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The Effect of a Cycloid Reducer Geometry on its Loading Capacity

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Abstract: The article addresses the effect of a k-h-v type cycloid reducer (CR) ring gear pin transmission geometry on indicators determining the gear unit loading capacity, particularly the satellite bearing load and contact stresses in the engagement area. An example of optimization of the said geometry with the specified transmission ratio and reducer dimensions is provided.

Key words: Cycloid reducer · Pin transmission · Conjugate profiles · Curvature radius · CAD

INTRODUCTION

K-h-v type cycloid reducers are widely used in mechatronic and robotic drive systems. The advantages of these speed reducers are small dimensions, low relative weight, wide range of transmission ratios and high efficiency. Packaging of certain actuating mechanisms is considerably simplified by the possibility of producing a speed reducer with a sufficiently big diameter of the axial hole.

Geometry and loading capacity of CRs have been the subject of many works [1-8].

The basic indicators determining the CR loading capacity are: satellite bearing [1] life and absence of fatigue pitting on teeth surface. Operating ability of the mechanism *v* formed by the satellite holes and output shaft pins (Fig. 1) are beyond the scope of this work.

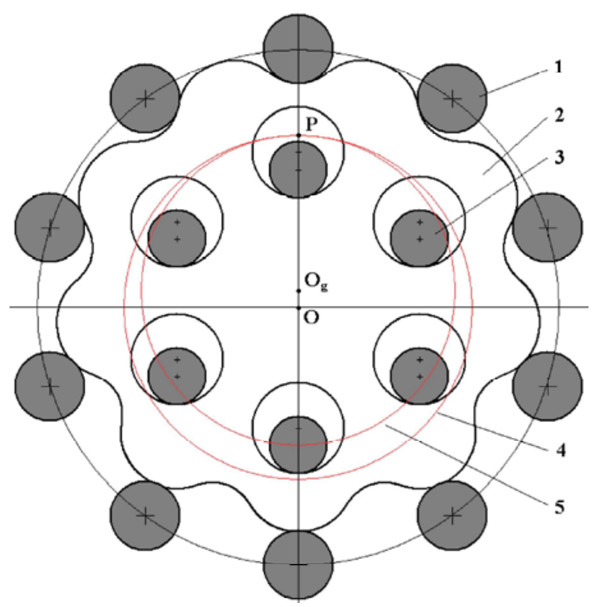

Fig. 1: Gear-pin cycloid reducer: 1 – ring gear pin; 2 – satellite; 3 – output shaft pin; 4 – wheel centroid; 5 – satellite centroid; **O** – central axis; $\mathbf{O_g}$ – satellite axis; **P** – engagement pole

Indicator Determining the Satellite Bearing Load: The bearing durability is determined by the load the bearing is exposed to. The satellite bearing is exposed to radial forces effecting from the driving wheel ring gear pins side and from the output shaft pins side.

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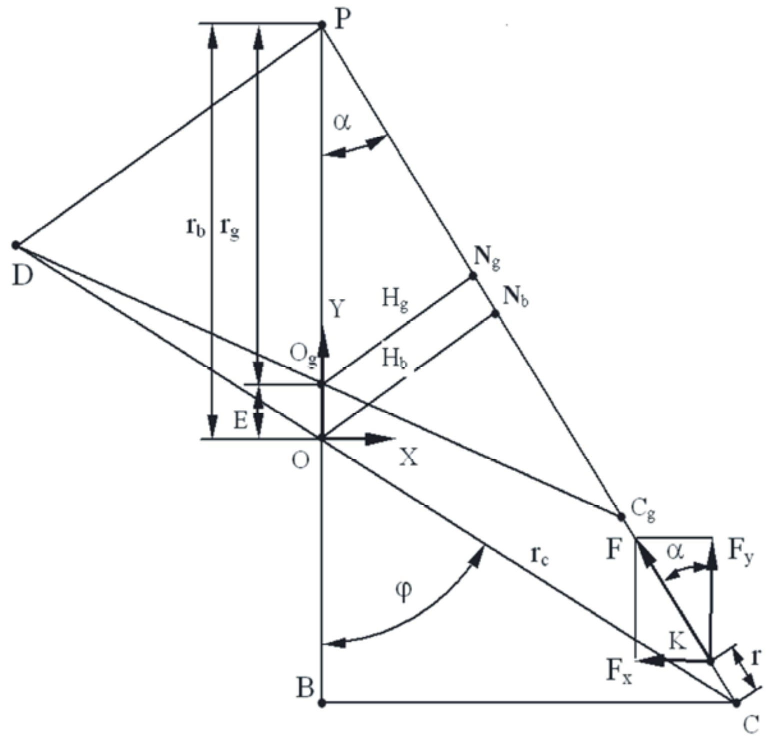

