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Solution of the Model for a Horizontal Cross Section of the Center Span by Using the Differential Transformation Method

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Abstract: In this paper, an aproximate analytical method called the differential transform method (DTM) is used as a tool to give approximate solutions of oscillation models of the Lazer-McKenna suspension bridge. The differential transformation method is described in a nuthsell. DTM can simply be applied to linear or nonlinear problems and reduces the required computational effort. The proposed scheme is based on the differential transform method (DTM), Laplace transform and Padé approximants. The results to get the differential transformation method (DTM) are applied Padé approximants. The reliability of this method is investigated by comparison with the classical fourth-order Runge–Kutta (RK4) method. Our the presented method showed results to analytical solutions of nonlinear ordinary differential equation. Some plots are gived to shows solutions of oscillation models of the Lazer-McKenna suspension bridge for illustrating the accurately and simplicity of the methods.

Key words: Padé approximants · The differential transform method · Oscillation models of the Lazer-McKenna suspension bridge

INTRODUCTION

The modified differential transform method (MDTM) will be employed in a straightforward manner without any need of linearization or smallness assumptions. DTM was first applied in the engineering domain by [1, 2]. DTM provides an efficient explicit and numerical solution with high accuracy, minimal calculations, avoidance of physically unrealistic assumptions. However, DTM has some drawbacks. By using DTM, we obtain a series solution, in practice a truncated series solution. This series solution does not exhibit the periodic behavior which is characteristic of oscillator equations and gives a good approximation to the true solution in a very small region. In order to improve the accuracy of DTM, we use an alternative technique which modifies the series solution for non-linear oscillatory systems as follows: we first apply the Laplace transformation to the truncated series obtained by DTM, then convert the transformed series into a meromorphic function by forming its Padé approximants [3] and finally adopt an inverse Laplace transform to obtain an analytic solution, which may be periodic or a better approximation solution than the DTM truncated series solution.

The goal of this paper is to extend the differential transformation method proposed by Zhou [1] to solve oscillation models of the Lazer-McKenna suspension bridge. The results of the differential transformation method are numerically compared with conclusions obtained by the modified differential transformation method and the fourth-order Runge–Kutta method. The MDTM is useful to obtain exact and approximate solutions of linear and nonlinear oscillations equations. No necessity to linearization or discretization, large computational work and round-off errors is bewared. It has been used to solve efficiently, easily and accurately a large class of nonlinear problems with approximations. These approximations converge rapidly to exact solutions [12-25].

For over seventy years, scientists in many disciplines have endeavoured to explain the cause of the dramatic and eventually destructive torsional oscillations of the Tacoma Narrows Bridge which preceded its collapse in 1940. The original Tacoma Narrows Bridge displayed significant vertical oscillations after it opened on July 1, 1940. Until Tacoma Narrows Bridge collapsed, had not made torsional oscillations. The amplitude of the vertical movement is large enough to bridge the transition was

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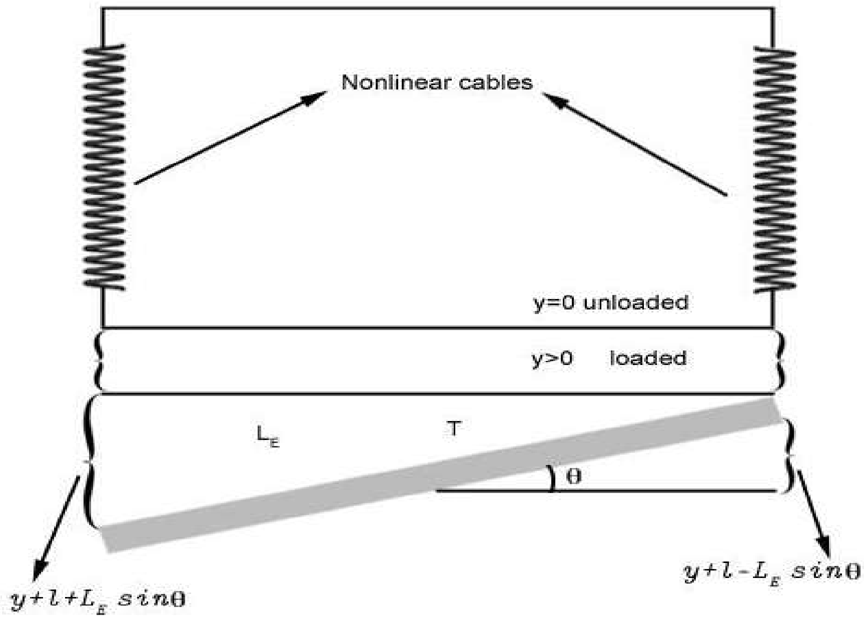

