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# Effect of Viscoelasticity on Entropy Generation in a Porous Medium over a Stretching Plate

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**Abstract:** In this article, boundary layer flow and heat transfer analysis of a second grade fluid over a stretching sheet through a porous medium has been discussed and effect of viscoelasticity on entropy generation has been taken in to account. Homotopy analysis method (HAM) is used to obtain the analytic solution for the equations and a comprehensive parametric study is presented.

**Key words:** Boundary layer flow · Second grade fluid · Stretching sheet · Homotopy analysis method · entropy generation

## INTRODUCTION

The study of boundary layer flow over a stretching sheet is of great importance and has many applications in fields of science and engineering such as liquid composite molding, extrusion of plastic sheets, gas blowing, wire drawing and hot rolling. The pioneer work on the boundary layer flows over stationary and continuously moving surfaces was initially done by Blasius [1], Sakiadis [2] and Crane [3]. Ali [4] carried out a study for a stretching surface subject to suction or injection for uniform and variable surface temperatures. Seshadri *et al*. [5] investigated non-Newtonian fluid flow and mass transfer effects on the hydrodynamic field over a stretching surface. S. J. Liao [6] presented an analytic solution of unsteady boundary layer flows caused by an impulsively stretching plate. A. Ali and A. Mehmood [7] studied the unsteady boundary layer flow adjacent to a permeable stretching surface in a porous medium. Furthermore, the effects of various heat transfer modes on velocity and temperature fields over stretching surface through porous medium were studied by various scientists and engineers. Elbashbeshy and Bazid [8] and [9] discussed the heat transfer over a continuously moving plate embedded in a non-darcian porous medium and in a porous medium with internal heat generation. Cortell [10] extended the same work by including the power law temperature distribution.

For the last few years, viscoelastic fluid is of great interest for engineers and scientists due to its applications in industry. Fox *et al*. [11] used both exact and approximate methods to examine the boundary layer flow of a viscoelastic fluid characterized bu a power law model. Vajravelu and Rollins [12] investigated the heat transfer of the boundary layer flow of second grade fluid. Char [13] studied the heat and mass transfer in a hydromagnetic flow of a viscoelastic fluid, the Walters' B liquid, over a stretching sheet. Mahantesh *et al*. [14] discussed the flow and heat transfer characteristics of a viscoelastic fluid in a porous medium over an impermeable stretching sheet with viscous dissipation.

In thermodynamical systems, thermal gradient, friction force, diffusion and chemical reactions give rise to energy losses, which induces entropy generation in the system. The entropy generation has been of great interest in the fields such as heat exchangers, turbomachinary, electronic cooling, porous media and combustion and several investigations have been made in this regard. Datta [15] studied the entropy generation in a confined laminar diffusion flame. Baytes [16] and [17] investigated the minimization of entropy generation in an inclined enclosure and inclined porous cavity. Bejan [18] showed that for a convective heat transfer, entropy generation is due to the temperature gradient and the viscous effect in the fluid. Recently Hooman and Bahrami [19] have studied the effects of entropy generation due to heat transfer in a porous medium over a stretching surface with suction and injection.

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The homotopy analysis method (HAM) is a powerful method to solve non-linear problems. This method is proposed by Liao [24] and in the recent few years, this method has been successfully employed to solve many types of nonlinear problems in science and engineering. Xu and Liao [25] used HAM to find a series solution of unsteady magnetohydrodynamic flow of non-Newtonian fluids caused by an impulsively stretching plate. Mehmood *et al*. [26] studied unsteady boundary-layer flow due to an impulsively started porous plate and obtained an analytical solution via homotopy analysis method. Kazemipour and Neyrameh [27] made use of HAM to retrieve an analytical solution of Goursat problem via homotopy analysis method. Bataineh *et al*. [28] obtained solution of singular higher order boundary value problems by Homotopy analysis method and its modified form.

In this study, flow of viscous incompressible second grade fluid over a stretching plate with porous medium having constant permeability is investigated. Heat transfer effects and entropy generation rate are taken into account and different physical properties are discussed regarding fluid flow and heat transfer analysis.