Fig. 2: Bobillier construction and force in the ring gear pin engagement with the satellite tooth: **O** is the central axis; $\mathbf{O_g}$ is the satellite axis; **P** is the engagement pole; **XOY** is a coordinate system tightly bound to the engagement pole; **φ** is the angular position of the ring gear pin; **CP** is the profile normal; **α** is the profile normal inclination angle; H_b,H_g are arms of the profile normal relative to the wheel and satellite axis; **C**,$\mathbf{C_g}$ are profile curvature centers of the ring gear pin and the satellite tooth; **D** is a crossing point of the straight lines connecting the curvature centers of conjugate profiles and curvature centers of centroids; **K** is the contact point; **F** is a force in engaged state; **E** is the satellite eccentricity; r is the ring gear pin radius; r_c is the radius of ring gear pin centers circumference; r_b, r_g are radiuses of the wheel and satellite centroids.

The force in the ring gear pin contact with the satellite tooth (Fig. 2) can be found from the assumption that it is proportional to the engagement strain, which, in turn, is proportional to the arm of contact normal relative to the satellite rotation axis 0_g [1].

The moment of force in engaged state relative to the central axis is determined by formula:

$$M = F \cdot H_b = k \cdot H_g \cdot H_b, \tag{2}$$

where

$$H_b = r_b \cdot \sin\alpha, \quad H_g = r_g \cdot \sin\alpha, \quad \alpha = arctg\left(\frac{r_c \cdot \sin\varphi}{r_b + r_c \cdot \cos\varphi}\right) \tag{3}$$

The k factor in formula (2) is determined from the condition of equilibrium of the output shaft – satellite – output shaft system:

$$M_1 + M_2 = \sum_{i=1}^{n} M_i, \tag{4}$$

where M_1, M_2 are input and output shaft torques; M_i is a moment of force in the area of the satellite engagement with an i –th ring gear pin; n is a number of ring gear pins simultaneously engaged with the satellite.

Taking into account (2-4), the force in the engagement an i –th ring gear pin is determined by formula:

$$F_i = \frac{M_1 + M_2}{r_b} \cdot \frac{\sin\alpha_i}{\sum_{j=1}^{n} \sin^2\alpha_j}. \tag{5}$$

Projections of resultant forces in engagements on the axis of the coordinate system tightly bound to the engagement pole (Fig. 2) are determined by formulas:

$$F_{\Sigma x} = -\frac{M_1 + M_2}{r_b}, \quad F_{\Sigma y} = \frac{M_1 + M_2}{r_b} \cdot \frac{\sum_{i=1}^{n} \sin\alpha_i \cdot \cos\alpha_i}{\sum_{j=1}^{n} \sin^2\alpha_j}. \tag{6}$$

Forces effecting the satellite from the output shaft pins side are directed along the OY axis. The magnitude of these forces depends on the number of pins and on their position on the circumference of the satellite hole centers, the radius of the circumference r_{co} satisfying the following condition:

$$r_{co} \le 0.5 \cdot \left(d_{gf} - d_p\right) - E - S, \tag{7}$$

where d_{gf} is a diameter of the satellite roots; $d_p$is a shaft pin diameter; E is the satellite eccentricity; S is the required distance between the bottom of a tooth root and a satellite hole. The magnitude r_{co} is set to be the same for all speed reducers of the given size. Therefore, the condition (7) is to be regarded in the first place as a limitation of the satellite eccentricity.

Similar to formula (6), the resultant of the forces effecting the satellite from the pins side is determined by formula:

$$F_{p\Sigma} = \frac{M_2}{r_{co}} \cdot \frac{\sum_{i=1}^{m} \sin\beta_i}{\sum_{j=1}^{m} \sin^2\beta_j}, \tag{8}$$

where β_i.is an angle between axis OY (Fig. 2) and a straight line connecting the satellite center with the center of an i-th hole. m is a number of the satellite holes simultaneously imparting torque to the output shaft.

