

Predicting State-Variables of Organic Rankine Cycle by a Generalized Mathematical Model

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Abstract: The paper deals with the study of a control strategy for a generalized thermodynamic ORC power plant utilizing waste heat by using dynamic modelling technique. As Organic Rankine Cycle (ORC) is particularly suitable for the recovery of energy from low-quality heat sources. Therefore a generalized proposed predictive control technique is used for predicting the efficiency of low quality heat recovery process operating on an Organic Rankine Cycle to obtain the optimal system performance as presented in this paper. The first order mathematical modelling has been proposed to obtain the equilibrium point, taking into account the ambient condition developed through the nonlinear behavior of the plant. The balancing condition of the system has been analyzed through linear model evaluation of the manipulated variable to reach the desired set point in order to balance the system.

Key words: Organic Rankine Cycle (ORC) • Generalized predictive control (GPC) • Moving boundary • Linear Quadratic regulator

INTRODUCTION

ORC has been widely applied in converting low temperature heat into electric power [1] and power harnessing [2]. Due to environmental constraints, the interest in low quality Heat recovery has been in highest demand in the past Decades. Control and optimization are major issues in ORC systems applied to waste heat sources. This provides an overview of the current state of the art and of the main areas of Research and Development. Several new solutions have been proposed to generate electricity from low-quality heat sources, such as solar thermal power, biological waste heat and engine exhaust gases. The Organic Rankine Cycle (ORC) is preferred because of the simplicity and the availability of its components [1]. From the view of the practical engineering, it is necessary to control and monitor the critical properties of ORC for avoiding unwanted conditions to occur. So far, classical control technique, PI controller was used for regulating the performance of ORC [9]. However, the physical model built in cannot characterize the disturbances or the dynamics of operation conditions in ORC process. The performance of

the dynamic control strategy proposed is limited when applied to heat exchanging process due to its inherent nonlinearity and time varying nature, especially when significant load change or other disturbances occur. Some models of ORC systems have been established for analyzing thermo-economic performance analysis [3, 4]. Modeling of key issues in ORCs has been investigated in [4, 5]. However, how to synchronize the inputs and outputs of each process in ORC systems is a key issue to be analyzed. Very few attempts have been made at implementation of control strategies for ORC systems. In [5], the evaporating temperature and the degree of superheat were controlled by manipulating evaporator speed and the pump flow rate respectively. It is necessary to investigate more process knowledge and advanced control strategies to ensure the ORC power plants operate well over a wide range. Among the different control strategies, generalized predictive control (GPC) algorithm got the most attention, which computed a sequence of manipulated variable adjustments in order to optimize the future behavior of a plant [10, 11]. GPC technology can now be found in a wide variety of application areas including chemicals, power plants and food processing

[8]. This paper aims at proposing a multivariable control algorithm for an ORC power plant utilizing waste heat based on a control-oriented model. The ORC dynamic model system description is described in Section II. The control methodology in ORC model equations in Section III. The output prediction horizon is designed for ORC system in Section IV. The conclusion is given in Section V. Organic Rankine Cycles are a favorable option for having high efficiency, cost effective and low geothermal flows for electricity production. Some studies in the literature searched for parameters which are most suitable for correct analysis in the overall system performance. Among them, the works [1], [2] and [3] focused on the importance of using a heat recovery efficiency a part to thermal efficiency to correctly enhance the system's capability of using the available energy content of the geothermal source. Other studies [4], [5] and [6] emphasize mainly on the best choice of the cycle operating control systems both in terms of efficiency and costs.

Dynamic Model of ORC: Consider the basic thermodynamic model of ORC where there are five major components namely Pump, Evaporator, Turbine, Generator and Condenser. The design model proposed for the ORC composed by the sequence of components shown in Fig. 1. The operating point of the system is the equilibrium point obtained at given ambient conditions such as addition of disturbances like white noise etc. The point of equilibrium is obtained by balancing the reactions that each system's component provides in reply to the thermodynamic conditions developed at the boundary of its control volume. Two key components of the system are the capacities at low and high pressure that can either exist in the real system as separate components or be only included in the model to calculate

the equilibrium point. The capacities are filled and Drained by the two main streams in the system: the stream flowing through the pump (from the low pressure capacity to the high pressure capacity) and the stream flowing through the turbine (from the high pressure capacity to the low pressure capacity).

