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The Extended Generalized Riccati Equation Mapping Method for the (1+1)-Dimensional Modified KdV Equation

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Abstract: In this article, the generalized Riccati equation mapping is extended by the (G'/G)-expansion method. In this method, the auxiliary equation $G' = r + pG + qG^2$ is used and called the generalized Riccati equation, where p,q and r are arbitrary constants. We construct twenty five exact traveling wave solutions of the (1+1)-dimensional modified KdV equation involving parameter by applying this method. The solutions are presented in terms of the hyperbolic, the trigonometric and the rational functional form including solitons and periodic solutions. Moreover, it is worth mentioning that one of our obtained solutions is in good agreement with the existing results which in turn validates our other solutions. In addition, some of newly obtained solutions are described in the figures.

Key words: The (G'/G)-expansion method · The generalized Riccati equation mapping · The modified KdV equation · Traveling wave solutions

INTRODUCTION

The investigation of exact traveling wave solutions of nonlinear partial differential equations (PDEs) has become one of the most exciting and incredible research fields in engineering sciences, applied mathematics and other technical arena [1-44]. In the recent years, various methods have been established, such as, the homogeneous balance method [1], the Jacobi elliptic function expansion method [2], the Backlund transformation method [3], the Hirota's bilinear transformation method [4], the homotopy analysis method [5-8], the F-expansion method [9], the variational iteration method [10-18], the Exp-function method [19-25] and others [26-33].

Recently, Wang *et al*. [34] presented a powerful and direct method which is called the (G'/G)-expansion method to construct traveling wave solutions for the nonlinear evolution equations (NLEEs). They used $G'' + \lambda G' + \mu G = 0$, as an auxiliary equation to the method, where λ and μ are arbitrary constants. Then, many researchers implemented the useful (G'/G)-expansion method to study different nonlinear PDEs for searching traveling wave solutions [35-38].

Zhu [39] introduced the generalized Riccati equation mapping to construct non-traveling wave solutions of the (2+1)-dimensional Boiti-Leon-Pempinelle equation. In the generalized Riccati equation mapping, the auxiliary equation $G' = r + pG + qG^2$ is used, where p, q and r are arbitrary constants. Li and Dai [40] applied the generalized Riccati equation mapping method to construct traveling wave solutions for the higher dimensional Jimbo-Miwa equation whilst Guo *et al*. [41] investigated the diffusion-reaction and mKdV equation with variable coefficient by using the extended Riccati equation mapping method. Naher and Abdullah [42] implemented the extended generalized Riccati equation mapping method to construct traveling wave solutions of the modified Benjamin-Bona-Mahony equation and so on.

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Many researchers constructed traveling wave solutions for the (1+1)-dimensional modified KdV equation by using different methods. For example, Wang *et al*. [34] investigated the same equation by using the basic (G'/G)-expansion method wherein they employed the second order linear ordinary differential equation as an auxiliary equation for obtaining traveling wave solutions. Zhang [43] used the direct algebraic method to find complex solutions for the same equation while Yinping and Zhibin [44] applied the homotopy analysis method to obtain some solutions for the modified KdV equation.

The aim of our present work is to apply the generalized Riccati equation mapping combined with the (G'/G)-expansion method to construct new results of the (1+1)-dimensional modified KdV equation involving parameter.

The Generalized Riccati Equation Mapping Combined with the (G'/G)-expansion Method: Suppose the general nonlinear partial differential equation:

$$F\left(u, u_t, u_x, u_{xt}, u_{tt}, u_{xx}, \ldots\right) = 0, \tag{1}$$

where $u = u(x, t)$ is an unknown function, F is a polynomial in $u = u(x,t)$ and the subscripts denote the partial derivatives.

The main steps of the generalized Riccati equation mapping combined with the (G'/G)-expansion method are as follows [34,39]:

Step 1: Consider the traveling wave variable:

$$u(x,t) = v(\xi), \quad \xi = x - Vt, \tag{2}$$

where V is the speed of the traveling wave. Now using Eq. (2), Eq. (1) is converted into an ordinary differential equation for $v(\xi)$

$$Q\left(v, v', v'', \ldots\right) = 0, \tag{3}$$

where the superscripts stand for the ordinary derivatives with respect to ξ

Step 2: According to possibility, integrates Eq. (3) term by term one or more times, yields constant(s) of integration. For simplicity, the integral constant(s) may be zero.

