

Modelling of Concrete Gravity Dam Including Dam-Water-Foundation Rock Interaction

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Abstract: A hybrid meta-heuristic optimization method is introduced to efficiently find the optimal shape of concrete gravity dams including dam-water-foundation rock interaction. For this purpose model of two-dimensional finite element that is included dam, reservoir and foundation be provided using the finite element in the most widely used APDL (Parametric Design Language) language programming. To considering 11 geometry variables, finite element analyses of gravity dams are carried. With regard to 11 geometric variables Can modeling each gravity dam and with each geometry. In order to check verify of modeling and ensured used assumption during the modeling, dam is intended in 4 different cases: 1. Dam with empty reservoir and rigid foundation. 2. Dam with empty reservoir and flexible foundation. 3. Dam with full reservoir and rigid foundation. 4. Dam with full reservoir and flexible foundation. To assess the accuracy of this modeling, the modal analysis and mode shapes of the Pine Flat, koyna and idealized triangular Dams is studied and the results are compared with other reference results. Numerical results show the merits of the suggested technique for gravity dam shape simulation. It is also found that considering the dam-water–foundation rock interaction has an important role for safely designing a gravity dam.

Key words: Concrete gravity dam-reservoir–foundation rock interaction • Geometry shape variables • Natural frequency • APDL/finite element method

INTRODUCTION

The performance of the dam in water management, as a storage dam and water controller in winter and early spring and gradual usage of the stored water in summer, is very critical. Gravity dams are fluid–structure–soil interaction problems [1]. It is obvious that the foundation soil and water reservoir affect the dynamic response of gravity dams during earthquakes. Many factors have influence the dynamic response concrete gravity dams against earthquake motion. Some of these factors included dam-reservoir-foundation interaction, sediments at the bottom of the reservoir and nonlinear behavior of concrete gravity dams. For numerical solution of interaction problems that have a large amount of calculations, using commercial standard finite elements software packages can be useful [2]. Usually to compute the dynamic response of the dam, the concrete dam and the foundation rock are modeled by standard finite elements, whereas for the interaction effects of the water, there are several methods to investigate the dynamic

response of the mentioned systems. The dam reservoir interaction problems can be analyzed using the three famous approaches: Westergaard approach: the dynamic effect of the reservoir is modeled as added masses. Eulerian approach: since in this approach the displacements are the variables in the structure and the pressures are the variables in the fluid, a special purpose computer program is required for the solution of the coupled systems. Lagrangian approach: In this approach the behavior of the fluid and structure is expressed in terms of displacements. For that reason, compatibility and equilibrium are automatically satisfied at the nodes along the interfaces between the fluid and structure. This makes a Lagrangian displacement based fluid finite element very desirable; it can be readily incorporated into a general purpose computer program for structural analysis; because special interface equations are not required. The first presented solution was based on the added mass method. In this approach, the only effect of fluid was the portion of fluid mass which was added to the solid. The stiffness and damping effects of the fluid was ignored. In

this state, the solid was solved without considering the fluid and the solid mass matrix was modified by a portion of fluid mass. This method was used to analyze stiff and flexible structures such as dams and water reservoir. In general, this method gives overestimated results, but is still useful for pre-analysis procedures. The first research on the analysis of concrete gravity dam has been done by Westergaard in 1930 and its analysis response for hydrodynamic pressure on the dam face was clear [3]. The original added mass concept is based on simplifying assumptions of vertical upstream face, rigid dam section and incompressible water but was modified by Kuo (1982) for other orientations of the upstream face and in the linear and nonlinear responses dam-reservoir system approximated dam equation by adding some mass [4]. Both approaches, however, ignore compressibility of water and the energy loss due to radiation of pressure waves in the upstream direction and due to reflection and refraction at the reservoir bottom. Chopra and his coworkers (1981) the complete system is considered as composed of three substructures, the dam, represented as a finite element system, the fluid domain, as a continuum of infinite length in the upstream direction and the foundation rock region as a viscoelastic half-plane. The foundation region may also be idealized as a continuum or as a finite element system. The continuum idealization permits the continuum idealization permits accurate modeling of the structure-foundation interaction when similar materials extend to large depths. For sites where soft rock or soil overlies harder rock at shallow depths, a finite element idealization of the foundation region is more appropriate, but at low depths the rock and rigid layer should be modeled with finite element method [5].