Methodology of Predictive Controller Applied in Orc System: Generalized predictive control (GPC) is one of the most popular predictive control algorithms developed. The GPC algorithm is a model-based strategy in order to maintain the outputs of the controlled plant close to their desired set-point referring to the equilibrium point ultimately. The future outputs are predicted at each instant based on the ORC model. Corresponding Faults such as transient either permanent or intermittent appearing during plant operation may result in logical errors, which can be critical for the realized applications [7]. Transient faults are especially critical as they dominate in contemporary technologies. Hence, an important practical issue is to evaluate dependability of applications in the presence of faults. It is particularly critical in many reactive systems (e.g. nuclear plants, satellites, aircrafts, chemical industry, medicine). In this paper, the plant model given by following Controlled Auto-Regressive Integrated Moving Average model which is a part of GPC algorithm and is used to predict the outputs of the ORC process using discrete samples (Z-transforms). For satisfying the control objectives, it makes the use of a CARIMA model and various horizons. This model is more appropriate in industrial applications where disturbances are non-stationary. A CARIMA model is used to obtain good output predictions and optimize a sequence of future control signals to minimize a multistage cost function defined over a prediction horizon. The inclusion of disturbance is necessary to deduce the correct controller structure.

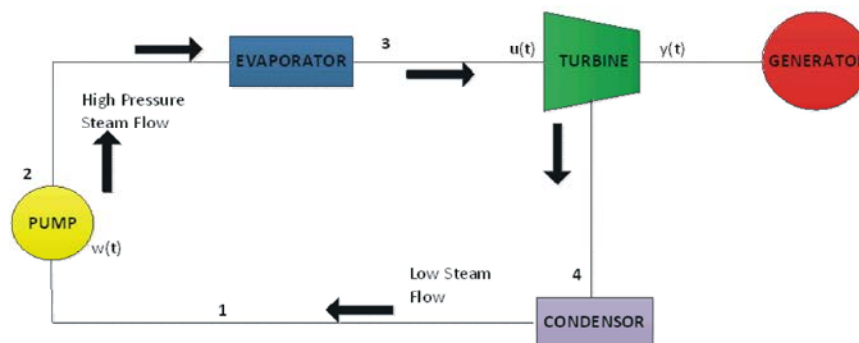


Fig. 1: Generalized Organic Rankine Cycle

$$A(z^{-1})y(t) = B(z^{-1})u(t-1) + 1/\Delta C(z^{-1})\epsilon(t) \quad (1) \quad 0_{n \times n} = [E_{j+1}(z^{-1}) - E_j(z^{-1})] \bar{A}(z^{-1}) + z^j F_{j+1}(z^{-1}) - F_j(z^{-1}) \quad (8)$$

where $y(t), u(t-1)$ are the output and manipulated variables of system, $\epsilon(t)$ is disturbance with zero mean, $\epsilon(t)$ is a differential operator, defined as $\Delta = 1 - z$, $A(z^{-1}), B(z^{-1}), C(z^{-1})$ are the polynomial matrices.

$$A(z^{-1}) = I_{n \times n} + A_1 z^{-1} + A_2 z^{-2} + \dots + A_{n_a} z^{-n_a} \quad (2)$$

$$B(z^{-1}) = B_0 + B_1 z^{-1} + B_2 z^{-2} + \dots + \quad (3)$$

$$C(z^{-1}) = I_{n \times n} + C_1 z^{-1} + C_2 z^{-2} + \dots + \quad (4)$$

The Cost function “ J ” is given as:

$$J = \sum_{j=1}^{N_y} \|\hat{y}(t+j|t) - w(t+j)\|_R^2 + \sum_{j=1}^{N_u} \|\Delta u(t+j-1)\|_Q^2 \quad (5)$$

Here the cost function is presented within the finite horizon, where $\hat{y}(t+j|t)$ is an optimal j -step prediction of the system output up to time t , $w(t+j)$ is the reference signal which is chosen for the system output to track, N_y is the costing horizon. N_u is the control costing horizon, R and Q are positive definite weighting matrices and Δu is the sequence of the control signal. The ultimate goal is to minimize the constraints of the cost function.