Step 3: Suppose that the traveling wave solution of Eq. (3) can be expressed in the form [34, 39]:

$$v(\xi) = \sum_{j=0}^{m} d_j \left(\frac{G'}{G}\right)^j \tag{4}$$

where $d_j (j = 0,1,2,...,m)$ and $d_m \neq 0$, with $G = G(\xi)$ express the solution of the generalized Riccati equation:

$$G' = r + pG + qG^2, \tag{5}$$

where p,q,r are arbitrary constants and $q \neq 0$.

Step 4: To determine the positive integer m, consider the homogeneous balance between the highest order nonlinear terms and the highest order derivatives appearing in Eq. (3).

Step 5: Substitute Eq. (4) along with Eq. (5) into the Eq. (3), then collect all the coefficients with the same order, the left hand side of Eq. (3) converts into polynomials in $G'(\xi)$ and $G^{-1}(\xi)$, $(i = 0,1,2,...)$. Then equating each coefficient of the polynomials to zero, yields a set of algebraic equations for $d_j (j = 0,1,2,...,m)$, p,q,r and V.

Step 6: Solve the system of algebraic equations which are obtained in step 5 with the aid of algebraic software Maple and we obtain values for $d_j (j = 0,1,2,...,m)$ and V Then, substituting obtained values in Eq. (4) along with Eq. (5) with the value of m, we obtain exact solutions of Eq. (1).

We have the following twenty seven solutions including four different families of Eq. (5).

Family 2.1: When $p^2 - 4qr > 0$ and $pq \neq 0$ or $qr \neq 0$, the solutions of Eq. (5) are:

$$G_1 = \frac{-1}{2q}\left(p + \sqrt{p^2 - 4qr}\,\tanh\frac{\sqrt{p^2 - 4qr}}{2}\xi\right),$$

$$G_2 = \frac{-1}{2q}\left(p + \sqrt{p^2 - 4qr}\,\coth\frac{\sqrt{p^2 - 4qr}}{2}\xi\right),$$

$$G_3 = \frac{-1}{2q}\left(p + \sqrt{p^2 - 4qr}\left(\tanh\sqrt{p^2 - 4qr}\ \xi \pm i\,\mathrm{sec}\,h\sqrt{p^2 - 4qr}\ \xi\right)\right),$$

$$G_4 = \frac{-1}{2q}\left(p + \sqrt{p^2 - 4qr}\left(\coth\sqrt{p^2 - 4qr}\ \xi \pm \mathrm{csc}\,h\sqrt{p^2 - 4qr}\ \xi\right)\right),$$

$$G_5 = \frac{-1}{4q}\left(2p + \sqrt{p^2 - 4qr}\left(\tanh\frac{\sqrt{p^2 - 4qr}}{4}\ \xi + \cot h\frac{\sqrt{p^2 - 4qr}}{4}\ \xi\right)\right),$$

$$G_6 = \frac{1}{2q}\left(-p + \frac{\pm\sqrt{\left(A^2 + B^2\right)\left(p^2 - 4qr\right)} - A\sqrt{p^2 - 4qr}\cosh\sqrt{p^2 - 4qr}\ \xi}{A\sinh\sqrt{p^2 - 4qr}\ \xi + B}\right),$$

$$G_7 = \frac{1}{2q}\left(-p - \frac{\pm\sqrt{\left(A^2 + B^2\right)\left(p^2 - 4qr\right)} + A\sqrt{p^2 - 4qr}\cosh\sqrt{p^2 - 4qr}\ \xi}{A\sinh\sqrt{p^2 - 4qr}\ \xi + B}\right),$$

where A and B are two non-zero real constants.