Taking into account that $M_1 = M_2 \frac{1}{u}$, , where u is the speed reducer transmission ratio, the indicator determining the load of the satellite bearing can be represented as follows:

$$\Pi_1 = \sqrt{\left(\frac{1+u}{r_b \cdot u}\right)^2 + \left(\frac{1}{r_{co}} \cdot \frac{\sum_{i=1}^{m} \sin\beta_i}{\sum_{j=1}^{m} \sin^2\beta_j} - \frac{1+u}{r_b \cdot u} \cdot \frac{\sum_{i=1}^{n} \sin\alpha_i \cdot \cos\alpha_i}{\sum_{j=1}^{n} \sin^2\alpha_j}\right)^2}. \tag{9}$$

To optimize the speed reducer in terms of the satellite bearing load, the Π_1 indicator needs be minimized.

Indicator Determining Contact Stresses in the Engagement Area: According to the Hertz formula, contact stresses in the engagement area depend on the force in the engagement area determined by formula (5), as well as on the equivalent radius of profiles curvature in the point of contact.

Using the Bobillier construction [9,10], the curvature radius of the satellite tooth profile can be found without building the profile (Fig. 2):

$$\rho_g = \frac{r_c \cdot \sin\varphi}{\sin\alpha} - \frac{1}{\frac{r_b - r_g}{r_g \cdot r_b \cdot \cos\alpha} + \frac{\sin\alpha}{r_c \cdot \sin\varphi}} - r. \tag{10}$$

According to the above, the indicator determining contact stresses in the engagement area can be represented as follows:

$$\Pi_2 = \max\left\{\sqrt{\frac{1}{r_b} \cdot \frac{\sin\alpha_i}{\sum_{j=1}^{n} \sin^2\alpha_j} \cdot \left(\frac{1}{r} + \left(\frac{r_c \cdot \sin\varphi_i}{\sin\alpha_i} - \frac{1}{\frac{r_b - r_g}{r_g \cdot r_b \cdot \cos\alpha_i} + \frac{\sin\alpha_i}{r_c \cdot \sin\varphi_i}} - r\right)^{-1}\right)}\right\}_{i=1}^{i=n}.$$

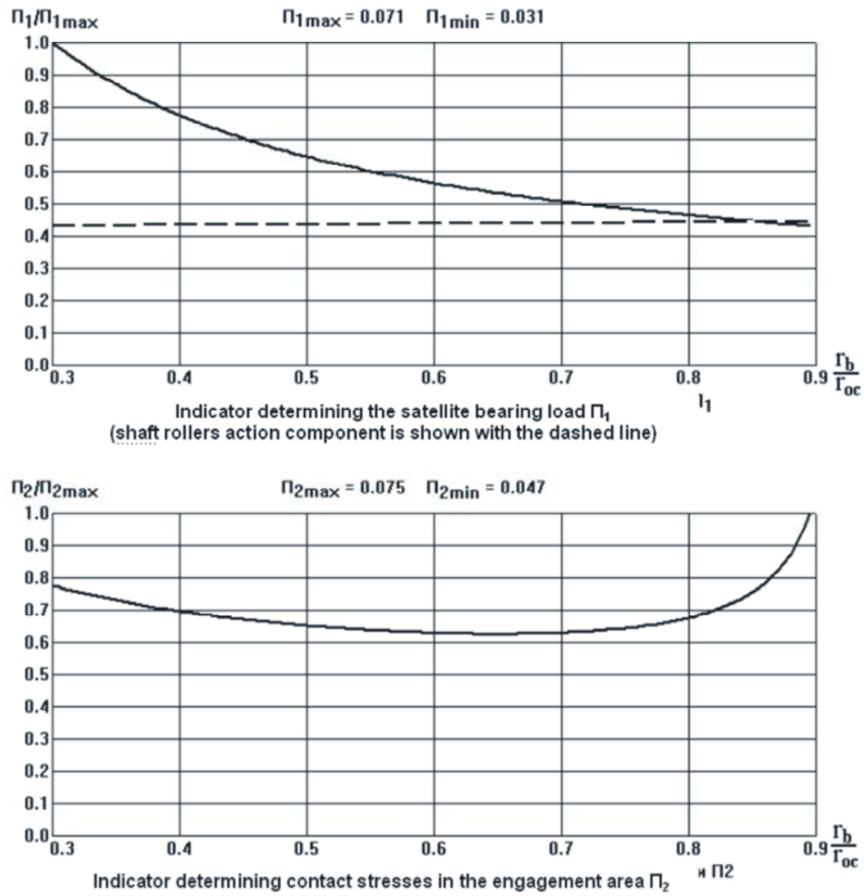

Fig. 3. Dependency of Π_1 и Π_2 indicators on the $\frac{r_b}{r_{oc}}$ ratio

To optimize the speed reducer in terms of contact stresses in the engagement area, the Π_2 indicator needs to be minimized.

