

Joule Heating Effect on Electroosmotic Flow (Analytical Approach)

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Abstract: In this study, we have examined a Joule heating effect on a temperature of electroosmotic flow in microchannels. We have performed a parametric study to investigate effects of ζ -potential (electric potential of an electric double layer on walls of the microchannel), an external electric field and the microchannel height on the temperature. Obtained results reveal that the Joule heating effect has a major impact on the temperature; it is also shown that increasing size of the microchannels intensifies the Joule heating effect. Energy dissipation has negligible influence on the temperature of the flow field. If the applied external electric field is small, the ζ -potential will have minor contribution on the temperature of the electroosmotic flow; however, by increasing the microchannels dimension, the ζ -potential no longer influences the temperature. Intensifying the externally applied electric field also increases the temperature of the flow field (Joule heating effect). An increase of the microchannel height magnifies the temperature of the electroosmotic flow field in the microchannels (the Joule heating effect).

Key words: Joule heating · Electroosmotic · Microchannel · Temperature

INTRODUCTION

By entering technology to the micro and nano dimensions, design and fabrication of miniaturized instruments, such as microelectronics devices, microfluidic instruments, MEMS, NEMS become very desirable. These miniaturized devices have variety of applications ranging from energy [1-4] to biomedical studies [5-7]. In these microscale devices, many undesired effects such as Joule heating may ruin the performance of these devices. To minimize the harmful influences of these phenomena, it is so important to optimize the design of those, such that the undesired harmful effects become less and less. By definition, electroosmosis is the motion of ionized liquid

relative to stationary charged surfaces in a presence of applied external electric field; the generated flow due to this effect is called electroosmotic flow, see Figure 1 [8, 5]. Microchannels are the channels with the smallest dimension of μm - $200\ \mu\text{m}$ [2]. In our recent paper, we have proposed an analytical approach to find the temperature distribution of the electroosmotic flow in the planar microchannels [9]. In that study, effects of the microchannel height and the external electric field on the temperature distribution of the flow field have been examined. It was revealed that, for the larger microchannels, the effect of the external electric field can be intensified on the temperature of the flow field. However, in the small microchannels, its effect may be negligible.

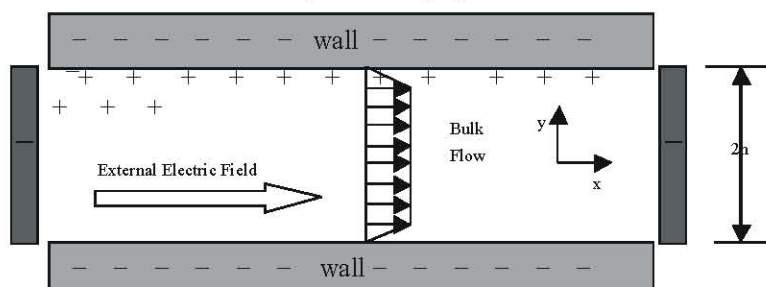


Fig. 1: Schematic of the electroosmotic flow in a planar microchannel channel with the finite EDL

The electroosmotic flow has many advantages compare with a pressure driven flow such as being vibration free, not requiring any external mechanical pumps or moving parts, being compact and stable. Few weak points also associate with this type of flow field. The Joule heating is one of them. The external electric field of the electroosmotic flow generates heat in the system due to an electrical resistance. If F is an electrical conductivity and E is the applied external electric field, the produced heat of the Joule heating effect can be calculated as $q = FE^2$. The Joule heating may harm the performance of the microfluidic devices; it may increase a local temperature of fluid flow that decreases the fluid viscosity by its turn and consequently increases the slip velocity on boundaries. Therefore, performing a comprehensive study on the Joule heating effect in the microfluidic systems seems to be essential.