**Mathematical Formulation of the Problem:** We consider a steady two dimensional flow of a viscous incompressible second grade fluid through a homogenous porous medium of permeability $K$ past a flat sheet coinciding with the plane $y$ = 0 and the flow being confined to $y$ > 0. The flow is generated due to sheet stretching with linear velocity distribution, i.e, $u_w = \frac{u_0 x}{L}$.

The origin is kept fixed while the wall is stretching and the y-axis is perpendicular to the surface as shown in Figure 1. Under the above assumptions and conditions, the governing boundary layer equations for the considered problem are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

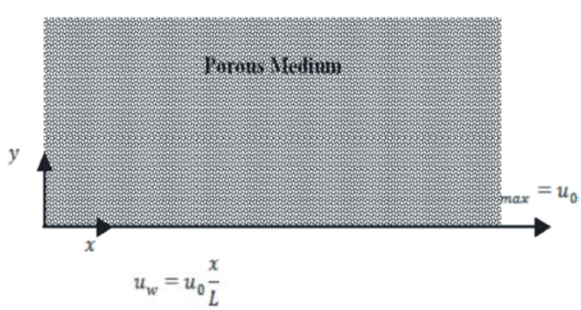


Fig. 1: Schematic figure of the problem.

Table 1: Convergence of HAM solution at different orders of approximation when $Re$ = 1, $Pr$ = 1, $De$ = 0.1, $Br$ = 1, $\hbar_1$ = −0.2628, $\hbar_2$ = − 1.4915

| Order | $f''(0)$ | $-\theta'(0)$ |
|---|---|---|
| 1 | -1.2903 | -0.2809 |
| 2 | -1.3396 | -0.2372 |
| 3 | -1.3470 | -0.2148 |
| 4 | -1.3482 | -0.2124 |
| 5 | -1.3483 | -0.2096 |
| 6 | -1.3484 | -0.2095 |
| 7 | -1.3484 | -0.2104 |
| 8 | -1.3484 | -0.2103 |
| 9 | -1.3484 | -0.2103 |
| 10 | -1.3484 | -0.2103 |

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + \frac{\alpha_1}{\rho}\left[\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right) + v\frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\right] - \frac{\nu}{K}u, \tag{2}$$

Where $\nu$ is the kinematic viscosity, $\alpha_1$ is the material constant and $K$ is the permeability coefficient of porous medium. The hydrodynamic boundary conditions are:

$$u(x^*,0) = u_0 x^*, \quad v(x^*,0) = 0, \quad u(x^*,\infty) = 0, \tag{3}$$

Where $x^* = x/L$ is the non-dimensional x-coordinate and L is the length of the porous plate. We define the following new variables

$$u = u_0 x^* f'(\eta), \quad v = -\frac{u_0}{L}\sqrt{K} f(\eta), \quad \eta = \frac{y}{\sqrt{K}}. \tag{4}$$

Substituting for $u$ and $v$ into Eq. (2) gives

$$f''' + \mathrm{Re}(ff'' - f'^2) + \mathrm{Re}\, De(2f'f''' - ff^{IV} - f''^2) - f', \tag{5}$$

Where $\mathrm{Re} = \frac{\rho u_0 K}{\mu L}$ is the Reynolds number and $De = \frac{\alpha_1}{\rho K}$ is the Deborah number. The corresponding boundary conditions are

$$f(0) = 1,\ f'(0) = 0,\ f'(\infty) = 0, \tag{6}$$

The local skin-friction coefficient or the frictional drag is given by

$$C_f = \frac{1}{\mathrm{Re}_x}(1 + 3\alpha^*) f''(0), \quad \alpha^* = \frac{\alpha_1 u_0}{\mu L}. \tag{7}$$

**Heat Transfer Analysis:** The governing boundary layer energy equation in the presence of viscous dissipation for the two-dimensional flow problem under consideration is given by

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\alpha_1}{\rho c_p}\left[u\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}+v\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\right]. \quad (8)$$

The following thermal boundary conditions are considered:

$$T(x^*, 0) = T_w, \quad T'(x^*, \infty) = T_\infty, \quad (9)$$

Where $T_w$ is the temperature of the sheet and $T_\infty$ is the temperature of the fluid far away from the sheet. Defining the non-dimensional temperature $\theta(\eta)$ as