In addition to dam-reservoir-foundation interaction, the effect of seismic waves absorption by the reservoir bottom sediments on the response of the dam have been studied. Dam-reservoir-foundation-sediment interaction has been investigated by many researchers. Among other, Fenves and Chopra (1984, 1985) presented a model which includes reservoir bottom absorption for the seismic analysis of gravity dam by the means of an absorbing boundary condition. The study concluded that the sediment could significantly reduce the hydrodynamic pressure effect on the seismic response of the dam [6, 7].

Singhal (1991), the effect wave reflection coefficient (α) on maximum values crest displacement and maximum stress at the heel of the dam investigated. The (α) is the ratio of the amplitude of the reflected hydrodynamic pressure wave to the amplitude of a vertical propagating pressure wave incident on the reservoir bottom ($0 < \alpha < 1$). A value of $\alpha = 1$ indicates that pressure waves are completely reflected and smaller values of α indicate

increasingly absorptive materials. The results show that increasing the wave reflection coefficient increases the maximum values crest displacement and maximum stress at the heel of the dam [8].

Many researches, studied this problem using the computer programs for analysis 2D Finite Element Method of gravity dams. For example the computer program EAGD-84 (Fenves and Chopra, 1984) is a two-dimensional finite element method of analysis for gravity dams which includes dam-water interaction with water compressibility, dam-foundation rock interaction and reservoir bottom absorption due to reservoir bottom sediments [9]. Lotfi (2003, 2007) a new technique is proposed for earthquake analysis of concrete gravity dams, which is referred to as decoupled modal approach. A special computer program "MAP-76" is used as the basis of this study. The program was already capable of analyzing a general dam-reservoir system by direct approach in the time domain and frequencies of the dam - reservoir found. The main advantage of this modal technique is that it employs eigenvectors of the decoupled system, which can be easily obtained by standard eigen-solution routines [10, 11]. Akkose (2010), the seismic nonlinear behavior of the concrete gravity dams to earthquake ground motion near and far fault including dam-reservoir-sediment-foundation rock interaction is investigated and using a computer program NONSAP modified System frequencies dam - reservoir received. The program is modified for elasto-plastic analysis of fluid-structure systems and employed in the response calculations [12].

In this paper, we study the dam-reservoir-foundation interaction during an earthquake. For this purpose model of two-dimensional finite element that is included dam, reservoir and foundation be provided using the finite element. In order to check verify of modeling and ensured used assumption during the modeling, dam is intended in 4 different cases: 1. Dam with empty reservoir and rigid foundation. 2. Dam with empty reservoir and flexible foundation. 3. Dam with full reservoir and rigid foundation. 4. Dam with full reservoir and flexible foundation. The modal analysis and mode shapes results of the Pine Flat, koyna and idealized triangular Dam is studied and the results obtained, verify the accuracy of the modeling against available reference results.

Finite Element Model of Dam-reservoir- Foundation System: To modeling concrete gravity dam-reservoir-foundation problem using the finite element procedure, the discretized dynamic equations of the fluid and structure including dam and its foundation need to be considered simultaneously to obtain the coupled fluid-structure-foundation.

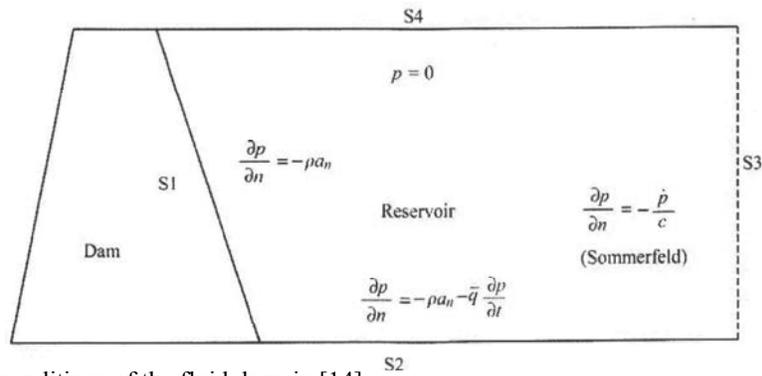


Fig. 1: The boundary conditions of the fluid domain [14]