In the present study, we have utilized the proposed analytical method of Ref. [9] to study the Joule heating effect in the electroosmotic flow of the planar microchannel. Figure one shows the schematic diagram of this type of flow field. We examine effects of the microchannel dimension (height) and the external electric field on the temperature distribution and investigate how the Joule heating effect contributes to the temperature distribution. In the next section, we have briefly introduced the proposed analytical method of Ref. [9]. Results and discussions have been explained in section 3 and the concluding remarks have been discussed at the end of the article.

Modeling: In the present study, we have assumed a uni-directional, steady state, incompressible and fully developed flow of the Newtonian fluid in the planar microchannel (Figure 1). The flow field is generated because of the electroosmotic effect. No-slip velocity and constant temperature are assumed on the walls of the microchannel. The ζ -potential is the considered potential of the electric double layer (EDL) on the walls of the microchannel. In this study, h is the half-height of the planar- microchannel. u and p represent velocity and pressure, respectively. T is temperature; E stands for external electric field; ϕ denotes the electric potential of the electric double layer (EDL). μ and ρ are the viscosity and density of the fluid; c and k are the specific heat capacity and thermal conductivity of the assumed fluid. ϵ and ϵ_0 are the relative and absolute permittivity. z is the valence of the ion, e is the elementary charge, k_b is Boltzmann constant and n_0 is the ionic bulk number concentration.

The detail of the analytical approach of finding temperature distribution of the electroosmotic flow can be found in Ref. [9]. In the following, we have presented a closed form equations of finding the temperature of the electroosmotic flow in the planar microchannels:

$$T_u(y) = H_1(y) + C_1y + C_2 \quad 0 \leq y \leq h \quad (1)$$

$$T_l(y) = H_2(y) + C_3y + C_4 \quad -h \leq y \leq 0 \quad (2)$$

In the above equations, the constants c_1 to c_4 are defined as:

$$c_1 = -\left. \frac{\partial H_1(y)}{\partial y} \right|_{y=0} \quad (3)$$

$$c_2 = T_s - H_1(h) - c_1h \quad (4)$$

$$c_3 = -\left. \frac{\partial H_2(y)}{\partial y} \right|_{y=0} \quad (5)$$

$$c_4 = T_s - H_2(-h) + c_3h \quad (6)$$

In Eqns. (1-2), functions H_1 and H_2 are defined as:

$$H_1(y) = g_1(y) + g_2(y) + g_3(y) + g_4(y) + g_5(y) \quad (7)$$

$$H_2(y) = g_6(y) + g_7(y) + g_8(y) + g_9(y) \quad (8)$$

$$g_1(y) = -\frac{1}{4}l_1^2l_6(Ln(1-f_1(y)) - 2yl_3) \quad (9)$$

$$g_2(y) = l_8y^2 + l_9y^4 \quad (10)$$

$$g_3(y) = l_{10}(y^2(f_3(y) + Ln(\frac{1-f_2(y)}{1+f_2(y)}))) \quad (11)$$

$$g_4(y) = l_{11}(y \times pl(2, f_1(y)) - 4y \times pl(2, f_2(y))) \quad (12)$$

$$g_5(y) = l_{12}(pl(3, f_1(y)) - 8pl(3, f_2(y))) \quad (13)$$

$$g_6(y) = -\frac{1}{4}l_1^2l_6(Ln(-1+f_4(y)) - 2yl_3) \quad (14)$$

$$g_7(y) = g_2(y) \quad (15)$$

$$g_8(y) = l_{11}(y \times pl(2, f_4(y)) - 4y \times pl(2, f_5(y))) \quad (16)$$

$$g_9(y) = l_{12}(pl(3, f_4(y)) - 8pl(3, f_5(y))) \quad (17)$$

$$f_1(y) = \frac{\exp(2(-h+y)l_3)}{l_2^2} \quad (18)$$

$$f_2(y) = \frac{\exp((-h+y)l_3)}{l_2} \quad (19)$$

$$f_3(y) = 2 \tanh^{-1}(\exp(-(-h+y)l_3)l_2) \quad (20)$$

$$f_4(y) = l_2^2 \exp(2(h+y)l_3) \quad (21)$$

$$f_5(y) = l_2 \exp((h+y)l_3) \quad (22)$$

For more simplicity, following constants and parameters are defined in the above equations:

$$A = \frac{1}{\rho_f c_p} \quad (23)$$

$$B = \frac{k_f}{\rho_f c_p} \quad (24)$$

$$C = \frac{\sigma_{eff} E^2}{\rho_f c_p} \quad (25)$$

$$D = \frac{\partial T}{\partial x} = \frac{(T_s - T)}{(T_s - T_m)} \frac{dT_m}{dx} \quad (26)$$

$$L_5 = \frac{D}{B} \quad (27)$$

$$L_6 = -\frac{\mu A}{B} \quad (28)$$

$$L_7 = -\frac{C}{B} \quad (29)$$

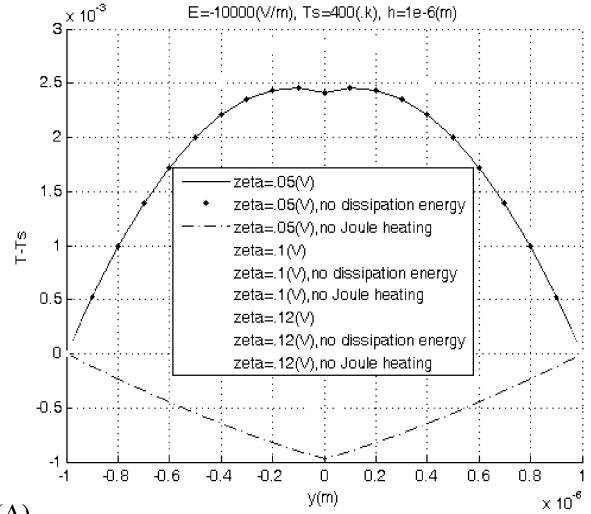
$$l_8 = \frac{(l_7 + l_5 - l_4 l_5)}{2} \quad (30)$$

$$l_9 = \frac{1}{12}(4l_6 l_4^2 + l_4 l_5) \quad (31)$$

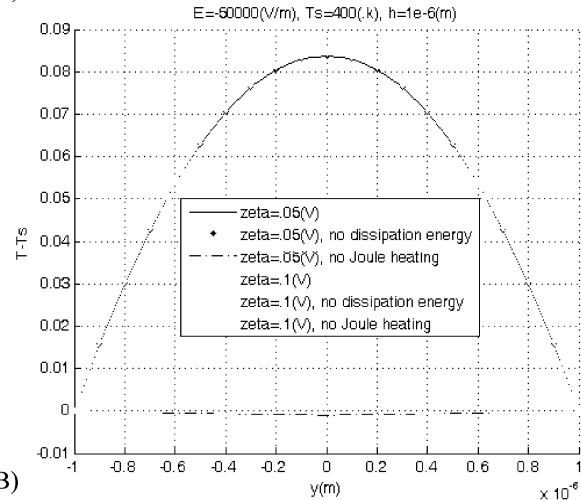
$$l_{10} = -\frac{l_1 l_5}{4} \quad (32)$$

$$l_{11} = \frac{l_1 l_4 l_6}{l_3} \quad (33)$$

$$l_{12} = \frac{l_1(l_5 - 8l_4 l_6)}{8l_3^2} \quad (34)$$



(A)



(B)

Fig. 2: Effects of Joule heating and dissipation energy on temperature distribution, microchannel height is 1 μ m, (A) $E=-10000$ V/m, (B) $E=-50000$ V/m.

RESULTS AND DISCUSSION

In this section, we aim to examine the Joule heating effect on the temperature of the electroosmotic flow in the planar microchannels. It is shown that how the Joule heating is influenced by the geometrical and electrical parameters. The assumed parameters of this study have been provided in Table 1.

