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Some New Type Iterative Methods for Solving Nonlinear Algebraic Equation

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Abstract: In This paper we present some of a new iterative numerical methods to solve non linear algebraic equations of the form f(x)=0, by using some integration methods. These methods use the Newton theorem, $f(x) = f(x_n) + \int_{x_n}^{x} f'(t).dt$ for y=f(x). we apply some numerical integration rules to compute the integral in this formula, as the trapezoidal integral [TI], composite trapezoidal integral [2-CTM] (of 2 Trapezes), Simpson's integral [SI], Romberg's integral [RI or $R_2^{(1)}$], Chebyshev's integral [CHI] and Gaussian quadratic integral [GQI]. these methods like the method of S.Weeraksoon and Fernand [1] given also by H.H.H. Homeir [2], also like widely methods as the method of Xing-Guo Luo [3], Nasr-Al-Din Ide [4, 6], Jishing [7], Masoud A and Nafiseh A [8] and Babajee [9]. By considering some examples we confirm that these new iterative method converge more quickly than Newton method [5,10], Hybrid iteration method [3], new Hybrid iteration method [4] and modified Newton method [8].

Key words: Algebraic equations · Newton method · Non linear equation · Iteration method · Hybrid method · New hybrid method · Modified Newton method

INTRODUCTION

Many of complex problems in Sciences and engineering contains the function of nonlinear equation of the form f(x)=0. Several numerical iterative methods [1-10] used to obtain the approximate solution of some problems because it is not always possible to obtain its solution by usual algebraic methods. In [S. Weeraksoon and T.G.I. Fernando, A variant of Newton's method with accelerated third-order convergence], given also by [H.H.H. Homeier, On Newton-type methods with cubic convergence] a new iteration method for solving algebraic equations has been proposed, by the Newton theorem:

$$f(x) = f(x_n) + \int_{x_n}^{x} f'(t).dt \tag{1}$$

by approximating the integral by the rectangular, trapezoidal rules. Frontini and Sormani [5] also generalized the approach of Weerakoon and Fernando by using general interpolator quadrate rules of order one at least. In this paper we proposed a new iteration methods for solving algebraic equations by using the approximating integral of the Newton theorem (1) by various integration methods .

New Methods: We consider the non linear algebraic equation:

$$f(x) = 0 \tag{2}$$

then, let us use the Newton theorem (1), by approximating the integral by numerical integration methods, we have the following:

By using trapezoidal rule, we see that,

$$f(x) = f(x_n) + \int_{x_n}^{x} f'(t)dt \approx \frac{x-x_n}{2}[f'(x) + f'(x_n)]$$

by using f(x)=0, also the substitution of x by x^{n+1} and by considering $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = y_n$, we have

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Table 1:

	TM	2-CTM	SM	RM(R_2(1))
α	2	4	6	6
β	0	2	4	3
γ	1	1	1	2

$$x_{n+1} = x_n - \frac{2.f(x_n)}{f'(x_n)+f'(y_n)} \tag{3}$$

This formula is the same of Arithmetic Newton's method [9].

By using composite Trapezoidal rule [2-CTM] (of 2-Trapezes) we have the iterative formula:

$$x_{n+1} = x_n - \frac{4.f(x_n)}{f'(x_n)+2.f'(\frac{x_n+y_n}{2})+f'(y_n)} \tag{4}$$

This formula is the same of Nedzhibov method [9].

In general, by using the integration rules of composite trapezoidal (for 2 trapezes) and Romberg's rule, we see the general iterative formulas:

$$x_{n+1} = x_n - \frac{\alpha.f(x_n)}{f'(x_n)+\beta.f'(\frac{x_n+y_n}{2})+\gamma.f'(y_n)} \tag{5}$$

where α, β are γ given for all of these methods given by Table (1):

By using the Chebyshev's Integral for N subinterval of $[x_n, x]$ we have the iterative formula:

$$x_{n+1} = x_n - \frac{N.f(x_n)}{\sum_{i=1}^{N} f'(\alpha_1+\beta_1 x_i)} \tag{6}$$

where α_1 are β_1 given by following relations, $\alpha_1 = \frac{x_n+y_n}{2}$ and $\beta_1 = \frac{y_n-x_n}{2}$

By using the Gaussian quadratic Integral, we have the iterative formula:

$$x_{n+1} = x_n - \frac{2.f(x_n)}{\sum_{i=1}^{N} c_i.f'(\alpha_1+\beta_1 x_i)} \tag{7}$$

where α_1 are β_1 given by following relations, $\alpha_1 = \frac{x_n+y_n}{2}$ and $\alpha_1 = \frac{y_n-x_n}{2}$. Here, the constants c_i are x_i given, see for example [11], specially, for N=5 by Chebyshev's method and for N=2 and $c_1 = c_2 = 1$ we trait the following examples that illustrate the results obtained by these new methods and we compare the results with Newton's method and with some other recently methods to compare the efficiency and accuracy of these methods by PASCAL software and show some examples.