The Discretized Fluid Equation: Assuming that water is linearly compressible and neglecting its viscosity, the small amplitude irrotational motion of the water is governed by the two-dimensional wave equation [13, 2]:

$$\nabla^2 p = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} \tag{1}$$

where c is the speed of pressure wave, p is the acoustic hydrodynamic pressure; t is time and ∇^2 is the two-dimensional Laplace operator.

As shown in Fig. 1, some boundary conditions may be imposed on the fluid domain as follows:

(S1), at the fluid–structure interface:

$$\frac{\partial p}{\partial n} = -\rho a_n \tag{2}$$

where n is a unit normal vector to the interface, a_n is the normal acceleration on the interface and ρ_w is the mass density of water.

(S2), at the bottom of the fluid domain:

$$\frac{\partial p}{\partial n} = -\rho a_n - \bar{q} \frac{\partial p}{\partial t} \tag{3}$$

where \bar{q} is the damping coefficient characterizing the effects of absorption of hydrodynamic pressure waves at the reservoir boundary [6] and α is the wave reflection coefficient, which represents the ratio of the amplitude of the reflected wave to that of the normally incident pressure wave. α is related to \bar{q} by the following expressions:

$$\alpha = \frac{1 - \bar{q}c}{1 + \bar{q}c} \tag{4}$$

It is believed that a Value from 1 to 0 would cover the wide range of materials encountered at the boundary of actual reservoirs. The value of the wave reflection coefficient α that characterizes the reservoir bottom materials should be selected based on their actual properties, not on properties of the foundation rock. Materials on the reservoir bottom has great influence in absorbing of earthquake waves and decreases the system response under the vertical component of the earthquake and this effect is also important for horizontal component. (S3), at the far-end of the fluid domain a Sommerfeld-type radiation boundary condition [13, 2] may be implemented, namely

$$\frac{\partial p}{\partial n} = -\frac{1}{c} \frac{\partial p}{\partial t} \tag{5}$$

(S4), at the free surface when the surface wave is neglected, the boundary condition is easily defined as:

$$p = 0 \tag{6}$$

Eqs. (2)– (6) can be discretized to get the matrix form of the wave equation as [15]:

$$M_f \ddot{P}_e + C_f \dot{P}_e + K_f P_e + \rho_w Q^T (\ddot{u}_e + \ddot{u}_g) = 0 \tag{7}$$

where M_f , C_f and K_f are the fluid mass, damping and stiffness matrices, respectively and P_e ; \ddot{u}_e and \ddot{u}_g are the nodal pressure, relative nodal acceleration and nodal ground acceleration vectors, respectively. The term $\rho_w Q^T$ is also often referred to as coupling matrix.

The Discretized Structural Equation: The discretized structural dynamic equation including the arch dam and foundation rock subject to ground motion can be formulated using the finite-element approach as:

$$M_s \ddot{u}_e + C_s \dot{u}_e + K_s u_e = -M_s \ddot{u}_g + QP_e \quad (8)$$

where M_s , C_s and K_s are the structural mass, damping and stiffness matrices, respectively, u_e is the nodal displacement vector with respect to ground and the term QP_e represents the nodal force vector associated with the hydrodynamic pressure produced by the reservoir.

