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Application of the Differential Transformation Method to Non-Linear Shock Damper Dynamics

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Abstract: This paper adopts the differential transformation method (DTM) to non-linear shock damper dynamics with non-linear spring and non-linear damping. The principle of differential transformation is briefly introduced and is then applied in the derivation of a set of difference equations for the problem. The solutions are subsequently solved by a process of inverse transformation. The time responses of the equations are presented for two cases and the current results are then compared with those derived from the established Runge-Kutta method in order to verify the accuracy of the proposed method. It is shown that there is excellent agreement between the two sets of results. This finding confirms that the proposed differential transformation method is a powerful and efficient tool for solving non-linear problems.

Key words: Differential transformation method (DTM) · Shock absorber · Non-linear damping · Non-linear spring · Runge-Kutta method

INTRODUCTION

Most of engineering problems, especially some equations of motion of vibratory systems are non-linear. Therefore some of them are solved using numerical solutions and some are solved using the analytical methods. In order to obtain the equation of motion of vibratory systems, we will need a mathematical description of the forces and moments involved, as function of displacement or velocity. The solution of vibration models to predict system behavior requires solution of differential equations. The differential equations based on linear model of the forces and moments are much easier to solve than the ones based on non-linear models. Sometimes a non-linear model is unavoidable; this is the case when a system is designed with non-linear spring and non-linear damping. Chi and Rosenberg [1] studied non-linear mass-spring-damper systems with many degrees of freedom in which all of the springs and dampers were strongly non-linear. Their results indicated that the ultimate state of such damped systems generally rests in the equilibrium position, or possibly exists as a normal mode vibration. The authors also identified the conditions necessary for the existence of classical normal modes in damped non-linear systems. Andrianov and Awrejcewicz [2, 3] developed an asymptotic approach for the analysis of strongly non-linear dynamic systems and compared the approximated results with those obtained by the fourth-order Runge-Kutta method. It was found that the asymptotic approximations were in good agreement with the Runge-Kutta solutions at high orders of non-linearity, but were less satisfactory at lower power-form orders. Jang and Chen [4] applied the differential transformation method to analyze the response of a strongly non-linear damped system. Lo and Chen [5] employed the differential transformation technique to investigate duffing oscillators with time-varying parameters. Chen and Ho [6] utilized the differential transform concept to solve the free vibration problem of a rotating twisted Timoshenko beam under axial loading.

The differential transformation method was first applied in the engineering domain by Zhou [7] and is commonly used for the solution of electric circuit problems. The differential transformation method is based on the Taylor series expansion and constructs an analytical solution in the form of a polynomial [8-12]. The traditional high-order Taylor series method requires symbolic computation. However, the differential transformation method obtains a polynomial series solution by means of an iterative procedure.

In this paper, the basic idea of DTM is described and then it is applied to non-linear shock damper dynamics with non-linear spring and non-linear damping. The study

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considers two examples and employs the proposed method to solve the corresponding differential equations in the form of a power series. A comparison of the present results with those yielded by the established Runge-Kutta method confirms the accuracy of the proposed method.

Problem Statement: The basic function of the shock absorber is to absorb and dissipate the impact kinetic energy to the extent that accelerations imposed upon the airframe are reduced to a tolerable level. Existing shock absorbers can be divided into two classes based on the type of the spring being used: those using a solid spring made of steel or rubber and those using a fluid spring with gas or oil, or a mixture of the two that is generally referred to as oleo-pneumatic. The high gear and weight efficiencies associated with the oleo-pneumatic shock absorber make it the preferred design for commercial transports.

A spring element exerts a reaction force in response to a displacement, either compression or extension, of the element. The linear spring relation $f = kx$ becomes less accurate with increasing deflection (for either compression or extension). In such case it is often replaced by [13]:

$$f = k_1 x \quad (1a)$$

$$f = k_1 x + k_2 x^3 \quad (1b)$$

Where $k_1 > 0$ and has dimension of N/m and x is thespring displacement from its free length. The spring elements is said to be hardening or hard if $k_2 > 0$ and has dimension of N/m^3 and softening or soft if $k_2 > 0$. These cases are shown in Fig. 1.

The spring stiffness k is the slope of force-deflection curve and is constant for the linear spring element. A non-linear spring does not have a single stiffness value since its slope is variable. A damper element is an element that resists relative velocity across it. A simple way to achieve damping is with a dashpot or damper, which is the basis of shock absorber. It consists of a piston moving inside a cylinder that is sealed and filled with a viscous fluid (Fig. 2) the piston has a hole or orifice through which the fluid can flow when the piston moves relative to the cylinder, but the fluid' viscosity resists this motion [13].

There are many applications where we can derive an expression for the spring force as a function of deflection or the damping force as a function of velocity [13]; in practice forms for the spring relation are the linear model and the cubic model as shown in Eq. (1a) and (1b) respectively.