Analysis of Convergence

Theorem 1: If n > 0, then the formula defined in equations (5), (6) and (7) converge to the simple zero of f defined by equation (2) which is called the root of the equation.

Proof: To prove the result it suffices to prove, as, $n \to \infty$, $f(x_n) \to 0$. Since, it is a iterative process, so $n \to \infty$ meant $x_{n+1} \approx x_n$, but as we consider (Newton method) $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = y_n$, hence, as $n \to \infty$, we have $f(x_n) \to 0$ that proves the required result.

Numerical Results and Discussion: The following examples illustrate the result obtained by suggested methods and verify the validity of the new iteration methods to solve non-linear equations. Some other proposed methods are presented to compare with the suggested methods. The comparison appear that these methods converge much more quickly than the Newton method [10], Hybrid iteration method [3], new Hybrid iteration method [4] and the modified Newton methods [8].

Some Examples

Examples (1): Consider the following equation [4, 8]: $f(x) = x^3 - e^{-x} = 0$.

Starting with x_0=1, the results obtained by Newton iteration [8], hybrid iteration [3], New Hybrid iteration [4], Modified Newton method [8] and present iterations are shown in Table 2.

Examples (2): Consider the following equation [7, 8]: $f(x) = \sin x - 0.5x = 0$.

Starting with x_0=1.5, the results obtained by Newton iteration [8], hybrid iteration [3], New Hybrid iteration [4], Modified Newton method [8] and present iterations are shown in Table 3.

Examples (3): Consider the following equation [7, 8]: $f(x) = x^3 - 2x - 5 = 0$.

Starting with x_0=2, the results obtained by Newton iteration [8], hybrid iteration [3], New Hybrid iteration [4], Modified Newton method [8] and present iterations are shown in Table 4.

Examples (4): Consider the following equation [7, 8]: $f(x) = x.\ \mathrm{lm}\ x - 1.2 = 0$.

Starting with x_0=2, the results obtained by Newton iteration [8], hybrid iteration [3], New Hybrid iteration [4], Modified Newton method [8] and present iterations are shown in Table 5.

Table 2: Comparison of the results obtained by different methods for solving $f(x)=x^3-e^{-x}=0$

Iterative method	n	x_n	$\|f(x_n)\|$
Newton	5	0.77288295914921012	0.00000000000000006456(6.5E-17)
Hybrid	6	0.77288295914921012	0.00000000000000006456(6.5E-17)
New Hybrid	6	0.772882959149210113	1.62630325872826E-19
Modified Newton	4	0.772882959149210680	1.27667E-14
TI	3	0.772882959149210113	2.71050543121376E-20
2-CTM	3	0.772882959149210113	2.71050543121376E-20
SI	3	0.772882959149210113	8.13151629364128E-20
RI	4	772882959149210113	2.71050543121376E-20
CHI	3	772882959149210113	1.62630325872826E-20
GQI	3	772882959149210113	1.62630325872826E-20

Table 3: Comparison of the results obtained by different methods $f(x) = \sin x - 0.5x = 0$ for solving.

Iterative method	n	x_n	$\|f(x_n)\|$
Newton	5	1.895494267033980	0.000000000
Hybrid	6	1.895494267033980	0.000000000
New Hybrid	5	1.8954942670339999	0.000000000
Modified Newton	4	1.895494267033980900	0.000000000
TI	3	1.89549426703398095 0	5.42101086242752E-20
2-CTM	3	1.89549426703398095 0	5.42101086242752E-20
SI	3	1.89549426703398095 0	5.42101086242752E-20
RI	4	1.89549426703398095 0	5.42101086242752E-20
CHI	3	1.89549426703398095 0	5.42101086242752E-20
GQI	3	1.89549426703398095 0	5.42101086242752E-20

Table 4: Comparison of the results obtained by different methods $f(x) = x^3 - 2x - 5 = 0$. for solving

Iterative method	n	x_n	$\|f(x_n)\|$
Newton	5	2.09455148151852778	8.8818E-12
Hybrid	9	2.09455148151852700	3.55271E-11
New Hybrid	5	2.09455148154232650	8.8818E-12
Modified Newton	4	2.094551481542326500	8.8818E-12
TI	3	2.0945514815542326590	1.30104260698261E-18
2-CTM	3	2.0945514815542326590	1.30104260698261E-18
SI	3	2.0945514815542326590	1.30104260698261E-18
RI	4	2.0945514815542326590	1.30104260698261E-18
CHI	4	2.0945514815542326590	1.30104260698261E-18
GQI	3	2.0945514815542326590	1.30104260698261E-18

Table 5: Comparison of the results obtained by different methods $f(x) = x.\ \ln x - 1.2 = 0$. for solving