Fig. 1: A simple model of the center span and its horizontal cross section

a sudden large amplitude torsional motion. As a result of the fact that occured slackening of the cables that suspended the roadbed. Because of suddenly lateral asymmetry is the loss of one or more hangers on one side of a bridge. Due to growing amplitude of the torsional oscillations, the bridge collapsed within an hour.

The Model for a Horizontal Cross Section of the Center Span: We deal with the center span of the bridge as a beam of length L_B and width $2L_E$ suspended by cables. To model the motion of a horizontal cross section of the beam, we consider it as rod of length $2L_E$ and mass m is the masser per unit length of the road bed. As we can see in Figure 1, $y(t)$ indicates the downward(upward) distance of the center of gravity of the rod from the loaded state and $\theta(t)$ defines the angle of the rod from horizontal at time t.

Here, we will assume that that the cables resist elongation according to Hooke's law with spring constant k, but do not resist compression [6-11]. Observe that the stretch in the right hand cable is given $y + l - L_E \sin\theta$ and the stretch in the other cable is $y + l + L_E \sin\theta$, see Figure 1.

Then the force exerted by the right hand cable is

$$F = \begin{cases} -k\left(y + l - L_E \sin\theta\right), & y + l - L_E \sin\theta \geq 0 \\ 0, & y + l - L_E \sin\theta < 0 \end{cases} = -k\left(y + l - L_E \sin\theta\right)^{+} \quad (1)$$

Where $v^{+} = \max\{v, 0\}$. Analogously, the force exerted by the left hand cable is $-k(y + l + L_E \sin\theta)^{+}$. Using Newton's second law, the torsional and vertical motions are governed by

$$\begin{cases} y'' + \delta_v y' + \dfrac{k}{m}\left[\left(y + l - L_E \sin\theta\right)^{+} + \left(y + l + L_E \sin\theta\right)^{+}\right] = g + \lambda_v \sin(wt) \\ \theta'' + \delta_t \theta' - \dfrac{3k}{mL_E}\cos\theta\left[\left(y + l - L_E \sin\theta\right)^{+} - \left(y + l + L_E \sin\theta\right)^{+}\right] = 0 \\ y(0) = y_0, \quad y'(0) = y_1, \quad \theta(0) = \theta_0, \quad \theta'(0) = \theta_1 \end{cases} \quad (2)$$

δ_v are δ_t damping constants, λ_v ; w are the amplitude and frequency of the external vertical forcing and is the $g \approx 9.8$ gravitational acceleration.

Let’s suppose that the cables never get loose, we have $y + l \pm L_E \sin\theta \geq 0$ and so $(y + l \pm L_E \sin\theta)^{+} = y + l \pm L_E \sin\theta$. Therefore, the equations (2) become uncoupled and torsional and vertical motion satisy

$$\begin{cases} y'' + \delta_v y' + \dfrac{2k}{m} y = \lambda_v \sin(wt) \\ \theta'' + \delta_t \theta' + \dfrac{6k}{m}\sin\theta\cos\theta = 0 \\ y(0) = y_0, \quad y'(0) = y_1, \quad \theta(0) = \theta_0, \quad \theta'(0) = \theta_1 \end{cases} \quad (3)$$

respectively. The first term in the equation for the vertical motion approaches $l = \frac{mg}{2k}$, the acceleration due to gravity [10].

