World Applied Sciences Journal 26 (10): 1335-1339, 2013
ISSN 1818-4952

DOI: 10.5829/idosi.wasj.2013.26.10.973

Fuzzy Bases

[1]*Shahryar Sorooshian and* [2]*Amir Azizi*

[1]Faculty of Technology, University Malaysia Pahang, Malaysia
[2]Faculty of Manufacturing Engineering, University Malaysia Pahang, Malaysia

Submitted: Sep 17, 2013; **Accepted:** Nov 19, 2013; **Published:** Nov 30, 2013

Abstract: This study tries to provide a comprehensive understanding of theories of fuzzy approach and their application methods. This paper can be a note for those who are interested to apply fuzzy theories in their researches. Text starts with theoretically review fuzzy theories and follows by an introduction to application methods of these theories.

Key words: Fuzzy · Zadeh · Fuzzifier · Diffuzifier

INTRODUCTION

Fuzzy sets, for the first time, was proposed by Zadeh [1] in an article with the title of “fuzzy set”. Zadeh was a famous man in the area of control engineering before he proposed fuzzy sets and fuzzy systems. He developed the concepts of “state” which has become the basic principle of modern control. In the early of 1960s, he taught the classical control theory which is concentrated on certainty that is not applicable in the complex systems. In 1962, he wrote for biologic system: “basically, we need a new mathematics, vague value mathematics or fuzzy which could be stated by probability distribution”. After that he proposed his idea in the article of fuzzy sets by proposing fuzzy theory, some challenge and discussion were started. Somebody accepted it and started their research on this topic but somebody claimed that this theory is in the contrast of science principles. The major challenge was from the mathematicians who declare that probability theory does the same as fuzzy theory and it is sufficient. Because in the first, application of fuzzy theory was not known, understanding the concepts of fuzzy theory was so hard and none of the research center accepted the fuzzy theory for their future research topics.

However, there were some advocators of fuzzy theory and they do their research on this topic seriously and by the end of 1960s, some new fuzzy methods such as fuzzy algorithm and fuzzy decision making were proposed.

It’s not exaggerated if we said that fuzzy became an independent topic by the outstanding attempts of Prof. Zadeh. The majority of the fuzzy theory concepts were proposed by Prof. Zadeh by the end of 1960s and first of 1970s. After introducing fuzzy sets in 1965, he proposed fuzzy ranking in 1971. In 1973 he published another article about fuzzy deciosion making. This article became the basic of fuzzy control. He introduced linguistic variable and fuzzy *if-then* rules in this article.

In 1970s, fuzzy controller for a real system was constructed. Mamdani & Assilian [2] determined the framework for controller and used it for a real steam engine, they understand that making a fuzzy controller is so easy and it works properly. Holmblad & ostergaard [3] applied the first fuzzy controller for a complete industrial process.

However, the basic of fuzzy theory was accepted in 1970s, by introducing the new concepts, the image of fuzzy theory as a new area became clear. The first application in the steam engine also helps to introduce fuzzy theory as a new area. Usually new area show supported by the research centers and university. Unfortunately, for fuzzy theory, it wasn’t happen. In 1980s, fuzzy theory had a slow improving in the theory. In fact, the application of fuzzy theory help it in that period to continuous its life.

Japanese engineer understand quickly that fuzzy controllers are so easy to design and could be used in many situations. As the fuzzy controllers don’t need any

Corresponding Author: Shahryar Sorooshian, Faculty of Technology, University Malaysia Pahang, Malaysia.

mathematical model, so they could be used in many situations where traditional control system can't apply. Sugeno [4-5] started to construct a fuzzy controller for purifying the water. In 1983, he started to work on a fuzzy robot, an automobile which could be controlled from distance and park by its own. Finally, in 1987 one of the progressive subways based on the fuzzy theory was constructed. In July of 1987, the second conference of fuzzy system was held in Japan. In this conference, a robot which could play ping pong was shown. After this conference, Japanese engineering, researchers and even government focused on fuzzy theory so that in the beginning of 1990s some goods with the base of fuzzy theory could be fined in the store.

