redundancy Matrix represents the connections between the sensors/detector themselves. This matrix represents the material redundancy between the sensors. This will help us to optimise the sensor placement and the redundancy information about the system.


Fig. 3: Digraph of the System
As a matter of fact, the redundancy matrix observed in the first $6 \times 6$ part of Table 1, where it shows a 1 ! in the column between the two detectors P and L , is interpreted as a material redundancy between these two detectors. It can be shown by the following equation: $\left(\mathrm{L}=\frac{\mathrm{p}}{\mathrm{p} \cdot \mathrm{g}}\right)$.

Based on BG given in Figure 2 and the obtained matrix given in table 1 we construct the digraph of the system which is given in Figure 3.

We can see from the Digraph, given in Figure 3, that the pressure sensor $P$ is connected to the tank while this latter is related to other components. Thus the pressure sensor P is a redundant element, because the level sensor L is an essential sensor for controller LC.

However, in this system we cannot add new sensors. But, we can prove that all faults can be detected and isolated theoretically, since each component has a unique fault signature in the fault signature matrix. The proof of this theoretical aspect was given in our previous work [11].

Thus we can say that we have an optimal sensor placement for fault detection and isolation on the thermo-fluid system.

## CONCLUSION

In this paper we have presented a new qualitative approach to convert the Bond graph to a Digraph which can be naturally associated with a structured system. This graph gives a visual representation of the internal structure and the solvability of several structural problems that can be stated in a very comprehension way in terms of this graph.

We have defined the structural adjacency matrix, a simple way to draw the Digraph. This matrix is a qualitative approach to solve the FDI problems and the
optimal sensors placement on system or components level. This matrix illustrates the propagation of the information and the flows through the system. So, this idea could be used in other contexts.

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