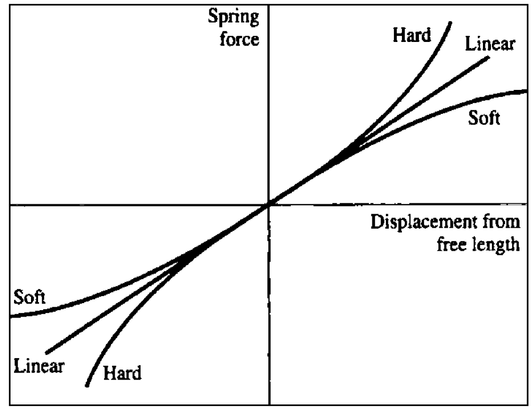

Fig. 1: Hard, soft and linear spring function [13]

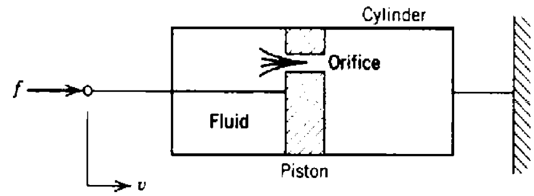

Fig. 2: A fluid damper consists of a piston moving inside a cylinder filled with a viscous fluid

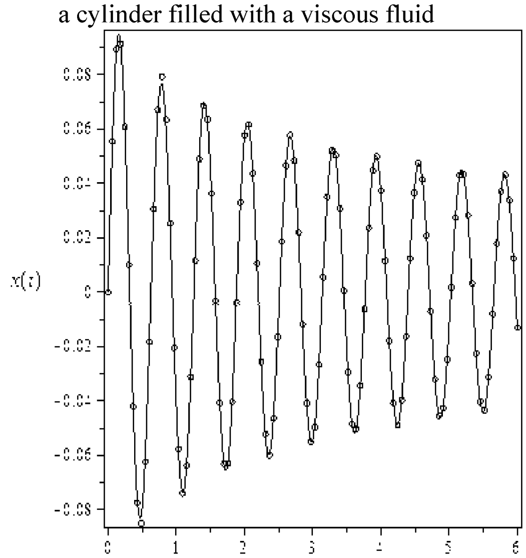

Fig. 3: Compare results of Example 1 obtained by the DTM (solid line) and the Runge-Kutta method (circle)

Also the most common model forms for the damping relation are:

$f(v) = cv$	The linear model	(2a)
$f(v) = cv^2$	The square-law model	(2b)
$f(v) = cv^n$, $n > 1$	The general progressive model	(2c)
$f(v) = cv^n$, $n < 1$	The degressive model	(2d)

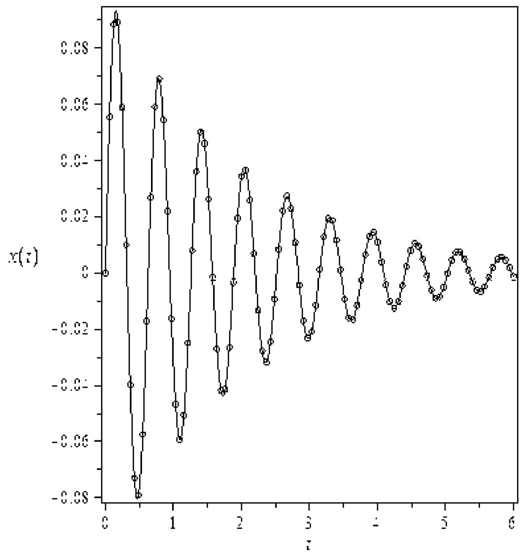

Fig. 4: Compare results of Example 2 obtained by the DTM (solid line) and and the Runge-Kutta method (circle)

Fundamentals of Differential Transformation Method: Let $x(t)$ be analytic in a domain D and let $t = t_i$ represent any point in D. The function $x(t)$ is then represented by one power series whose center is located at t_i. The Taylor series expansion function of $x(t)$ is in form of:

$$x(t)=\sum_{k=0}^{\infty}\frac{(t-t_i)^k}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=t_i} \qquad \forall t\in D \tag{3}$$

The particular case of Eq. (3) when $t_i = 0$ is referred to as the Maclaurin series of $x(t)$ and is expressed as:

$$x(t)=\sum_{k=0}^{\infty}\frac{(t)^k}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=0} \tag{4}$$

As explained in [14] the differential transformation of the function $x(t)$ is defined as follows:

$$X(k)=\frac{(H)^k}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=0} \tag{5}$$

Where $x(t)$ is the original function and $X(k)$ is the transformed function. The differential spectrum of $X(k)$ is confined within the interval $t\in[0,H]$, where H is a constant. The differential inverse transform of $X(k)$ is defined as follows:

Table 1: The fundamental operations of the differential transform method

Original function	Transformed function
$x(t)=\alpha f(t)\pm\beta g(t)$	$X(k)=\alpha F(k)\pm\beta G(k)$
$x(t)=\frac{df(t)}{dt}$	$X(k)=(k+1)F(k+1)$
$x(t)=\frac{d^2f(t)}{dt^2}$	$X(k)=(k+1)(k+2)F(k+2)$
$x(t)=t^m$	$X(k)=\delta(k-m)=\begin{cases}1 & k=m\\ 0 & k\neq m\end{cases}$
$x(t)=\exp(\lambda t)$	$X(k)=\frac{\lambda^k}{k!}$
$x(t)=f(t)g(t)$	$X(k)=\sum_{l=0}^{k}F(l)G(k-l)$

$$x(t)=\sum_{k=0}^{\infty}\left(\frac{t}{H}\right)^k X(k) \tag{6}$$

It is clear that the concept of differential transformation is based upon the Taylor series expansion. The values of function $X(k)$ at values of argument k are referred to as discrete, i.e. $X(0)$ is known as the zero discrete, $X(1)$ as the first discrete, etc. the more discrete available, the more precise it is possible to restore the unknown function. The function $x(t)$ consists of T-function $X(k)$ and its value is given by the sum of the T-function with $\left(\frac{t}{H}\right)^k$ as its coefficient. In real applications, at the right choice of constant H, the larger values of argument K the discrete of spectrum reduce rapidly. The function K is expressed by a finite series and Eq. (6) can be written as:

$$x(t)=\sum_{k=0}^{n}\left(\frac{t}{H}\right)^k X(k) \tag{7}$$

Mathematical operations performed by differential transform method are listed in Table 1.

Applications of the DTM: We applied the differential transformation method to non-linear shock damper dynamics with non-linear spring and non-linear damping and assess the advantage and the accuracy of proposed method. We will consider the two examples as follows:

Example 1: We consider the equation of motion and the initial conditions as [15]:

$$\frac{d^2x}{dt^2}+\left(\frac{dx}{dt}\right)^3+100x=0, \tag{8}$$

With initial conditions:

$$x(0) = 0 \tag{9a}$$

$$\frac{dx}{dt}(0) = 1 \tag{9b}$$

We applied the DTM for the Eq. (8) and taking the differential transform Eq. (8) with respect to t gives:

$$(k+2)(k+1)X(k+2)+100X(k)+\sum_{w=0}^{k}(k-w+1)X(k-w+1)\left(\sum_{j=0}^{w}(w-j+1)X(w-j+1)\times(j+1)X(j+1)\right)=0 \tag{10}$$

From a process of inverse differential transformation, it can be shown that the solutions of each sub-domain take n+1 term for the power series like Eq. (8), we can write:

$$x_i(t)=\sum_{k=0}^{n}\left(\frac{t}{H_i}\right)^k X_i(k) \qquad 0\le t\le H_i \tag{11}$$

Where k represents the number of term of the power series, i = 0.,1,2... expresses the i^{th} sub-domain and H_i is the sub-domain interval. We calculated $X(k+2)$ from Eq. (10) as following:

$X(2) = -0.5$ (12a)
$X(3) = -16.16$ (12b)
$X(4) = 16.04$ (12a)
$X(5) = 56.7$ (12b)
$\vdots$ (12a)
$X(21) = -2.26\times10^{11}$ (12b)

Substituting Eq. (12) into the main equation based on the DTM, it can be obtained that the closed form of the solutions is:

$$\begin{aligned} x_1(t)={}&t-0.5t^2-16.16t^3+16.04t^4+56.7t^5-299.36t^6+641.8t^7\\ &+3042.87t^8-17495.72t^9+14315.98t^{10}+2.24\times10^5t^{11}-1.1\times10^6t^{12}\\ &-1.02\times10^5t^{13}+1.96\times10^7t^{14}-6.94\times10^7t^{15}-1\times10^8t^{16}\\ &+1.67\times10^9t^{17}-4.3\times10^9t^{18}-1.58\times10^{10}t^{19}+1.41\times10^{11}t^{20}-2.26\times10^{11}t^{20}\end{aligned} \tag{13}$$

In the similar manner, we will obtain another sub-domain's series solution and we can present the solution of Eq. (8) accurately

Example 2: We consider the equation of motion and the initial conditions as [15]:

$$\frac{d^2x}{dt^2}+\frac{dx}{dt}-0.1\left(\frac{dx}{dt}\right)^3+x^3+100x=0 \tag{14}$$