Differential Transformation Method: As in [14-25], the basic definition of the differential transformation method are gived as follows:

Definition 3.1: If $y(t)$ is analytic in the domain T, then it will be differentiated continuously with respect to time t,

$$\frac{\partial^k y(t)}{\partial t^k} = \varphi(t,k), \quad \text{for all } t \in T \tag{4}$$

for $t = t_i$, then $\varphi(t,k) = \varphi(t_i, k)$, where k belongs to set of nonnegative integers, denoted as the K-domain. Therefore, Eq. (4) can be rewritten as

$$Y(k) = \varphi(t_i,k) = \left[\frac{\partial^k y(t)}{\partial t^k}\right]\Bigg|_{t=t_i}, y(t) \text{ at } t = t_i \tag{5}$$

Where $Y(k)$ is called the spectrum of $y(t)$ at $t = t_i$

Definition 3.2: If $y(t)$ can be described by Taylor's series, then $y(t)$ can be shown as

$$y(t) = \sum_{k=0}^{\infty}\left[\frac{(t-t_i)^k}{k!}\right]Y(k). \tag{6}$$

Eq.(6) is called the inverse of $y(t)$, with the symbol D denoting the differential transformation process. Upon combining (5) and (6), we attain

$$y(t) = \sum_{k=0}^{\infty}\left[\frac{(t-t_i)^k}{k!}\right]Y(k) \equiv D^{-1}Y(k). \tag{7}$$

Using the differential transformation, a differential equation in the domain of interest can be transformed to an algebraic equation in the K-domain and the $y(t)$ can be obtained by finite-term Taylor's series plus a remainder, as

$$y(t) = \sum_{k=0}^{n}\left[\frac{(t-t_i)^k}{k!}\right]Y(k) + R_{n+1}(t). \tag{8}$$

From the definitions (5) and (7), it is easy to obtain the following mathematical operations:

Trigonometric nonlinearity: $y(x)$ = sin (αx), $w(x)$ = cos(αx) By definition,

$$\begin{aligned} Y(0) &= \left[\sin(\alpha x(t))\right]_{t=0} = \sin(\alpha x(0)) = \sin(\alpha X(0)) \\ W(0) &= \left[\cos(\alpha x(t))\right]_{t=0} = \cos(\alpha x(0)) = \cos(\alpha X(0)) \end{aligned} \tag{9}$$

To find other transformed functions, we differentiate $y(x)$ = sin (αx), $w(x)$ = cos(αx) obtaining:

$$\begin{aligned} \frac{dy(x)}{dt} &= \alpha\cos(\alpha x)\frac{dx(t)}{dt} = \alpha w(x)\frac{dx(t)}{dt} \\ \frac{dw(x)}{dt} &= -\alpha\sin(\alpha x)\frac{dx(t)}{dt} = -\alpha z(x)\frac{dx(t)}{dt} \end{aligned} \tag{10}$$

Applying the differential transform to Eq. (10) obtain:

$$\begin{aligned} (k+1)Y(k+1) &= \alpha\sum_{r=0}^{k}(k+1-r)W(r)X(k+1-r) \\ (k+1)W(k+1) &= -\alpha\sum_{r=0}^{k}(k+1-r)Y(r)X(k+1-r) \end{aligned} \tag{11}$$

Similarly, replacing k–1 by k gives:

$$\begin{aligned} Y(k) &= \alpha\sum_{r=0}^{k-1}\left(\frac{k-r}{k}\right)W(r)X(k-r), \quad k \ge 1 \\ W(k) &= -\alpha\sum_{r=0}^{k-1}\left(\frac{k-r}{k}\right)Y(r)X(k-r), \quad k \ge 1 \end{aligned} \tag{12}$$

Combine Eqs. (9) and (12) to give the recursive relation:

$$Y(k) = \begin{cases} \sin(\alpha X(0)), & k = 0, \\ \alpha\sum_{r=0}^{k-1}\left(\frac{k-r}{k}\right)W(r)X(k-r), & k \ge 1. \end{cases} \tag{13}$$

and

$$W(k) = \begin{cases} \cos(\alpha X(0)), & k = 0, \\ -\alpha\sum_{r=0}^{k-1}\left(\frac{k-r}{k}\right)Y(r)X(k-r), & k \ge 1. \end{cases} \tag{14}$$

are given by [4].