Iterative method	n	x_n	$\|f(x_n)\|$
Newton	5	1.88808675302834340	2.2204E-12
Hybrid	10	1.88808675302834340	2.2204E-12
New Hybrid	6	1.88808675302834340	2.2204E-12
Modified Newton	4	1.888086753028343600	2.2204E-12
TI	3	1.888086753028343520	0.00000000
2-CTM	3	1.888086753028343520	0.00000000
SI	3	1.888086753028343520	0.00000000
RI	3	1.888086753028343520	0.00000000
CHI	3	1.888086753028343520	0.00000000
GQI	3	1.888086753028343520	0.00000000

Table 6: Comparison of the results obtained by different methods $f(x) = x^3 + 4x^2 - 10 = 0$. for solving

Iterative method	n	x_n	$\|f(x_n)\|$
Newton	5	1.365230013414096850	0.0(<E-17)
Hybrid	6	1.365230013893928000	7.923642E-9
New Hybrid	4	1.36523001344889900	5.565379E-9
Modified Newton	3	1.365230013414096900	9.010747E-11
TI	4	1.365230013414096850	0.0000000000
2-CTM	3	1.365230013414096850	0.0000000000
SI	3	1.365230013414096850	0.0000000000
RI	4	1.365230013414096850	0.0000000000
CHI	3	1.365230013414096850	0.0000000000
GQI	3	1.365230013414096850	0.0000000000

Table 7: Comparison of the results obtained by different methods $f(x) = x^2 - 5 = 0$. for solving

Formula	n	x_n	$\|f(x_n)\|$
Newton	7	2. 23606898849978980	8.8818E-12
Hybrid	4	2.23606898849978980	8.8818E-12
New Hybrid	6	2.236068988499789800	8.8818E-12
Modified Newton	5	2.236068988499789800	8.8818E-12
TI	4	2.236067977499789700	0.0000000000
2-CTM	3	2.236067977499789700	0.0000000000
SI	3	2. 236067977499789700	0.0000000000
RI	3	2.236067977499785120	2.04671349313124E-14
	4	2.236067977499789700	0.0000000000
CHI	3	2.236067977499789700	0.0000000000
GQI	3	2.236067977499789700	0.0000000000

Examples (5): Consider the following equation [7, 8]: $f(x) = x^3 + 4x^2 - 10 = 0$.

Starting with x_0=2, the results obtained by Newton iteration [8], hybrid iteration [3], New Hybrid iteration [4], Modified Newton method [8] and present iterations are shown in Table 6.

Examples (6): Consider the following equation [6, 8]: $f(x) = x^2 - 5 = 0$.

Starting with x_0=2, , the results obtained by Newton iteration [8], hybrid iteration [3], New Hybrid iteration [4], Modified Newton method [8] and present iterations are shown in Table 7.

CONCLUSION

We have shown a comparison of convergence for Newton's method, Hybrid iteration method, new Hybrid iteration method and modified Newton method with the suggested methods, we show by numerical results that these present methods converge more quickly than these other methods.

REFERENCES

1. Weeraksoon, S. and T.G.I. Fernando, 2000. A variant of Newton's method with accelerated third-order convergence, Appl. Math. Lett., 13: 87-93.
2. Homeier, H.H.H., 2005. On Newton-type methods with cubic convergence, Journal of Computational and Applied Mathematics, 176(2): 425-432.
3. Xing-Guo Luo, 2005. A note on the new iteration method for solving algebraic equation, Applied Mathematics and Computation, 171(2): 1177-1183.
4. Nasr-Al-Din Ide, 2008. A new Hybrid iteration method for solving algebraic equations, Applied Mathematics and Computation, 195(2): 772-774.
5. Frontini, M. and E. Sormani, 2003. Some variant of Newton's method with third-order convergence, Applied Mathematics and Computation, 140(2-3): 419-426.
6. Nasr-Al-Din Ide, 2008. On modified Newton methods for solving a non linear algebraic equations, Applied Mathematics and Computation, 198(1): 138-142.

7. Jishing Kou, Yitian Li and Xinhua Wang, 2006. On modified Newton methods with cubic convergence. Applied Mathematics and Computation, 176: 123-127.
8. Masoud Allame and Nafiseh Azad, 2012. On Modified Newton Method for Solving a Nonlinear Algebraic Equations by Mid-Point, World Applied Sciences Journal, 17(12): 1546-1548.
9. Babajee, D.K.R. and M.Z. Dauhoo, 2006. An analysis of the properties of the variants of Newton's method with third order convergence Applied Mathematics and Computation, 183(1): 659-684.
10. Avram Sidi, 2006. Unified treatment of regulafalsi, Newton-Raphson, Secent and Steffencen methods for nonlinear equations. Journal of Online Mathematic and Its Applications, pp: 1-13.
11. Éléments De Calcule Numérique Par B. Demidovitch and I. Maron, 1973. Traduction française Edition MIR Moscou.