Figures 2 and 3 show the temperature distribution across the microchannel. From these figures, one can conclude that:

When the applied electric field is 10000 V/m, for the smaller microchannels ($h=1$ μ m), ζ -potential has a small effect on the temperature. However, as the height of the microchannel increases ($h=10$ μ m), this effect become negligible. By increasing the applied electric field from

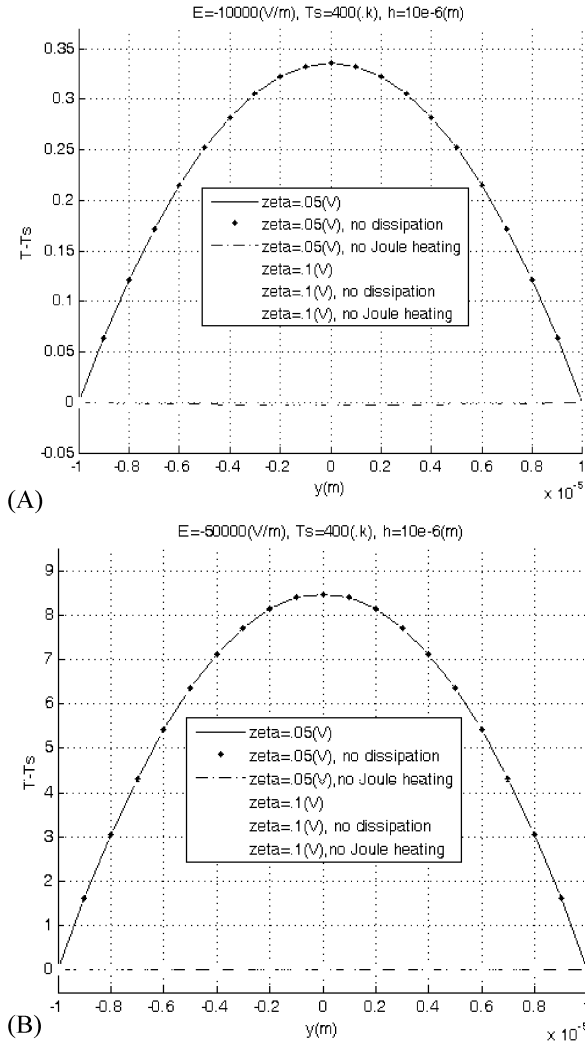


Fig. 3: Effects of Joule heating and dissipation energy on temperature distribution, microchannel height is 10 μ m, (A) $E = -10000$ V/m, (B) $E = -50000$ V/m.

Table 1: The values for constants and parameters used in the simulations

Parameter	Value/range	Unit
ϵ_r	80	-
ϵ_0	8.85×10^{-12}	F/m
ρ (liquid density)	1000	kg/m ³
μ	1×10^{-3}	Pa·s
F	96 485.3415	A·s / mol
k_B	1.381×10^{-23}	J / .K
h (microchannel height)	1~50	μ m
E_z (external electric field)	-10 ~ -100	kV / m
ζ -potential	0.05	V
e (elementary charge)	1.602×10^{-19}	C
z (valence)	1	-
c_p (specific heat capacity)	4.1813	kJ / kg·K
k (Thermal conductivity)	0.6	W / m·K
T_s (Temperature at the walls)	300	.K

10000 V/m to 50000 V/m, the effect of ζ -potential becomes negligible for all microchannels. This can be explained by considering the Joule heating effect. As the applied electric field increases, the Joule heating effect becomes stronger and dominates the other parameters affecting the temperature of the electroosmotic flow. Physically, it can be explained by considering the energy equation:

$$\rho c_p \left[u \frac{\partial T}{\partial x} \right] = k \frac{\partial^2 T}{\partial y^2} + \sigma E_x^2 + \phi_{inc}. \quad (35)$$

When the external electric field is small, the Joule heating effect does not dominate the other ones (convection, conduction and dissipation energy). For the smaller microchannels, the EDL thickness is not negligible compare with the microchannel height. Here, changing the ζ -potential alters the EDL thickness which affects the convective term of the energy equations; thus, variation of the ζ -potential impacts the temperature of the flow field. By increasing the microchannel height, the EDL thickness is negligible compare with the microchannel height and no longer influences the flow field. By increasing the external electric field, the convective term of the energy equations is dominated by the Joule heating term and thus velocity field do not considerably affect the temperature anymore. Therefore, the variation of the ζ -potential does not influence the temperature anymore.