Basic Theories

Fuzzy Set Theory: In fuzzy set in contrast to the crisp set, items don't classify into just two categories. In fuzzy sets items with a membership degree between 0 to 1 belong to a set. Suppose X is a place of items. A fuzzy subset like $\tilde{A}$ from X, determined by a membership function like $\mu(x)$ which consider for each items in X areal number from interval [0, 1]. The value of $\mu(x)$, called membership degree of A in X. So, if the value of $\mu(x)$ is near to 1, it means that A belongs to X by a stronger evidence. When, A supposed to be a crisp set, A could have just two value, 0 and 1. So, crisp set is an especial case of fuzzy set.

Linguistic Variable: In our life there are some words which usually use to describe a variable, such as 'good', 'bad, 'young', 'old', 'tall', short', 'strong', 'poor', 'hot', 'cold' and some adverb such as 'usually', 'always', 'almost', 'seldom'. It's obvious that we could not find a clear boundary for these words. A linguistic variable differ from a numerical variable, because the value of them is not scalar and it's a linguistic term. In the fuzzy textbook, a linguistic variable like L introduced by its terms which are shown by T(L).T(L) is a set consists of linguistic values which L could get. Each linguistic term in T(L) set are shown by Li and introduced by a fuzzy set like Fi which have a membership function like $\mu(x)$. As an example, suppose a quality inspector wants to determine the softness of a good mentally, in this case, softness of the good is a linguistic variable which could have the terms like 'good', 'medium' and 'poor'. For each of the terms, a membership function could consider as $\mu_i(x)$. Figure 1 depicted the membership function belonging to these terms.

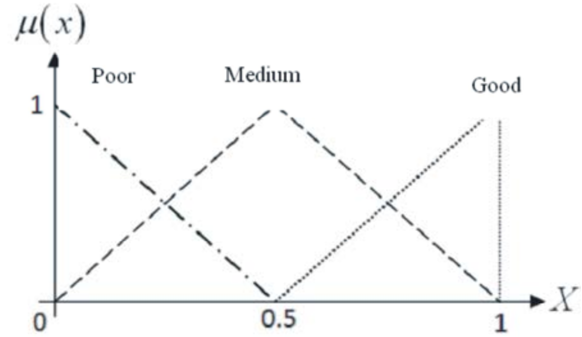

Fig. 1: The membership function for linguistic variables

In this example, base variable, *X*, which is shown in the horizontal axis could be measured quantitative. For using linguistic variable, usually the base variable could not be measured quantitatively. For such case, it's better to consider the range of the base variable standard in the interval of [0, 1]. By considering Figure (1), one can find that whenever *X=0, 1* the value of the softness belongs to two fuzzy sets simultaneously. It means that by applying fuzzy sets theory we could consider the vagueness of the mental inspections in our calculation. One could done calculations and mathematical function on linguistic variables by applying the definition and fuzzy mathematics which have been developed widely.

Fuzzy System: Fuzzy system is knowledge based or rule based systems. The heart of fuzzy system is a knowledge base which consists of fuzzy if-then rules. A fuzzy if-then rule is an if-then term which its terms are shown by membership function. As an example, consider this rule:

If the speed of the car is high, then push the accelerator slowly.

The term *high* and *slowly* are shown by membership functions.

The first step for constructing a fuzzy system is making a set of if-then rules by the knowledge and experiments of the experts. The second step is to combine these rules in one system. Different fuzzy systems use different methods for combining these rules.