$$G_8 = \frac{2r\cosh\frac{\sqrt{p^2 - 4qr}}{2}\ \xi}{\sqrt{p^2 - 4qr}\sinh\frac{\sqrt{p^2 - 4qr}}{2}\ \xi - p\cosh\frac{\sqrt{p^2 - 4qr}}{2}\ \xi},$$

$$G_9 = \frac{-2r\sinh\frac{\sqrt{p^2 - 4qr}}{2}\ \xi}{p\sinh\frac{\sqrt{p^2 - 4qr}}{2}\ \xi - \sqrt{p^2 - 4qr}\cosh\frac{\sqrt{p^2 - 4qr}}{2}\ \xi},$$

$$G_{10} = \frac{2r\cosh\sqrt{p^2 - 4qr}\ \xi}{\sqrt{p^2 - 4qr}\sinh\sqrt{p^2 - 4qr}\xi - p\cosh\sqrt{p^2 - 4qr}\ \xi \pm i\sqrt{p^2 - 4qr}},$$

$$G_{11} = \frac{2r\sinh\sqrt{p^2 - 4qr}\ \xi}{-p\sinh\sqrt{p^2 - 4qr}\xi + \sqrt{p^2 - 4qr}\cosh\sqrt{p^2 - 4qr}\ \xi \pm \sqrt{p^2 - 4qr}},$$

Family 2.2: When $p^2 - 4qr<0$ and $pq \neq 0$ or $qr \neq 0$, the solutions of Eq. (5) are:

$$G_{12} = \frac{1}{2q}\left(-p + \sqrt{4qr - p^2}\ \tan\frac{\sqrt{4qr - p^2}}{2}\xi\right),$$

$$G_{13} = \frac{-1}{2q}\left(p + \sqrt{4qr - p^2}\ \cot\frac{\sqrt{4qr - p^2}}{2}\xi\right),$$

$$G_{14} = \frac{1}{2q}\left(-p + \sqrt{4qr - p^2}\left(\tan\sqrt{4qr - p^2}\ \xi \pm \sec\sqrt{4qr - p^2}\ \xi\right)\right),$$

$$G_{15} = \frac{-1}{2q}\left(p + \sqrt{4qr - p^2}\left(\cot\sqrt{4qr - p^2}\ \xi \pm \csc\sqrt{4qr - p^2}\ \xi\right)\right),$$

$$G_{16} = \frac{1}{4q}\left(-2p + \sqrt{4qr - p^2}\left(\tan\frac{\sqrt{4qr - p^2}}{4}\xi - \cot\frac{\sqrt{4qr - p^2}}{4}\xi\right)\right),$$

$$G_{17} = \frac{1}{2q}\left(-p + \frac{\pm\sqrt{\left(A^2 - B^2\right)\left(4qr - p^2\right)} - A\sqrt{4qr - p^2}\cos\sqrt{4qr - p^2}\,\xi}{A\sin\sqrt{4qr - p^2}\,\xi + B}\right),$$

$$G_{18} = \frac{1}{2q}\left(-p - \frac{\pm\sqrt{\left(A^2 - B^2\right)\left(4qr - p^2\right)} + A\sqrt{4qr - p^2}\cos\sqrt{4qr - p^2}\,\xi}{A\sin\sqrt{4qr - p^2}\,\xi + B}\right),$$

where A and B are two non-zero real constants and satisfies $A^2 - B^2 > 0$.

$$G_{19} = \frac{-2r\cos\frac{\sqrt{4qr - p^2}}{2}\xi}{\sqrt{4qr - p^2}\sin\frac{\sqrt{4qr - p^2}}{2}\xi + p\cos\frac{\sqrt{4qr - p^2}}{2}\xi},$$

$$G_{20} = \frac{2r\sin\frac{\sqrt{4qr - p^2}}{2}\xi}{-p\sin\frac{\sqrt{4qr - p^2}}{2}\xi + \sqrt{4qr - p^2}\cos\frac{\sqrt{4qr - p^2}}{2}\xi},$$

$$G_{21} = \frac{-2r\cos\sqrt{4qr - p^2}\,\xi}{\sqrt{4qr - p^2}\sin\sqrt{4qr - p^2}\,\xi + p\cos\sqrt{4qr - p^2}\,\xi \pm \sqrt{4qr - p^2}},$$

$$G_{22} = \frac{2r\sin\sqrt{4qr - p^2}\,\xi}{-p\sin\sqrt{4qr - p^2}\,\xi + \sqrt{4qr - p^2}\cos\sqrt{4qr - p^2}\,\xi \pm \sqrt{4qr - p^2}},$$

Family 2.3: when $r = 0$ and $pq \neq 0$, the solution Eq. (5) become:

$$G_{23} = \frac{-pd}{q\left(d + \cosh(p\xi) - \sinh(p\xi)\right)},$$

$$G_{24} = \frac{-p\left(\cosh(p\xi) + \sinh(p\xi)\right)}{q\left(d + \cosh(p\xi) + \sinh(p\xi)\right)},$$

where d is an arbitrary constant.