The microchannel dimension has a great influence on the temperature of the flow field. If the ζ -potential and the applied electric field are kept constant, the difference of temperature at the center and the wall of the microchannel is intensified by enlarging the microchannel height. It can physically be explained by considering the Joule heating effect. The Joule heating is defined as $Q = IV$, where I and V are electric current and electric potential, respectively. The electric current (I) can be calculated as follows:

$$I = \iint F z_i c_i U^{bulk} dA + \iint F^2 z_i^2 v_i c_i E dA \quad (36)$$

Here, F is Faraday's constant, z_i , v_i and c_i are the valence, mobility and concentration of the ion type i , respectively; U^{bulk} is the bulk velocity and E is the external electric field. From this equation, it is clear that for the same applied electric field, the smaller microchannels have lower electric current. Thus, the effect of Joule heating decreases by reduction in size. By decreasing the effect of Joule heating, the generated heat and consequently temperature decreases in the microchannel.

External electric field has a great influence on the temperature distribution. It can also be explained by considering the Joule heating effect. As the external electric field strengthens, the generated heat due to the Joule heating is also intensified. Therefore, the temperature of the flow field builds up in the microchannel.

Figures 2 and 3 also show that energy dissipation has negligible influence on the temperature of the electroosmotic flow in microchannels. The heat energy dissipation is the energy lost due to deformation and turbulence. For the incompressible flow, it can be express as follows [10]; in microfluidic devices, Reynolds number is low and the flow field is Laminar. Thus, the dissipated energy due to the turbulence is substantially diminished.

$$\phi_{inc.} = 2\mu(D_{11}^2 + D_{22}^2 + D_{33}^2 + 2D_{12}^2 + 2D_{13}^2 + 2D_{23}^2) \quad (35)$$

$$D_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \quad (36)$$

CONCLUSION

In this study, an analytical and closed form solution was utilized to examine Joule heating effect on temperature of an electroosmotic flow in planar microchannels. A parametric study was performed to investigate effects of the ζ -potential (electric potential of the electric double layer on the walls of the microchannel), an external electric field and a microchannel height on the temperature. Obtained results revealed that the Joule heating effect has major impact on the temperature; it was also shown that increasing the size of the microchannels intensified the Joule heating effect. The energy dissipation has negligible influence on the temperature of the flow field. If the applied external electric field was small, the ζ -potential had minor contribution on the temperature of the electroosmotic flow; however, by increasing the microchannels dimension, the ζ -potential no longer influences the temperature. Intensifying the externally applied electric field also increases the temperature of the flow field (the Joule heating effect). Increase of the microchannel height magnifies the temperature of the electroosmotic flow field in the microchannels (the Joule heating effect). The results of this study show that the Joule heating effect has considerable influence on the temperature of the electroosmotic flow in the microchannels. Therefore, its effect should be carefully considered to avoid its harmful impact on the performance of the microfluidic devices.

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Nomenclature:

u :	Velocity
p :	Pressure
μ :	Dynamic viscosity
ρ :	Liquid density
ε :	Relative permeability
ε_0 :	Absolute permeability
E_e :	External electric field
ϕ :	Potential of electric double layer
ϕ_0 :	Potential of electric double layer at the walls
z :	Valance
e :	Elementary charge
n_0 :	Ionic bulk number concentration
k_b :	Boltzmann constant
T :	Temperature
T_s :	Temperature at the walls
k :	Thermal conductivity
c_p :	Specific heat capacity
σ :	Electric conductivity
h :	Half channel height

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