The Coupled Fluid–structure–foundation Equation:

Eqs. (7) and (8) describe the complete finite-element discretized equations for the dam-water–foundation rock interaction problem and can be written in an assembled form as:

$$\begin{bmatrix} M_s & 0 \\ M_{fs} & M_f \end{bmatrix} \begin{Bmatrix} \ddot{u}_e \\ \dot{p}_e \end{Bmatrix} + \begin{bmatrix} C_s & 0 \\ 0 & C_f \end{bmatrix} \begin{Bmatrix} \dot{u}_e \\ \dot{p}_e \end{Bmatrix} + \begin{bmatrix} K_s & K_{fs} \\ 0 & K_f \end{bmatrix} \begin{Bmatrix} u_e \\ p_e \end{Bmatrix} = \begin{Bmatrix} -M_s \ddot{u}_g \\ -M_{fs} \ddot{u}_g \end{Bmatrix} \quad (9)$$

where $K_{fs} = -Q$ and $M_{fs} = \rho\omega Q^T$.

Eq. (9) expresses a second order linear differential equation having unsymmetrical matrices and may be solved by Means of direct integration methods. In general, the dynamic equilibrium equations of systems modeled in finite elements can be expressed as:

$$M_c \ddot{u}_c + C_c \dot{u}_c + K_c u_c = F(t) \quad (10)$$

where M_c , C_c , K_c and $F(t)$ are the structural mass, damping, stiffness matrices and dynamic load vector, respectively.

Modelling of Dam-Reservoir-Foundation System:

The objective of this work is to study the effects of dam-reservoir-foundation interaction on modal behavior of gravity dams. The computer program used to modelling and analyzes the dam–reservoir– foundation system was APDL language programming. Pine flat, koyna and idealized triangular Dams are analyzed to evaluate the accuracy and efficiency of the present model finite element. For dam body modelling four nodes element of Plane 42 (structural 2D solids) is used. The dam and foundation elements are in a state of plane-stress. The reservoir is assumed to be of uniform shape and four-noded FLUID29 element is used to discretize the fluid medium and the interface of the fluid–structure interaction problem. The element has four degrees of freedom per node: translations in the nodal x, y and z directions and pressure. The translations, however, are active only at the nodes that are on the interface. In order to consider the damping effect arising from the propagation of

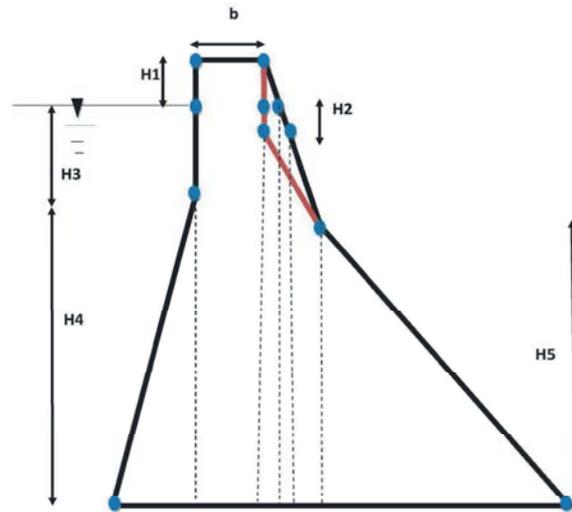


Fig. 2: The geometry variables of gravity dam [17]

pressure waves in the upstream direction, instead of a Sommerfield-type radiation boundary condition, the reservoir length is selected as one and a half times the reservoir depth and zero pressure is imposed on all nodes of the far end boundary. In this study, foundation rock treating as a linearly elastic structure is represented via a four-noded Plane 42 element as well. The foundation rock is assumed to be massless in which only the effects of foundation flexibility are considered and the inertia and damping effects of the foundation rock are neglected. The foundation rock is extended to one and a half times dam height in upstream, downstream and downward directions [16].

The dam body is assumed to be homogeneous, isotropic and elastic properties for mass concrete. The foundation rock is idealized as a homogenous, isotropic media. The foundation model was constructed using solid elements arranged on semicircles having a radius one and a half times base of the dam. The impounded water is taken as inviscid and compressible fluid.

In the present study, to create the gravity dam geometry, 11 geometry variables are considered. With the defined geometry variables in APDL a 2D shape of gravity dam body is created. Shape of the dam with 11 geometry variables is presented in Fig. (2) [17].