Family 2.4: when $q \neq 0$ and $r = p = 0$, the solution of Eq. (5) becomes:

$$G_{25} = \frac{-1}{q\xi + e_1},$$

where e_1 is an arbitrary constant.

Remark: Zhu [39] tabulated 27 solutions of the generalized Riccati equation. But, unfortunately, Zhu's solution ϕ_9 and solution ϕ_{12}; and solution ϕ_{21} and solution ϕ_{24} are identical. In this article, we have tabulated only distinct twenty five solutions.

Applications of the Method: In this section, we construct new exact traveling wave solutions for the ((1+1)-dimensional modified KdV equation. In this work, we consider equation followed by Wang *et al*. [34]:

$$u_t - u^2 u_x + \delta u_{xxx} = 0, \quad \text{where} \quad \delta > 0. \tag{6}$$

Now, we use the transformation Eq. (2) into the Eq. (6), which yields:

$$-V v' - v^2 v' + \delta v''' = 0, \tag{7}$$

Eq. (7) is integrable, so, integrating Eq. (7) with respect ξ yields:

$$C - V v - \frac{1}{3} v^3 + \delta v'' = 0, \tag{8}$$

where C is an integral constant that could be determined later.

Taking the homogeneous balance between v'' and v^3 in Eq. (8), we obtain $m = 1$.
Therefore, the solution of Eq. (8) is of the form:

$$v(\xi) = d_1 (G'/G) + d_0, \quad d_1 \neq 0. \tag{9}$$

Using Eq. (5), Eq. (9) can be re-written as:

$$v(\xi) = d_1 \left(p + rG^{-1} + qG\right) + d_0, \tag{10}$$

where p,q and r are free parameters.

By substituting Eq. (10) into Eq. (8), collecting all coefficients of G^i and $G^i (i = 0,1,2,...)$ and setting them equal to zero, we obtain a set of algebraic equations for d_0,d_1,p,q,r,C and V (for simplicity, the algebraic equations are not displayed). Solving the system of algebraic equations with the aid of algebraic software Maple 13, we obtain

$$d_0 = \mp \frac{1}{2} p\sqrt{6\delta}, \quad d_1 = \pm\sqrt{6\delta}, \quad V = \frac{-1}{2}\delta p^2 - 4\delta qr, \quad C = \pm 2\delta rpq\sqrt{6\delta},$$

where p,q and R are free parameters.

Family 3.1: The soliton and soliton-like solutions of Eq. (6) (when $p^2 - 4qr > 0$ and $pq \neq 0$ or $qr \neq 0$) are:

$$v_1 = \pm\sqrt{6\delta}\,\frac{2\Omega^2 \operatorname{sec}h^2(\Omega\xi)}{p + 2\Omega\tanh(\Omega\xi)} \mp \frac{1}{2} p\sqrt{6\delta},$$

where $\Omega = \frac{1}{2}\sqrt{p^2 - 4qr}$, $\xi = x + \left(\frac{1}{2}\delta p^2 + 4\delta qr\right)t$ and p,q,r are arbitrary constants.

$$v_2 = \mp\sqrt{6\delta}\,\frac{2\Omega^2 \operatorname{csc}h^2(\Omega\xi)}{p + 2\Omega\coth(\Omega\xi)} \mp \frac{1}{2} p\sqrt{6\delta},$$

$$v_3 = \pm\sqrt{6\delta}\,\frac{4\Omega^2 \operatorname{sec}h(2\Omega\,\xi)(1 \mp i\sinh(2\Omega\xi))}{p\cosh(2\Omega\,\xi) + 2\Omega\sinh(2\Omega\xi) \pm i2\Omega} \mp \frac{1}{2}p\sqrt{6\delta},$$

$$v_4 = \mp\sqrt{6\delta}\,\frac{2\Omega^2 \operatorname{csc}h(\Omega\,\xi)}{p\sinh(\Omega\,\xi) + 2\Omega\cosh(\Omega\,\xi)} \mp \frac{1}{2}p\sqrt{6\delta},$$