Optimization of the CR Design Using Π_1 and Π_2 Indicators: The source data for calculation in the design of a speed reducer are normally the radius of gear ring pins centers circumference, which determines the speed reducer dimensions. The number of wheel gear pins and the number of satellite teeth altogether determine the reducer transmission ratio. Other parameters are found basing on the condition of obtaining the specified operating characteristics of the speed reducer, primarily high loading capacity. There are only two independent parameters among them: the gear ring pins radius and the radius of one of the centroids or the satellite eccentricity. Since roller bearing rollers are most often used as ring gear pins in CR, the ring gear pin diameters vary incrementally and the number of acceptable values of this parameter is rather limited. Optimization of the CR design is mainly brought to determining the second independent parameter. It is convenient to take the wheel centroid radius or the relation of that radius to the radius of the ring gear pin centers circumference as the said second independent parameter.

Dependency of Π_1 и Π_2 indicators on the $\frac{r_b}{r_{oc}}$ ratio shown on graphs in Fig.3, is given for a CR with the following parameters: $r_c = 50$ mm; $z_b = 52$; $z_g = 51$; $r = 1.75$ mm; $r_{oc} = 30$ mm.

The rise of the satellite bearing load (Π_1 indicator) in the range of low wheel centroid radius values is related to the reduction of arms of profile normals, which results in increased forces in the areas of the satellite teeth engagement with the ring gear pins. This is also a reason for the rise of contact stresses (Π_2 indicator). The faster rise of contact stresses in the range of high wheel centroid radius values is related to the reduction of the satellite teeth profile curvature radiuses.

The above graphs show that the centroid radius of the gear pin ring, optimal by both criteria for the given transmission ratio ($u = 51$), is within $(0.7÷0.8)\ r_c$.

CONCLUSION

Optimization of the cycloid gear geometry is of great importance for increasing the load-bearing capacity of the CR. Introduction of the composite indicators Π_1 and Π_2 allows simplifying and formalizing the task of optimal design of a speed reducer using the increased loading capacity criterion. The obtained results were used in the development of CAD systems for cycloid reducers.

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REFERENCES

1. Kudryavtsev, V.N., 1966. Planetary gears – M-L. Mashinostroyenie Publishers, pp: 307.
2. Dascalescu, A., 2005. Contribution to the Kinematics and Dynamics Studys of the Planetary Gearswith Cycloid Toothing and Roller Teeth, PhD Theses, Cluj-Napoca Technical University, Cluj-Napoca, Romania.
3. Dascalescu, A., 2007. The determination of the cycloid profile' limits of the cycloid reducer' satellite gear. Annals of the Oradea university. Fascicle of Management and Technological Engineering, VI(XVI): 697-700.
4. Gorla, C., *et al*., 2008. Theoretical and experimental analysis of a cycloidalspeed reducer. Journal of Mechanical Design, Transactions of the ASME, 130(11): 1126041-1126048.
5. Malhotra, S.K. and M.A. Parameswaran, 1983. Analysis of a cycloid speed reducer. Mechanism and Machine Theory, 18(6): 491-499.
6. Meng, Y., C. Wu and L. Ling, 2007. Mathematical modeling of the transmission performance of 2K-H pin cycloid planetary mechanism. Mechanism and Machine Theory, 42(7): 776-790.
7. Sensinger, J.W., 2010. Unified approach to cycloid drive profile, stress and efficiency optimization. Journal of Mechanical Design, Transactions of the ASME, 132(2): 0245031-0245035.
8. Sumitomo, Cyclo 6000 speed reducer. Catalog 03.601.50.007, 2010.
9. Litvin, F.L., 1968. Theory of gear-tooth systems. M., Nauka Publishers, pp: 584.
10. Litvin, F.L., 2004. Gear geometry and applied theory. Cambridge University Press, pp: 800.