RESULTS AND DISCUSSION

For analyzed of the selected dam four cases related to various conditions of dam-water-foundation rock interaction problem are considered as follows:

Table 1: The geometry variables of Pine Flat dam

Parameter	b	b1	b2	b3	b4	b5	H1	H2	H3	H4	H5
Value (ft)	32	16.75	0	0	31.57	234	19	14	46	335	300
Value (m)	9.7536	5.1054	0	0	9.6225	71.3232	5.7912	4.2672	14.0208	102.108	91.44

Table 2: The material properties of the dam, water and foundation rock

Dam body	Elasticity modulus of concrete	22,400 MPa
	Poisson's ratio of concrete	0.20
	Mass density of concrete	2430 kg m ⁻³
Water	Mass density of water	1000 kg m ⁻³
	Speed of pressure wave	1440 m s ⁻¹
	Wave reflection coefficient	0.817
Foundation rock	Elasticity modulus of foundation rock	68,923 MPa
	Poisson's ratio of foundation rock	0.3333
	Mass density of foundation rock	0.00

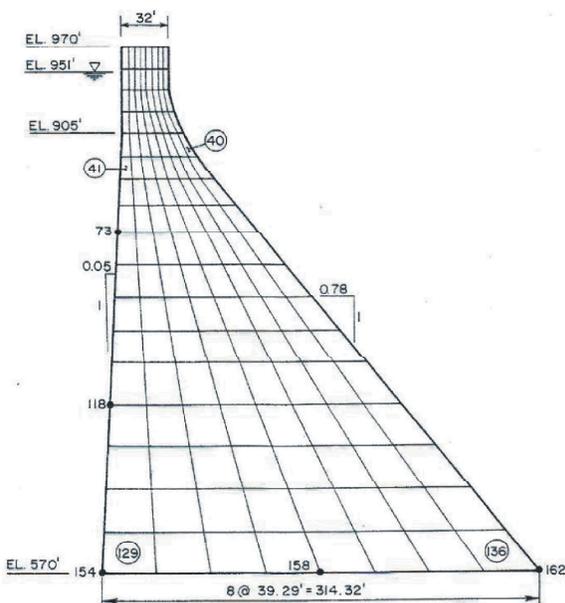


Fig. 3: Dimensions of the tallest monolith of Pine Flat dam [18, 19]

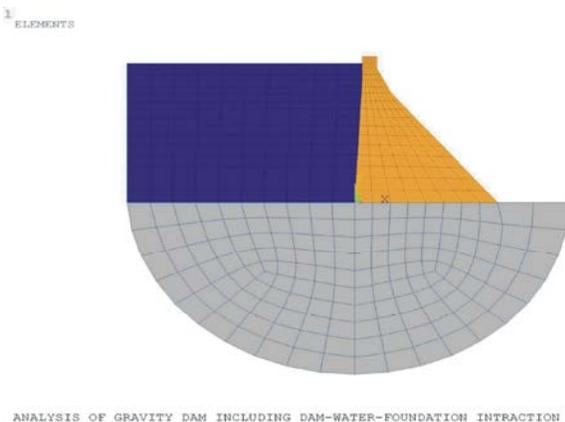


Fig. 4: Finite-element model of Pine Flat dam-water-foundation rock system [17]

Case 1: Dam with empty reservoir and rigid foundation.

Case 2: Dam with empty reservoir and flexible foundation.

Case 3: Dam with full reservoir and rigid foundation.

Case 4: Dam with full reservoir and flexible foundation.

In order to validate the finite-element model with the employed assumptions, the first natural frequency of the symmetric mode of the dam for Cases 1–4 are determined. The results of the present work are compared with those reported in the literature and other reference. The errors between exact and approximate frequencies are also calculated using the following equation:

$$error = \left| \frac{fr_{ap} - fr_{ex}}{fr_{ex}} \right| \times 100 \quad (11)$$

where fr_{ap} and fr_{ex} represent the approximate and exact frequencies, respectively.

Finite-Element Model of Pine Flat Dam: In this section, the analysis of Pine Flat Dam is considered as a verification example. The dam is 121.92 m high, with the crest length of 560.83 m and its basis has a length of 96.80 m. It is located on the King's River near Fresno, California. A (2D) finite element model with 162 nodes and 136 plane elements (PLANE 42) is used to model body dam. (Fig.3).