Solutions v_4 and v_5 are identical. Therefore, we do not display the solution v_5.

$$v_6 = \mp\sqrt{6\delta}\,\frac{4A\Omega^2\left(A - B\sinh(2\Omega\xi) - \sqrt{A^2+B^2}\cosh(2\Omega\xi)\right)}{\left(A\sinh(2\Omega\xi)+B\right)\left(pA\sinh(2\Omega\xi) + pB - 2\Omega\sqrt{\left(A^2+B^2\right)} + 2A\Omega\cosh(2\Omega\xi)\right)} \mp \frac{1}{2}p\sqrt{6\delta},$$

$$v_7 = \mp\sqrt{6\delta}\,\frac{4A\Omega^2\left(A - B\sinh(2\Omega\xi) + \sqrt{A^2+B^2}\cosh(2\Omega\xi)\right)}{\left(A\sinh(2\Omega\xi)+B\right)\left(pA\sinh(2\Omega\xi) + pB + 2\Omega\sqrt{\left(A^2+B^2\right)} + 2A\Omega\cosh(2\Omega\xi)\right)} \mp \frac{1}{2}p\sqrt{6\delta},$$

where A and B are two non-zero real constants.

$$v_8 = \mp\sqrt{6\delta}\,\frac{2\Omega^2 \operatorname{sec}h(\Omega\xi)}{2\Omega\sinh(\Omega\xi) - p\cosh(\Omega\xi)} \mp \frac{1}{2}p\sqrt{6\delta},$$

$$v_9 = \pm\sqrt{6\delta}\,\frac{2\Omega^2 \operatorname{csc}h(\Omega\xi)}{2\Omega\cosh(\Omega\xi) - p\sinh(\Omega\xi)} \mp \frac{1}{2}p\sqrt{6\delta},$$

$$v_{10} = \pm\sqrt{6\delta}\,\frac{4\Omega^2 \operatorname{sec}h(2\Omega\xi)(1 \mp i\sinh(2\Omega\xi))}{p\cosh(2\Omega\xi) - 2\Omega\sinh(2\Omega\xi) \mp i2\Omega} \mp \frac{1}{2}p\sqrt{6\delta},$$

$$v_{11} = \pm\sqrt{6\delta}\,\frac{4\Omega^2 \operatorname{csc}h(2\Omega\xi)(1 \pm \cosh(2\Omega\xi))}{2\Omega\cosh(2\Omega\xi) - p\sinh(2\Omega\xi) \pm 2\Omega} \mp \frac{1}{2}p\sqrt{6\delta},$$

Family 3.2: The periodic form solutions of Eq. (6) (when $p^2 - 4qr < 0$ and $pq \neq 0$or $qp \neq 0$) are:

$$v_{12} = \pm\sqrt{6\delta}\,\frac{2\Psi^2 \sec^2(\Psi\xi)}{-p + 2\Psi\tan(\Psi\xi)} \mp \frac{1}{2}p\sqrt{6\delta},$$

where $\Psi = \frac{1}{2}\sqrt{4qr - p^2}$, $\xi = x + \left(\frac{1}{2}\delta p^2 + 4\delta qr\right)t$ and p,q,r are arbitrary constants.