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}. \quad (10)$$

Using (10), Eq. (8) can be written in the form

$$\theta''+\mathrm{Re}\,\mathrm{Pr}(f\theta')+Br\,f''^2+Br\,\mathrm{Re}\,De(f'f''-ff''f'''), \quad (11)$$

Where $\mathrm{Pr}=\frac{c_p\mu}{k}$ is the Prandtl number and $Br$ is the brinkman number. Consequently the boundary conditions (9) take the form

$$\theta(0) = 1, \quad \theta(\infty) = 0. \quad (12)$$

The local Nusselt number is given by

$$Nu_x=-\frac{\theta'(0)x}{\sqrt{K}} \quad (13)$$

**Entropy Generation:** The entropy generation in a fluid is caused by exchange of momentum and energy within the fluid and at the boundaries. One part of entropy generation is due to heat transfer in the direction of finite temperature gradients and other part takes place due to fluid friction irreversibility. Following Bejan [21], the entropy generation rate per unit volume is given by

$$S_{gen}=\frac{k}{T^2}(\nabla T)^2+\frac{\mathbf{t.L}}{T}, \quad (14)$$

The entropy generation rate per unit volume for viscoelastic fluid is given by

$$S_{gen}=\frac{k}{T^2}\left(\frac{\partial T}{\partial y}\right)^2+\frac{1}{T}\left[\mu\left(\frac{\partial u}{\partial y}\right)^2+\alpha_1 u\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}+\alpha_1 v\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\right]+\frac{\mu}{KT}u^2, \quad (15)$$

Using the non-dimensional quantities, we obtain the entropy generation number in non-dimensional form

$$Ns=\frac{S_{gen}}{S_{0_{gen}}}=\frac{1}{(1+T^*\theta)^2}+Br\frac{1}{(1+T^*\theta)}\left[f''^2+f'^2+\mathrm{Re}\,De(f'f''^2-ff''f''')\right], \quad (16)$$

Where $S_{0_{gen}}=\frac{k(T_w-T_\infty)^2}{kT_\infty{}^2}$ (17)

In order to solve equations (5), (6) and (11), (12), we use an analytic technique Homotopy analysis method (HAM).

**Solution of the Problem:** Our boundary data (7) and (13) suggest that the velocity and temperature distributions, viz. $f(\eta, \xi)$, $\theta(\eta, \xi)$ can be expressed by the set of following base functions

$$\{\eta^k Exp(-n\eta): k\geq 0, n\geq 0\}, \quad (18)$$

Taking into account the boundary conditions, we choose the initial guess and the linear operators as:

$$f_0(\eta) = 1-e^{-\eta} \quad \theta_0(\eta) = e^{-\eta} \quad (19)$$

and

$$L[F(\eta;p)]=\frac{\partial^3 f}{\partial\eta^3}+\frac{\partial^2 f}{\partial\eta^2}, \qquad L[\Theta(\eta;p)]=\frac{\partial^2\theta}{\partial\eta^2}-\theta, \quad (20)$$

The remaining facet of the method is renowned and is therefore concealed for simplicity (see for instance [24-30]).

**Convergence and validity of HAM Solution:** In order to show that $f(\eta)$ and $\theta(\eta)$ are approximate solutions of the systems (6), (7) and (12), (13), it is necessary to show the convergence of the solution series (43), (44) which strongly depends upon the auxiliary parameters $\hbar_1$ and $\hbar_2$ once the initial guess and the linear operator have been selected. To find out the appropriate values of $\hbar_1$ and $\hbar_2$ for our problem we have plotted the $\hbar$-curves of velocity and temperature at 10th order of approximation in Figure 2 and Figure 3 respectively. To further ensure the convergence of the series solutions, relative errors between two consecutive iterations are plotted in Figure 4 and Figure 5.

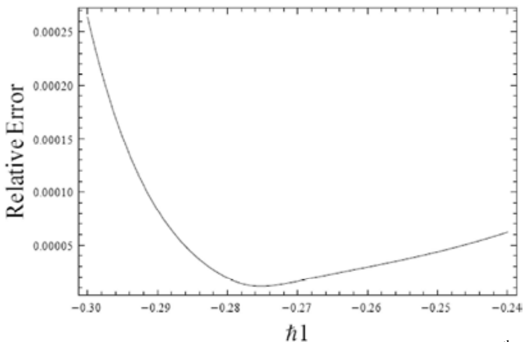


Fig. 2: Relative error of velocity profile between 11th and 12th iterations when $Re = 1, Pr = 1, De = 0.1, Br = 1$.