An idealized model of Pine Flat dam-water-foundation rock system is simulated using the finite-element method as shown in Fig. 4.

The geometry variables of dam are given in Table 1. The material properties of the dam, water and foundation rock are given in Table 2 [18, 19].

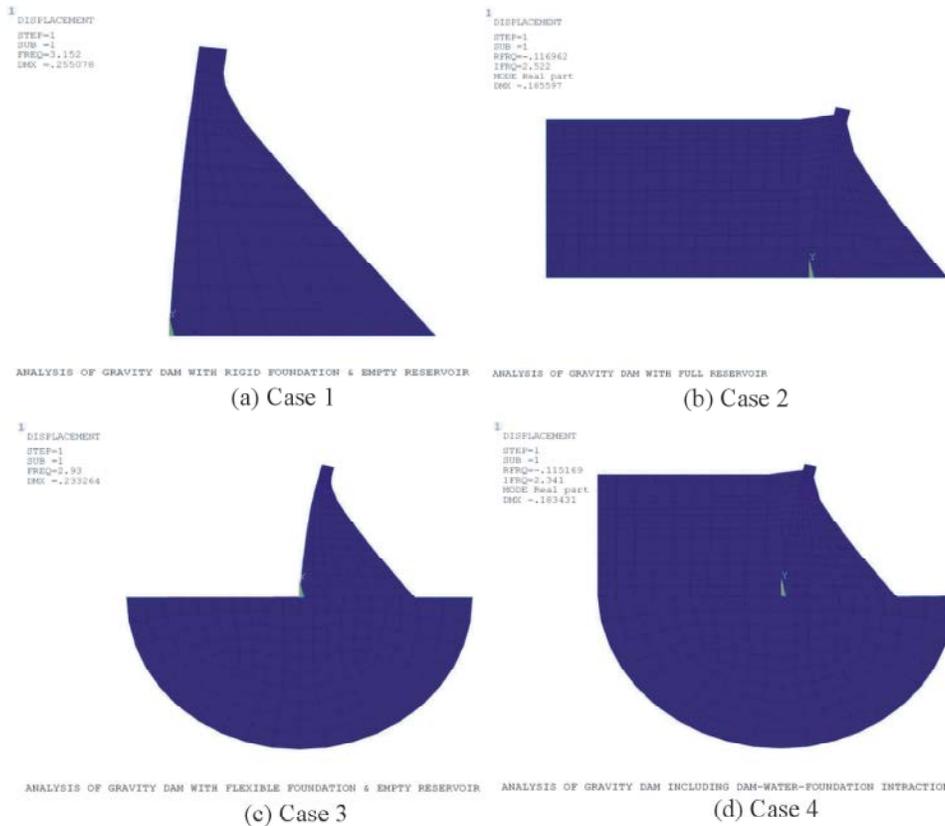


Fig. 5: The first mode shape of the dam for different Cases[17]

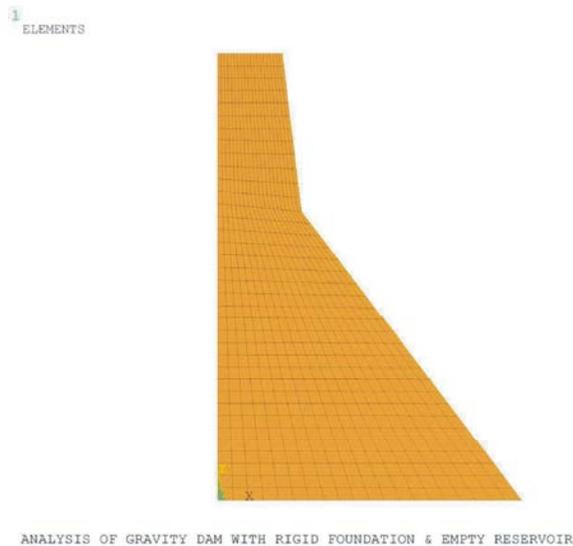


Fig. 6: Finite-element model of Koyna dam (Dam with empty reservoir and rigid foundation) [17]

The natural frequencies for Cases 1-4 from the finite element model [17] and the literature are given in Table 3 [17]. It can be observed that a good conformity has been

achieved between the results of present work with those of reported in the literature [19]. Also, the very small percentage error showed excellent accuracy of the proposed model for dam-reservoir-foundation system.