$$v_{13} = \mp\sqrt{6\delta}\,\frac{2\Psi^2 \csc^2(\Psi\xi)}{p + 2\Psi\cot(\Psi\xi)} \mp \frac{1}{2}p\sqrt{6\delta},$$

$$v_{14} = \pm\sqrt{6\delta}\,\frac{4\Psi^2 \sec(2\Psi\,\xi)(1 \pm \sin(2\Psi\xi))}{-p\cos(2\Psi\,\xi) + 2\Psi\sin(2\Psi\xi) \pm 2\Psi} \mp \frac{1}{2}p\sqrt{6\delta},$$

$$v_{15} = \mp\sqrt{6\delta}\,\frac{2\Psi^2 \sec(\Psi\xi)}{2\Psi\sin(\Psi\xi) + p\cos(\Psi\xi)} \mp \frac{1}{2}p\sqrt{6\delta},$$

$$v_{16} = \mp\sqrt{6\delta}\,\frac{2\Psi^2 \csc(\Psi\,\xi)}{p\sin(\Psi\,\xi) + 2\Psi\cos(\Psi\xi)} \mp \frac{1}{2}p\sqrt{6\delta},$$

$$v_{17} = \pm\sqrt{6\delta}\left(\frac{4A\Psi^2\left(\sqrt{A^2-B^2}\cos(2\Psi\xi) - B\sin(2\Psi\xi) - A\right)}{\left(A\sin(2\Psi\xi)+B\right)\left(pA\sin(2\Psi\xi)+2A\Psi\cos(2\Psi\xi)+pB-2\Psi\sqrt{A^2-B^2}\right)}\right) \mp \frac{1}{2}p\sqrt{6\delta},$$

$$v_{18} = \mp\sqrt{6\delta}\left(\frac{4A\Psi^2\left(\sqrt{A^2-B^2}\cos(2\Psi\xi) + B\sin(2\Psi\xi) + A\right)}{\left(A\sin(2\Psi\xi)+B\right)\left(pA\sin(2\Psi\xi)+2A\Psi\cos(2\Psi\xi)+pB+2\Psi\sqrt{A^2-B^2}\right)}\right) \mp \frac{1}{2}p\sqrt{6\delta},$$

where A and B are two non-zero real constants and satisfies $A^2 - B^2 > 0$.

Our obtained solutions v_{15} and v_{19} are identical. Therefore, we do not write the solution v_{19} Moreover, our solutions v_{16} and v_{20} are coincided. Therefore, the solution v_{20} is not shown.

$$v_{21} = \mp\sqrt{6\delta}\,\frac{2\Psi^2\sec(2\Psi\xi)\left(1\pm\sin(2\Psi\xi)\right)\left(p\cos(2\Psi\xi)+2\Psi\sin(2\Psi\xi)\pm 2\Psi\right)}{\left(p^2-2qr\right)\cos^2(2\Psi\xi)+2\Psi\left(1\pm\sin(2\Psi\xi)\right)\left(2\Psi\pm p\cos(2\Psi\xi)\right)} \mp \frac{1}{2}p\sqrt{6\delta},$$

$$v_{22} = \pm\sqrt{6\delta}\,\frac{2\Psi^2\csc(2\Psi\xi)\left(-p\sin(2\Psi\xi)+2\Psi\cos(2\Psi\xi)\pm 2\Psi\right)}{\left(2qr-p^2\right)\cos(2\Psi\xi)-2p\Psi\sin(2\Psi\xi)\pm 2qr} \mp \frac{1}{2}p\sqrt{6\delta},$$

Family 3.3: The soliton and soliton-like solutions of Eq. (6) (when $r = 0$ and $pq \neq 0$) are:

$$v_{23} = \pm\sqrt{6\delta}\,\frac{p\left(\cosh(p\xi)-\sinh(p\xi)\right)}{d+\cosh(p\xi)-\sinh(p\xi)} \mp \frac{1}{2}p\sqrt{6\delta},$$

$$v_{24} = \pm\sqrt{6\delta}\,\frac{pd}{d+\cosh(p\xi)+\sinh(p\xi)} \mp \frac{1}{2}p\sqrt{6\delta},$$

where d is an arbitrary constant and $\xi = x + \left(\frac{1}{2}\delta p^2 + 4\delta qr\right)t.$

Family 3.4: The rational function solution (when $q \neq 0$ and $r = p = 0$) is:

$$v_{25} = \mp\sqrt{6\delta}\,\frac{q}{q\xi + e_1},$$

where e_1 is an arbitrary constant $\xi = x + \left(\frac{1}{2}\delta p^2 + 4\delta qr\right)t.$