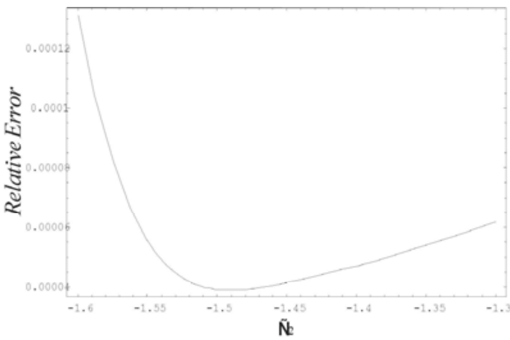


Fig. 3: Relative error of temperature profile between 11th and 12th iterations when $Re = 1$,

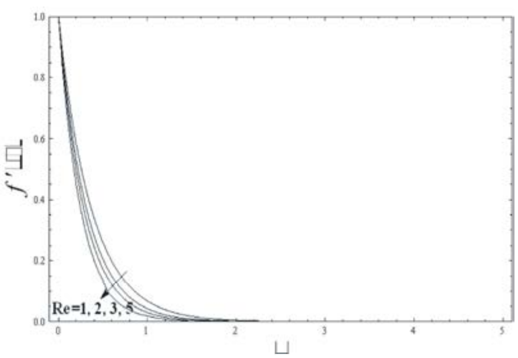


Fig. 4: Effects of Reynolds number on velocity profile when $Pr = 1, De = 0.1, Br = 1$.

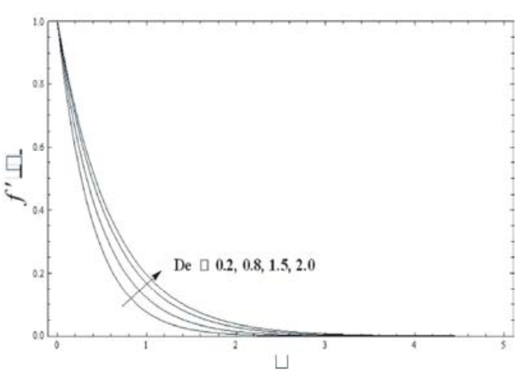


Fig. 5: Effects of Deborah number on velocity profile when $Re = 1, Pr = 1, Br = 1$.

**Graphical Illustration and Discussion:** To understand the physics of the problem we have investigated the effects of different parameters on the solution through graphs. In Figures 6 and 7, velocity profiles are plotted for different values of Reynolds number*Re* and Deborah number*De*. It is clear that boundary layer thickness decreases for large values of Reynolds number and increases with increase in Deborah number. Figure 8 shows the effects of Deborah number on the skin friction against the Reynold number. It is quite evident that there is a decrease in skin friction with the increase in Deborah number. To examine the effects of different parameters on the temperature profiles we have plotted the graphs in Figure 9-12. Figures 9 and 10 show the effects of Reynolds number and Prandtl number on the temperature profile from which it can be seen that thermal boundary layer thickness decreases with the increase in Reynolds number and Deborah number. Figure 11 shows that the thermal boundary layer thickness decreases with the increase in Deborah number. Similarly the thermal boundary layer thickness increases with Brinkman number shown in Figure 12. In Figs. 13-15 local Nusselt number has been plotted against Reynolds number for different parameters. In Figure 13 we have plotted $-\theta'(0)$ against Re for different values of parameter Pr. It shows that Nusselt number increases by increasing Prandtl number. Figure 14 shows that Nusselt number increase with Deborah number. Figure 15 shows that Nusselt number decrease with increase in Brinkman number.