The first mode shape of the dam for different Cases is displayed in Fig. 5.

Finite-Element Model of Koyna Dam: Koyna concrete dam (Fig. 6) In Maharashtra, India, has been chosen for the finite element modeling only in case 1 (Dam with empty reservoir and rigid foundation). The dam is 103m high; width of the dam base is 70m and crest width is 14.8m [20].

The material properties of the dam are modulus of elasticity, mass density and Poisson's ratio which are 31027MPa, 2354 kg/m³ and 0.2, respectively. The geometry variables of dam are given in Table 4.

The first four natural frequencies of dam from the finite element model [17] and the Reference are listed in Table 5 [18].

Table 3: A comparison of the natural frequencies from the FE model with the literature

Case	Foundation	Reservoir	Natural frequency (Hz)		
			Chopra [19]	The present work	Error (%)
1	Rigid	Empty	3.1546	3.152	0.082
2	Rigid	Full	2.5189	2.522	0.123
3	Flexible	Empty	2.9325	2.930	0.085
4	Flexible	Full	2.3310	2.383	2.180

Table 4: The geometry variables of Koyna dam

Parameter	b	b1	b2	b3	b4	b5	H1	H2	H3	H4	H5
Value (m)	14.80	0.00	1.3713	1.45987	1.61837	50.75	11.25	11.975	52.75	39.0	66.50

Table 5: A comparison of first four natural frequencies from the FE model with the literature

Mode number	Natural frequency (Hz)		
	Reference [20]	The present work	Error (%)
1	3.002	3.01	0.026
2	7.953	8.00	0.590
3	10.848	10.855	0.064
4	15.640	15.803	1.042

Table 6: The geometry variables of idealized triangular Dam

Parameter	b	b1	b2	b3	b4	b5	H1	H2	H3	H4	H5
Value (m)	97.536	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	121.92

Table 7: A comparison of first four natural frequencies from the FE model with the literature

Mode number	Natural frequency (Hz)		
	Reference [24]	The present work	Error (%)
1	3.797	3.805	0.210