Output Prediction of Dynamic Model of ORC: In order to predict the equilibrium j step ahead prediction of the system output, the following Diophantine equation can be considered:

$$I_{n \times n} = E_j(z^{-1})\bar{A}(z^{-1}) + z^{-j}F_j(z^{-1}) \quad (6)$$

where $\bar{A}(z^{-1}) = A(z^{-1})\Delta$

$$E_j(z^{-1}) = E_{j,0} + E_{j,1}z^{-1} + E_{j,2}z^{-2} + \dots + E_{j,j-1}z^{j-1},$$

$$F_j(z^{-1}) = F_{j,0} + F_{j,1}z^{-1} + F_{j,2}z^{-2} + \dots + F_{j,n_a}z^{-n_a}$$

Diophantine equation presented here allows the variables to take integer values only. Diophantine problems have fewer equations than unknown variables and involve finding integers that work correctly for all equations. The Diophantine equation corresponding to the prediction for $y(t+j+1|t)$ can be written as follow:

$$I_{n \times n} = E_{j+1}(z^{-1})\bar{A}(z^{-1}) + z^{-(j+1)}F_{j+1}(z^{-1}) \quad (7)$$

Subtracting Equation (6) from Equation (7), it leads to:

The recursive solution of Diophantine equation can be written as:

$$\begin{cases} K_j = F_{j,0} \\ E_{j+1}(z^{-1}) = E_j(z^{-1}) + K_j z^{-1} \\ F_{j+1,i} = F_{j,i+1} - K_j \bar{A}_{i+1} \end{cases} \quad (9)$$

The initial value is:

$$\begin{cases} E_1 = I \\ F_1 = z(I - \bar{A}(z^{-1})) \end{cases}$$

The j -step prediction output can be obtained as follows:

$$y(t+j|t) = G_j(z^{-1}) \Delta u(t+j-1) + f_j \quad (10)$$

where,

$$E_j(z^{-1})B(z^{-1}) = G_j(z^{-1}) + z^{-1}G_{jp}(z^{-1}) \quad (11)$$

$$f_j = F_j(z^{-1})y(t) + G_{jp}(z^{-1})\Delta u(t-1) \quad (12)$$

A set of N_y ahead predictions can be written as:

$$\begin{bmatrix} G_0 & 0 & \dots & 0 & \dots & 0 \\ G_1 & G_0 & \dots & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ G_{j-1} & G_{j-2} & \dots & G_0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ G_{N_y-1} & G_{N_y-2} & \dots & \dots & \dots & G_0 \end{bmatrix} x = \begin{bmatrix} \Delta u(t) \\ \Delta u(t+1) \\ \vdots \\ \Delta u(t+j-1) \\ \vdots \\ \Delta u(t+N_y-1) \end{bmatrix} + \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_j \\ \vdots \\ f_{N_y} \end{bmatrix} \quad (13)$$

The predicted value of the output Equation (13) can be expressed in condensed form as:

$$Y = G \Delta u + f \quad (14)$$

where G = system dynamics matrices which is made up of N_y columns of the systems step response appropriately shifted down in order. F is the flow rate matrices which is also equivalent to K matrices. Let $\Delta u(t+j-1) = 0$, ($j > N_u$) the optimized control sequence obtained by optimizing the cost function of Equation (5) can be written as:

$$\Delta u = (G^T R G + Q)^{-1} G^T R (w - f) \quad (15)$$

This derived equation enables to find the control sequence Δu . The control sequence is used to minimize the cost function of the system. This minimization of cost function will eventually achieve the optimality of the system in which all the state variables will reach an equilibrium point. By reaching the equilibrium point means that all the process variables have constant values.

CONCLUSION

A predictive mathematical approach of the control system of an ORC has been presented considering one case of GPC which is CARIMA modelling to predict the balanced output of the plant. It has been analyzed in this paper that for prediction, use of Diophantine equation is important. Therefore it is concluded that by having Proper choice of predictive control horizon, satisfactory performance can be achieved.

Nomenclature:

$y(t)$	= Plant output
$u(t)$	= Plant input
\hat{y}	= Vector of system predictions
G	= Systems dynamic matrix
u	= Vector of control increments
$\hat{y}(t + j / t)$	= Predicted value of output
f	= Part of response that does not depend on disturbance
J	= Cost function
w	= Reference trajectory
G	= Matrix containing coefficients of the system response
N_y	= Control horizon

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