The variations in total entropy generation number *Ns* are plotted against $\eta$ for different parameters in figures 16, 17, 18 and 19. It is seen in figure 16 that that entropy generation number *Ns* increases with the increase in Reynolds number. The reason is that the heat transfer rate between two energy sources with different temperature increases with*Re*. Figure 17 shows that with the increase in Prandtl number, there is an increase in entropy generation number. In figure 18 and 19 the effects of Deborah number and Brinkman number on entropy generation number have been examined which shows that there is an increase in entropy generation number with increasing values of Deborah number and Brinkman number. The Bejan number is, *Be* is defined as:

$$Be = \frac{Entropy\ generation\ due\ to\ heat\ transfer}{Total\ entropy\ generation} \tag{45}$$

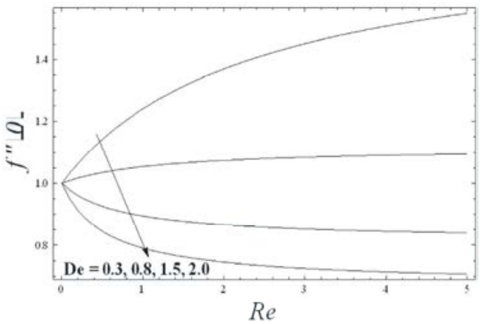


Fig. 6: Effects of Reynolds number on temperature profile when $Pr = 1$, $De = 0.1$, $Br = 1$.

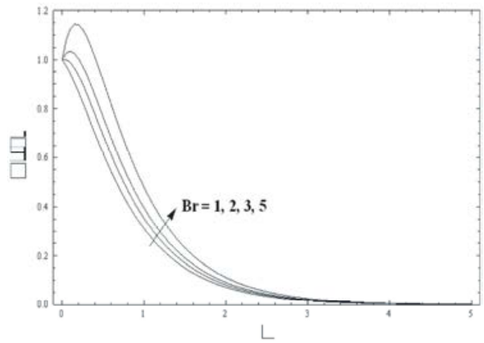


Fig. 10: Effects of Deborah number on skin friction plotted against Reynolds number.

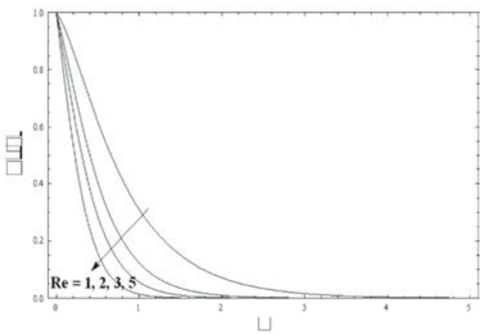


Fig. 7: Effects of Prandtl number on temperature profile when $Re = 1$, $De = 0.1$, $Br = 1$.

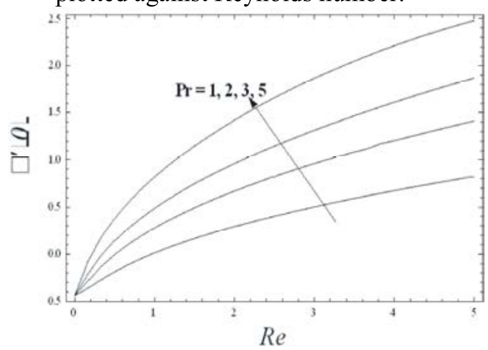


Fig. 11: Nusselt number plotted against Reynolds number for different Prandtl numbers with $De = 0.1$, $Br = 1$.

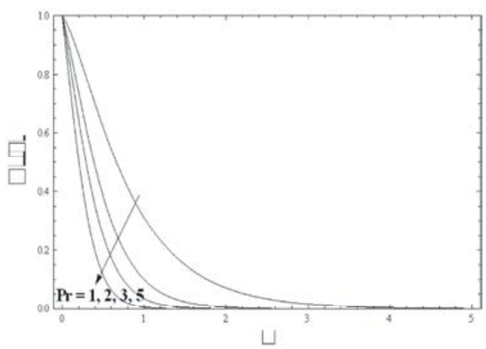


Fig. 8: Effects of Deborah number on temperature profile when $Re = 1$, $Pr = 1$, $Br = 1$.

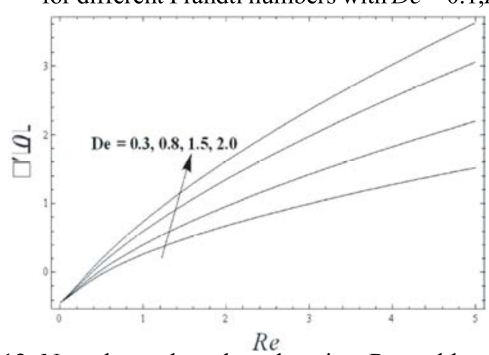


Fig. 12: Nusselt number plotted against Reynolds number for different Deborah numbers with $Pr = 1$, $Br = 1$.