In the literature these are three different type of fuzzy system:

- Pure fuzzy system
- Takagi-Sugeno-Kang fuzzy system (TSK)
- Fuzzy system with fuzzifier and diffuzifier

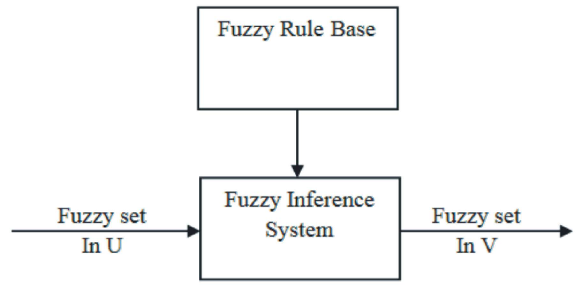

Fig. 2: The basic structure of a pure fuzzy system

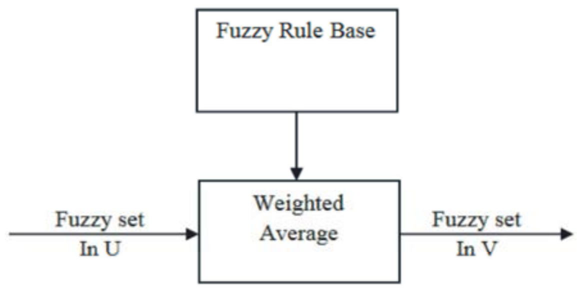

Fig. 3: The basic structure of a TSK system

Figure 2 depicted the basic structure of a pure fuzzy system. Fuzzy rule base shows a set of fuzzy if-then rules. Fuzzy inference engine combines these rules based on a function from input space to the output space. The disadvantage with pure fuzzy system is the input and outputs which are fuzzy sets. While, in the engineering systems, the inputs and outputs are variable with crisp value. For solving this disadvantage, Takagi *et al*. introduced another type of fuzzy system which its inputs and outputs are real numbers.

Chapter's text nonindent: Punctuation rules also applyy before or after formulas.

TSK system uses the rule as:

If the speed of the car is high, then the power on the accelerator is Y=cX.

where "*c*" is a fixed real coefficient. By comparing this rule with the previous rule, one can find that a mathematical function put instead of a linguistic variable. It makes the combination of the fuzzy rule easier. In fact, TSK system is a weighted mean of the value in the then part of the rules. Figure (3) shows the basic structure of a TSK system.

Fuzzy Inference System: A fuzzy inference system is a popular computing framework based on the concept of fuzzy set theory, fuzzy if-then rules and fuzzy reasoning. The basic structure of a fuzzy inference system consists of three conceptual components: a rule based, which contains a selection of fuzzy rules; a database, which defines the membership functions used in the fuzzy rules; and a reasoning mechanism which performs the inference procedure upon the rules and given facts to derive a reasonable output or conclusion. Figure 4 shows a fuzzy inference engine with fuzzifier and defuzzifier.

The central core of a fuzzy inference engine is based on fuzzy rules. The rules are constructed by the designer of the system, such that the input linguistic variables are connected to the output linguistic variable by means of their terms. The antecedent and consequence of these rules include terms of linguistic variable $LV_1, LV_2, ..., LV_n$ that described by their membership function $\mu_i^j(x)$, where $i(i = 1,2,...,n)$ indicates the linguistic variables, $j(j = 1,2,...,m(i))$ indicates the terms of linguistic variable i and