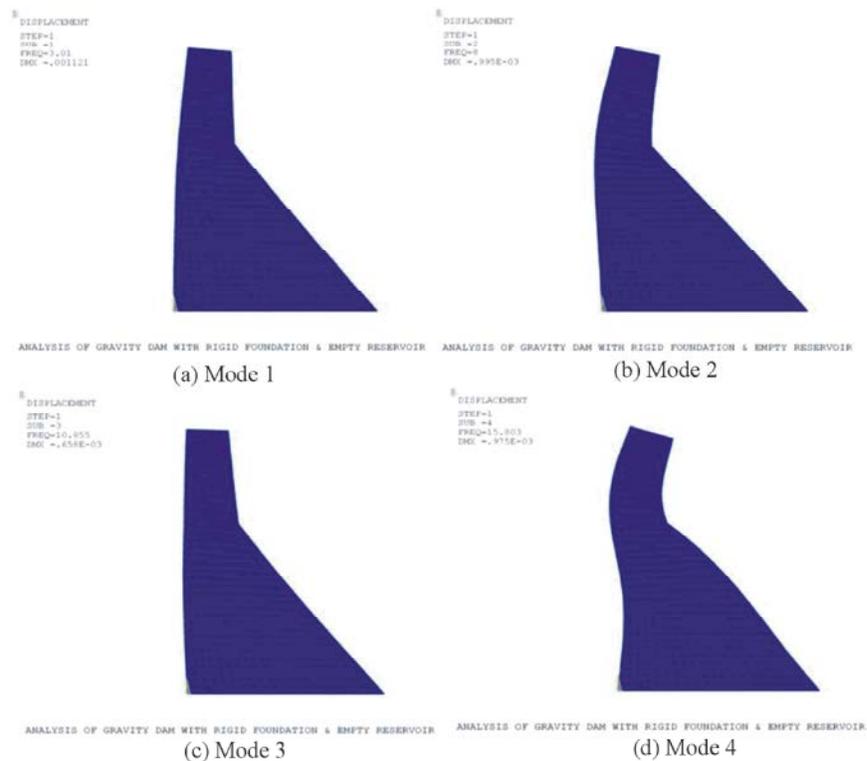


Fig. 7: The first four mode shape of the dam for Case 1[17]

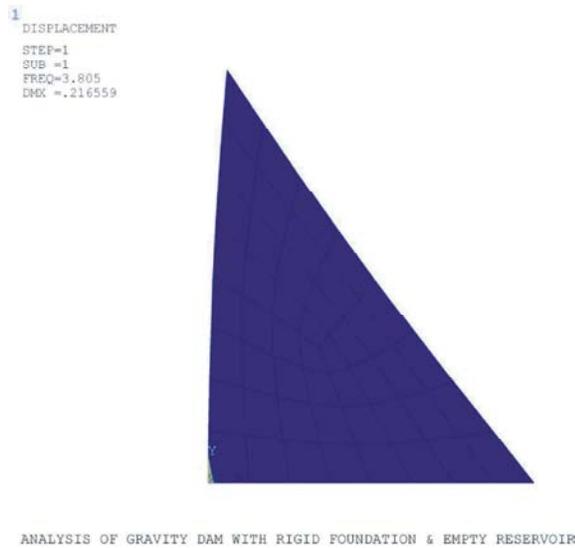


Fig. 8: The first mode shape of the dam for Case 1 [17]

The first four mode shape of the dam for Case 1 (Dam with empty reservoir and rigid foundation) is displayed in Fig. 7.

Finite-Element Model of Idealized Triangular Dam:

An idealized triangular dam with vertical upstream face and a downstream slope of 1:0.8 is considered on a rigid foundation under an empty reservoir condition [21]. The physical and mechanical properties involved here are the concrete mass density (2643 kg/m^3), the concrete poisson's ratio (0.2) and the concrete Elasticity modulus (27570MPa). The geometry variables of dam are given in Table 6 [22, 23].

The first four natural frequencies of dam from the finite element model [17] and the Reference are listed in Table 7[24].

The first mode shape of the dam for Case 1 (Dam with empty reservoir and rigid foundation) is showed in Fig. 8.

It can be observed that a good conformity has been achieved between the results of present work with those of reported in the literature in the previous sections (4-2 and 4-3). Also, the very small percentage error showed excellent accuracy of the proposed model for dam with empty reservoir and rigid foundation.

CONCLUSION

In the present study, an efficient procedure is developed to modeling the geometry shape of Concrete gravity dams considering dam-reservoir-foundation rock interaction with employing real values of the geometry

variables. To create the gravity dam geometry, 11 geometry variables are considered. With the defined geometry variables in APDL/FINITE ELEMENT, any 2D shape of gravity dam body is created. To achieve this aim, a 2D finite element model has been established for the modal analysis of Concrete gravity dams - reservoir–foundation rock system with APDL language.

Numerical results demonstrate the high performance of the hybrid meta-heuristic optimization for optimal shape design of concrete gravity dams. In order to assess the high capability of the proposed methodology for gravity dam shape modelling, an actual gravity dam is selected and is implemented for four design cases involved the various conditions of the interaction problem. The results of first natural frequency for four design cases are compared with those of reported in literature and its performance is verified.

Numerical results show that the proper optimal design can be achieved for the gravity dam. It is observed that both the gravity dam-water and gravity dam–foundation rock interactions have an important role in the design of arch dams and neglecting these effects can lead to an improper design. Also, it can be observed that when the reservoir is empty and the foundation is rigid (Case 1) main frequency of the dam is maximal. Furthermore, a minimum value for the main frequency is obtained when the dam-water–foundation rock interaction (Case 4) is considered.

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