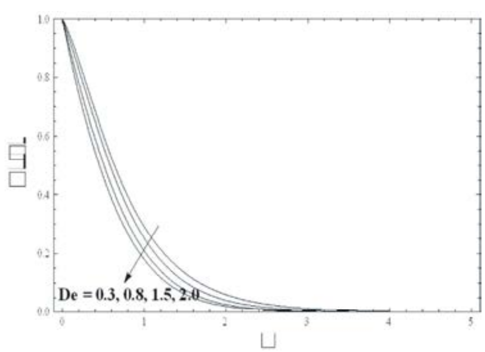


Fig. 9: Effects of Brinkman number on temperature profile when $Re = 1$, $Pr = 0.1$, $De = 1$.

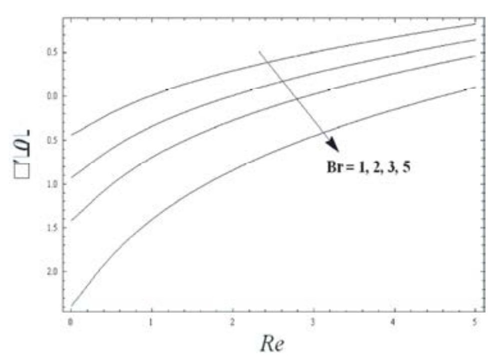


Fig. 13: Nusselt number plotted against Reynolds number for different Brinkman numbers with $Pr = 1$, $De = 0.1$.

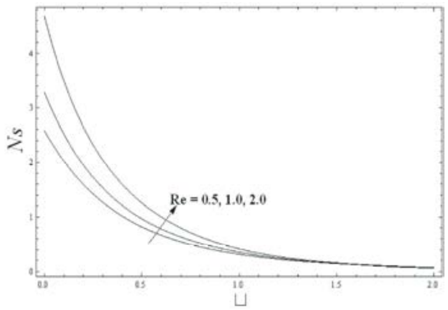


Fig. 14: The effects of Reynold number on non-dimensional entropy generation number *Ns* with $Pr = 1$, $De = 0.1$, $Br = 1$.

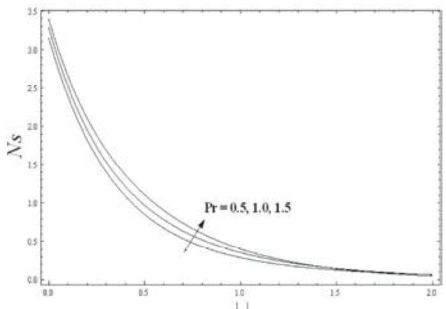


Fig. 15: The effects of Prandtl number on non-dimensional entropy generation number *Ns* with $Re = 1$, $De = 0.1$, $Br = 1$.

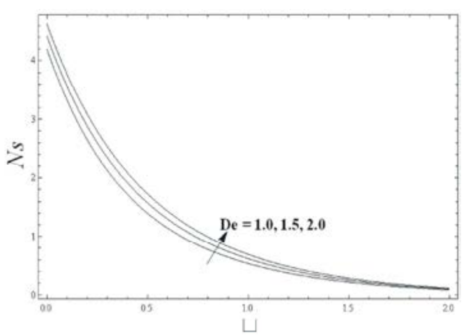


Fig. 16: The effects of Deborah number on non-dimensional entropy generation number *Ns* with $Re = 1$, $Pr = 1$, $Br = 1$.

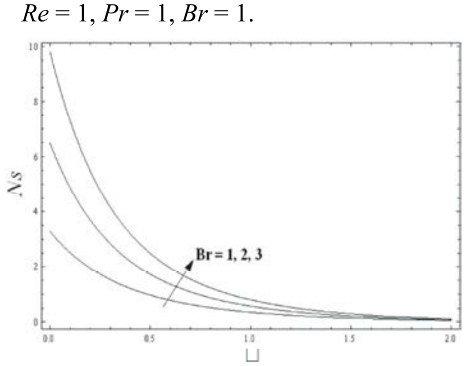


Fig. 17: The effects of Brinkman number on non-dimensional entropy generation number *Ns* with $Re = 1$, $Pr = 1$, $De = 1$.