Table 1: Comparison between Wang *et al*. [34] solutions and New solutions

Wang *et al.* [34] solutions	New solutions
I. If $C_1 = 1$, $C_2 = 1$ and $\delta = 1$, in section 4 solution becomes: $u_{5,6}(x,t) = \pm\sqrt{6}\frac{1}{1+x}.$	i. If $q = 1$, $e_1 = 1$, $\delta = 1$ and $v_{27}(\xi) = u_{5,6}(x,t)$ solution becomes: $u_{5,6}(x,t) = \pm\sqrt{6}\frac{1}{1+x}.$
ii. If $C_1 = 1$, $C_2 = 1$ and $\delta = -1$, in section 4 solution becomes: $u_{5,6}(x,t) = \pm i\sqrt{6}\frac{1}{1+x}.$	ii. If $q = 1$, $e_1 = 1, \delta = -1$ and $v_{27}(\xi) = u_{5,6}(x,t)$ solution v_{27} becomes: $u_{5,6}(x,t) = \pm i\sqrt{6}\frac{1}{1+x}.$

RESULTS AND DISCUSSION

It is important to mention that one of our solutions is in good agreement with already published results which are shown in the Table 1. Furthermore, graphical presentations for some newly constructed solutions are depicted in the following subsection in Figure 1 to Figure 6.

Outside of the above mentioned table, we have obtained new exact traveling wave solutions v_1 to v_{26} which are not informed in the earlier literature.