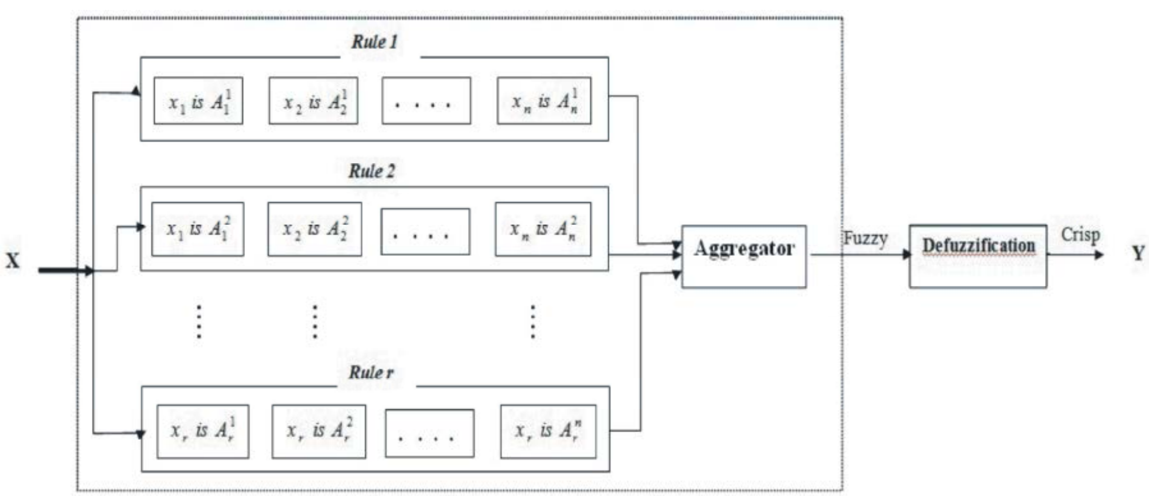

Fig. 4: Fuzzy inference engine

$m(i)$ is the number of terms of linguistic variable i. By using the number of linguistic variable and the number of terms of each linguistic variable, we can determine the number of possible rules. A rule can be shown as below:

$$Rule\ r:\ if\ x_1\ is\ A_1^{j_1}\ and\ x_2\ is\ A_2^{j_2}\ and\ ...and\ x_n\ is\ A_n^{j_n}\ ,then\ u\ is\ A^j.$$

where $A_i^{j_i}$ is the j^{th} term of linguistic variable i corresponding to membership function $\mu_i^{j_i}(x_i)$ and A^j corresponds to the membership function $\mu^j(u)$ representing a term of the output variable.

In general, there are three types of fuzzy inference systems that have been widely employed in various applications, namely Mamdani [2], Sugeno [4-5] and Tsukamoto [6]. The differences between these three fuzzy inference systems lie in the consequents of their fuzzy rules and thus their aggregation. In the following , we Shall introduce the Mamdany fuzzy model.

Mamdani Fuzzy Model: The definition of linguistic variables and rules are the main design steps when implementing a Mamdani controller. Before elaborating on the last design step, which is the choice of an appropriate defuzzification procedure, we show how input values trigger the computation of the control action. The computational core can be described as a three-step process consisting of

- Determination of the degree of membership of the input in the rule antecedent,
- Computation of the rule consequences and
- Aggregation of rule consequences to the fuzzy set "control action".

The first step is to compute the degree of membership of the input values in the rule antecedents. Employing the minimum-operator as a model for the "and", we compute the degree of match of rule r as

$$\alpha_r = \min_{i=1,....,n}\left\{\mu_i^{j_i}\left(x_i^{input}\right)\right\} \tag{1}$$

This concept enables us to obtain the validity of the rule consequences. We assume that rules with a low degree of membership in the antecedent also have little validity and therefore clip the consequence fuzzy sets at the height of the antecedent degree of membership. Formally,

$$\mu_r^{conseq}(u) = \min\left\{\alpha_r, \mu^j(u)\right\} \tag{2}$$

The result of this evaluation process is obtained by aggregation of all consequences using the maximum operator. We compute the fuzzy set of the control action:

$$\mu^{conseq}(u) = \max_r\left\{\mu_r^{conseq}(u)\right\} \tag{3}$$

This computation is a special case of an inference process described in chapter 10 and other inference methods can be applied. It is important to note that Mamdani's method takes into account all rules in a single stage and that no chaining occurs. Thus the inference process in fuzzy control is much simpler than in most expert systems.

Constructing the Membeship Function: The validation of any fuzzy inference systems directly due to the type and parameters of the membership function of terms of linguistic variables in the antecedent and consequent of fuzzy rules. It has pointed out [4] as: "it is dangerous to neglect clarifying where the membership came from, because it is one of the most important aspects in application (a fuzzy system)".

We prefer sigmoid membership function for the terms in the antecedent due to its flexibility and triangular membership function for the terms in the consequent due to its computational efficiency.