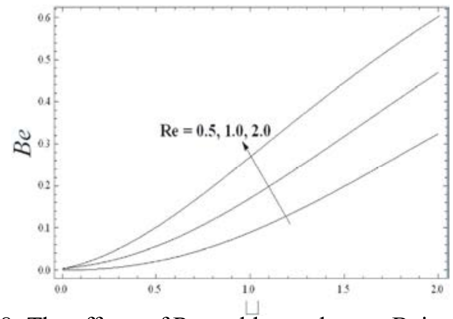


Fig. 18: The effects of Reynolds number on Bejan number *Be* with $Pr = 1$, $De = 0.1$, $Br = 1$.

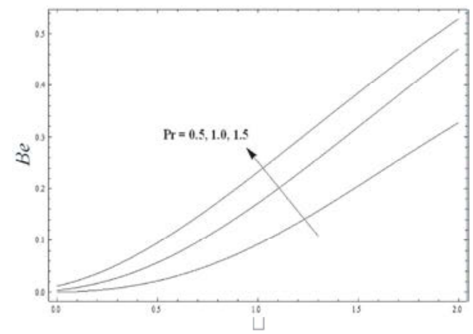


Fig. 19: The effects of Prandtl number Bejan number *Be* with $Re = 1$, $De = 0.1$, $Br = 1$.

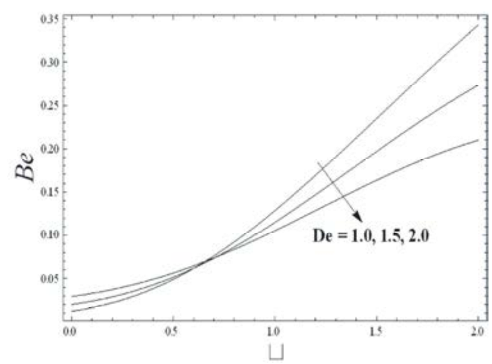


Fig. 20: The effects of Deborah number on Bejan number *Be* with $Re = 1$, $Pr = 1$, $Br = 1$.

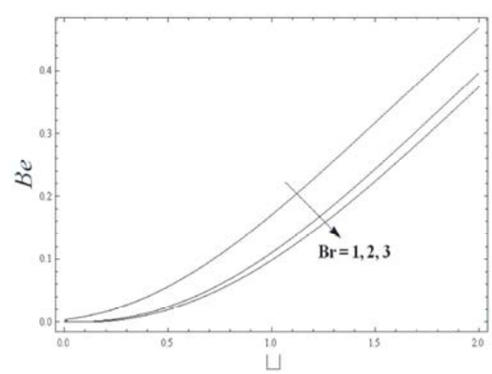


Fig. 21: The effects of Brinkman number on Bejan number *Be* with $Re = 1$, $Pr = 1$, $De = 0.1$.

Bejan number ranges from 0 to 1 and when the entropy generation due to heat transfer is dominant, *Be* is close to 1. Figures 20 – 23 shows the effects of different physical parameters on the Bejan number. It can be seen that Bejan number increases with Reynolds number, Prandtl number and Deborah number and decreases with Brinkman number.

**Summary and Conclusion:** Boundary layer second grade fluid flow over a stretching surface is studied. Similarity solution technique is applied and boundary layer equations are transformed into ordinary differential equations and are solved with homotopy analysis method. Introducing non-dimensional numbers, a parametric study is performed. Moreover thermal analysis of the problem has been made. The highlights of this study are:

- Viscous boundary layer thickness decreases with Reynolds number and increases with.

Deborah number whereas the thermal boundary layer thickness decreases with Reynolds, Prandtl number and Deborah number and increases with Brinkman number.

- The Nusselt Number *Nu* increases with increase in Prandtl number and Deborah number and decreases with increase in Brinkman number.
- Increasing Reynolds number, Prandtl number and Deborah number enhances total entropy generation number *Ns* significantly.
- Brinkman number has a reverse effect on total entropy generation number as *Ns* decreases with increase in Brinkman number.

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