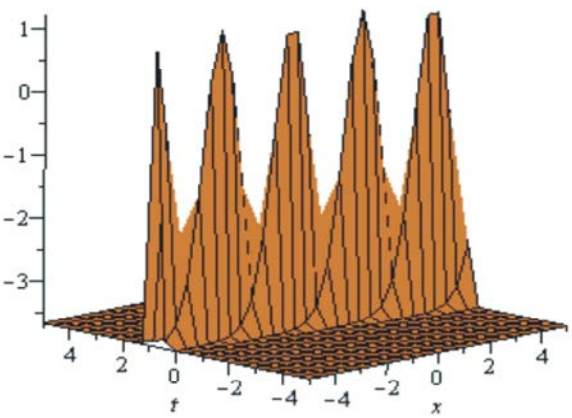

Fig. 1: Solitons solution for
$p = 3, q = 1, r = 0.25, \delta = 1$

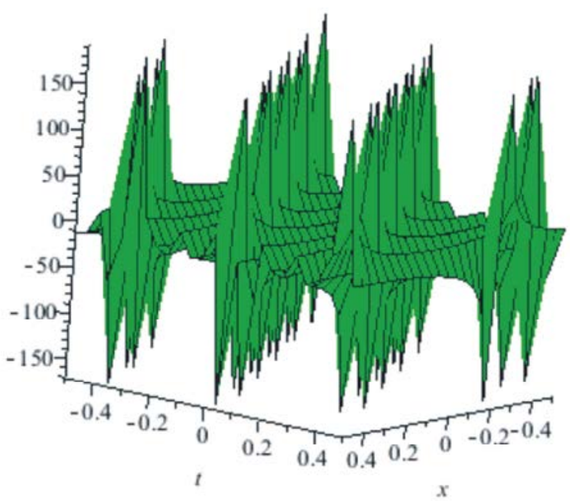

Fig. 2: Periodic solution for
$p = 1, q = 2, r = 1, \delta = 2$

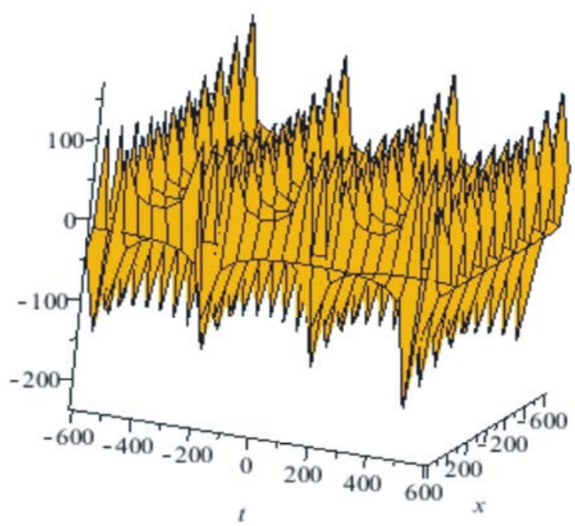

Fig. 3: Periodic solution for
$p = 3, q = 3, r = 2, \delta = 3$

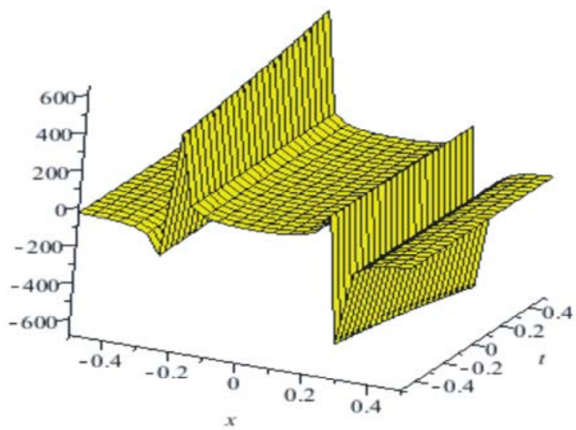

Fig. 4: Periodic solution for
$p = 1, q = 2, r = 1, \delta = 2$

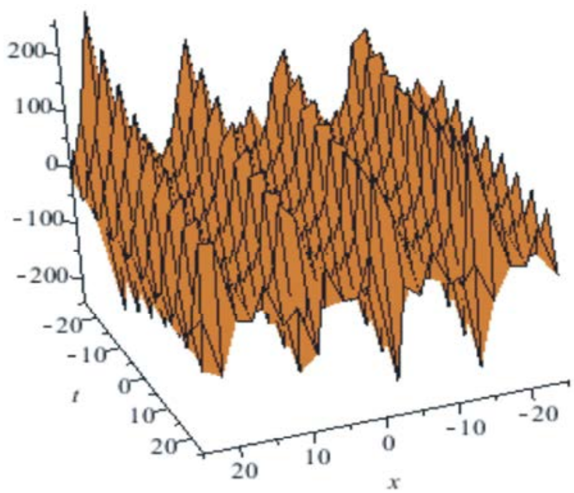

Fig. 5: Solitons solution for
$p = 3, q = 3, r = 3, \delta = 3$

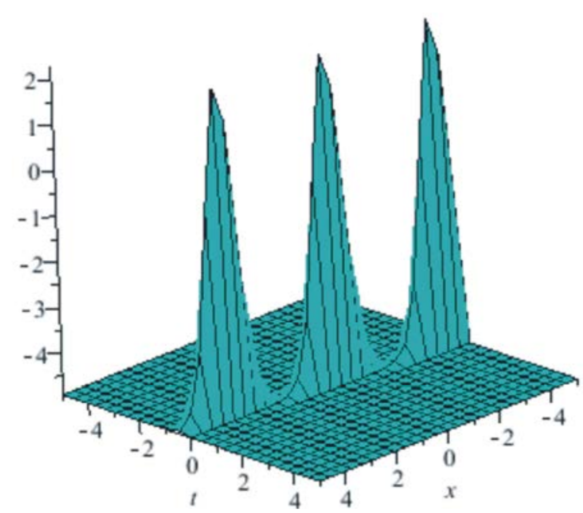

Fig. 6: Solitons solution for
$p = 4, q = 1, r = 0.25, \delta = 1$

Graphical Representations of the Solutions: The graphical illustrations of the solutions are depicted in Figures 1-6 with the aid of Maple:

CONCLUSIONS

In this article, the generalized Riccati equation mapping combined with the (G'/G)-expansion method is successfully applied to the (1+1)-dimensional modified KdV equation for constructing twenty five exact traveling wave solutions. The obtained traveling wave solutions are expressed in the hyperbolic, the trigonometric and the rational functions solutions including soliton and periodic solutions. Moreover, it is important to declare that one of our obtained solutions is in good agreement with the published results which validates our other solutions. Therefore, the generalized Riccati equation mapping combined with the (G'/G)-expansion method will be effectively used to investigate nonlinear evolution equations which arise in mathematical physics, engineering sciences and other technical arena.

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