With initial conditions:

$$x(0) = 0 \tag{15a}$$

$$\frac{dx}{dt}(0) = 1 \tag{15b}$$

We applied the DTM for the Eq. (14) and taking the differential transform Eq. (14) with respect to t gives:

$$(k+2)(k+1)X(k+2)+(k+1)X(k+1)+100X(k)+\sum_{w=0}^{k}X(k-w)\left(\sum_{j=0}^{w}X(w-j)X(j)\right)-0.1\left(\sum_{w=0}^{k}(k-w+1)X(k-w+1)\left(\sum_{j=0}^{w}(w-j+1)X(w-j+1)\times(j+1)X(j+1)\right)\right)=0 \tag{16}$$

Similar to example 1 we have:

$X(2) = -0.45$ (17a)
$X(3) = 16.56$ (17b)
$X(4) = 6.66$ (17a)
$X(5) = 83.16$ (17b)
$\vdots$ (17a)
$X(21) = -4.42\times10^8$ (17b)

Substituting Eq. (17) into the main equation based on the DTM, it can be obtained that the closed form of the solutions is:

$$\begin{aligned} x_1(t)={}&t-0.45t^2-16.56t^3+6.66t^4+83.16t^5-8.08t^6-236.02t^7\\ &-387.58t^8+894.83t^9+4734.05t^{10}-4689.05t^{11}-3709668t^{12}\\ &+2071.17t^{13}+2.54\times10^5t^{14}+3.37\times10^5t^{15}-1.69\times10^6t^{16}-4.94\times10^6t^{17}\\ &+9.47\times10^6t^{18}-1.58\times10^{10}t^{19}+5.02\times10^7t^{20}-2.4\times10^7t^{20}-4.42\times10^8t^{21}\end{aligned} \tag{18}$$

In the similar manner, we will obtain another sub-domain's series solution and we can present the solution of Eq. (14) accurately.

Table 2: The results of the DTM and fourth order Runge-Kutta method for example 1.

t	$x(t)_{DTM}$	$x(t)_{RK}$	Error = $\lvert x(t)_{DTM}-x(t)_{RK}\rvert$
0	0	0	0
1	-0.042389180	-0.041037598	0.001351582
2	0.057676656	0.0576663784	0.000010277
3	-0.054189824	-0.054836214	0.000646390
4	0.036444190	0.037277259	0.000833069
5	-0.011053633	-0.011992513	0.000938880
6	-0.013778616	-0.013040705	0.000737911

Table 3: The results of the DTM and fourth order Runge-Kutta method for example 2.

t	$x(t)_{DTM}$	$x(t)_{RK}$	Error = $\lvert x(t)_{DTM}-x(t)_{RK}\rvert$
0	0	0	0
1	-0.033224341	-0.033223959	3.82E-07
2	0.034385642	0.034385873	2.31E-07
3	-0.023020243	-0.023020481	2.38E-07
4	0.010940017	0.010940186	1.69E-07
5	-0.002746481	-0.002746541	6.00E-08
6	-0.012087954	-0.001208841	0.010879113

RESULTS AND DISCUSSION

The present study employs the differential transformation method to generate a number of numerical results for the response of non-linear shock damper dynamics with non-linear spring and non-linear damping. The calculations presented in this paper adopt a value of $n = 20$. Having determined the various values of $X(k+2)$ from Eqs. (10) and (16) with the transformed initial conditions of Eqs. (9) and (15), the first sub-domain solutions of Eqs. (8) and (14) can be obtained by means of the inverse transformed equations of Eq. (11). The final values of the first sub-domain, i.e. the solutions of the previous calculation, are then taken as the initial condition of the second sub-domain, which is subsequently calculated using the same procedure as that described above. By repeatedly adopting the final values of one sub-domain as the initial condition of the following sub-domain, the differential equation can be solved from its first sub-domain to its final sub-domain. Therefore, the proposed method enables the solutions of Eqs. (8) and (14) to be solved over the entire time domain.

The response of $x(t)$ are plotted in Figs. 1 and 2 for two examples. Also these results are seen in tables 2 and 3 for more clearly. In order to verify the effectiveness of the proposed differential transformation method, the fourth-order Runge-Kutta numerical method is used to compute the displacement response of the non-linear oscillator for a set of initial amplitudes. It is noted that the present results are in excellent agreement with the numerical results obtained from the fourth-order Runge-Kutta approach.

CONCLUSION

The current study has applied the differential transformation method to non-linear shock damper dynamics with non-linear spring and non-linear damping. The non-linear equations are first expressed as algebraic relationships. These relationships are transformed into a set of difference equations, which are then solved via a process of inverse transformation. It has been shown that the results of the differential transformation method are in good agreement with those obtained from the fourth-order Runge-Kutta numerical method. The present study has confirmed that the DTM offers significant advantages in terms of its straightforward applicability, its computational effectiveness and its accuracy.

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