Sigmoid membership function as shown in equation (1) has two parameters a and c. If the sign of a is negative the function would be Z-shape, otherwise it would be S-shape.

Defuzzifcation Methods: Defuzzification refers to the way a crisp value is extracted from a fuzzy set as a representative value. In general, there are five methods for defuzzifying a fuzzy set A of a universe of discourse Z, as shown in Figure 5. A brief explanation of each defuzzification strategy follows.

Centroid of area (COA):

$$z_{COA} = \frac{\int_Z \mu_A(z)\, z\, dz}{\int_Z \mu_A(z)\, dz} \tag{4}$$

where $\mu_A(z)$ is the aggregated output MF. This is the most widely adopted defuzzification strategy, which is reminiscent of the calculation of expected values of probability distribution.

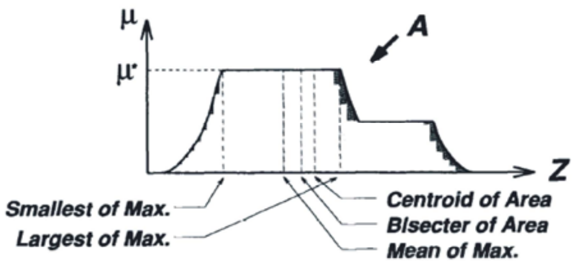

Fig. 5: Different defuzzification value

Bisector of area (BOA): Z_{BOA} satisfies

$$\int_{\alpha}^{z_{BOA}} \mu_A(z)\,dz = \int_{z_{BOA}}^{\beta} \mu_A(z)\,dz, \tag{5}$$

where $a = \min\{z|z \in Z\}$ and $\beta = \max\{z|z \in Z\}$. That is, the vertical line $z = z_{BOA}$ partitions the region between $z = \alpha, z = \beta$, $y = 0$ and $y = \mu_A(z)$ into two regions with the same area.

Mean of maximum (MOM): z_{MOM} is the average of the maximizing z at which the MF reach a maximum μ^*. In symbols,

$$z_{MOM} = \frac{\int_{z'} z\,dz}{\int_{z'} dz}, \tag{6}$$

where $Z' = \{z|\mu_A(z) = \mu^*\}$. In particular, if μ_A(iz) has a single maximum at $z = z^*$, then $z_{MOM} = Z^*$.

Smallest of Maximum (SOM): z_{SOM} is the minimum (in terms of magnitude) of the maximizing z.

Largest of Maximum (LOM): z_{LOM} is the maximum (in terms of magnitude) of the maximizing z. Because of their obvious bias, z_{SOM} and z_{LOM} are not used as often as the other three defuzzification methods. Other more flexible defuzzification methods can be found in different references.

CONCLUSION

Fuzzy Theory is a widely used approach in many studies. Its areas of applications include: Engineering, Management, medicine, physics, education, sports and etc. It ranks as one of the most important theories used in these disciplines. This paper reviewed the bias of fuzzy approach. Provided review in this paper can be a guideline for scholars to start their studies using fuzzy approach.

ACKNOWLEDGEMENT

This study is supported by University Malaysia Pahang research grant (RUD130375).

REFERENCES

1. Zadeh, L.A., 1965. Fuzzy sets. Information and control, 8(3): 338-353.
2. Mamdani, E.H. and S. Assilian, 1975. An experiment in linguistic synthesis with a fuzzy logic controller. International Journal of Man-machine Studies, 7(1): 1-13.
3. Holmbland, L.P. and J.J. Ostergaard, 1982. Control of a cement kiln by fuzzy logic, Smidth.
4. Sugenu, M. and M. Nishida, 1985. Fuzzy control of model car. Fuzzy Sets and Systems, 16(2): 103-113.
5. Sugenu, M. and G. Kang, 1988. Fuzzy modelling and control of multilayer incinerator. Fuzzy Sets and Systems, 25(2): 259-260.
6. Tsukamoto, Y., 1979. An approach to fuzzy reasoning method. Advances in fuzzy set Theory and Applications, 137: 149.