Padé Approximation: A rational approximation to $f(x)$ on $[a, b]$ is the quotient of two polynomials $P_N(x)$ and $Q_M(x)$ of degrees N and M, respectively. We use the notation $R_{N,M}(x)$ to denote this quotient. The $R_{N,M}(x)$ Padé approximations to a function $f(x)$ are given by [3].

$$R_{N,M}(x) = \frac{P_N(x)}{Q_M(x)} \quad \text{for } a \le x \le b. \tag{15}$$

Table 1: The fundamental operations of the differential transformed method

Original function	Transformed function
$y(t) = cw(t)$	$Y(k) = cW(k)$
$y(t)=\frac{dw(t)}{dt}$	$Y(k) = (k+1)W(k+1)$
$y(t)=\frac{d^i w(t)}{dt^i}$	$Y(k) = (k+1)(k+2)\ldots(k+i)W(k+i)$
$y(t) = u(t)v(t)$	$Y(k)=\sum_{r=0}^{k} U(r)V(k-r)$
$y(t) = u(t)v(t)w(t)$	$Y(k)=\sum_{s=0}^{k}\sum_{m=0}^{k-s} U(s)V(m)W(k-s-m)$
$y(t) = u(t)\mp v(t)$	$Y(k) = U(k)\mp V(k)$
$y(t) = \sin(wt+\phi)$	$Y(k)=\frac{w^k}{k!}\sin\left(\frac{\pi k}{2}+\phi\right)$
$y(t) = \cos(wt+\phi)$	$Y(k)=\frac{w^k}{k!}\cos\left(\frac{\pi k}{2}+\phi\right)$
$y(t) = Ce^{at}\sin(wt+\phi)$	$Y(k)=C\sum_{l=0}^{k}\frac{a^l}{l!}\sin\left(\frac{\pi(k-l)}{2}+\phi\right)\frac{w^{(k-l)}}{(k-l)!}$
$y(t) = t^m$	$Y(k)=\delta(k-m)=\begin{cases}1, & k=m\\ 0, & k\neq m\end{cases}$

The method of Padé requires that $f(x)$ and its derivative be continuous at $x = 0$. The polynomials used in (15) are

$$P_N(x) = p_0 + p_1x + p_2x^2+\ldots+p_Nx^N \tag{16}$$

$$Q_N(x) = 1 + q_1x + q_2x^2+\ldots+q_Mx^M \tag{17}$$

The polynomials in (16) and (17) are constructed so that $f(x)$ and $R_{N,M}(x)$ agree at $x = 0$ and their derivatives up to $N + M$ agree at $x = 0$. In the case $Q_0(x) = 1$, the approximation is just the Maclaurin expansion for $f(x)$. For a fixed value of $N + M$ the error is smallest when $P_N(x)$ and $Q_M(x)$ have the same degree or when $P_N(x)$ has degree one higher than $Q_M(x)$.

Notice that the constant coefficient of Q_M is $q_0 = 1$. This is permissible, because it notice be 0 and $R_{N,M}(x)$ is not changed when both $P_N(x)$ and $Q_M(x)$ are divided by the same constant. Hence the rational function $R_{N,M}(x)$ has $N + M + 1$ unknown coefficients. Assume that $f(x)$ is analytic and has the Maclaurin expansion.

$$f(x) = a_0 + a_1x + a_2x^2 + \ldots + a_kx^k + \ldots, \tag{18}$$

And from the difference $f(x)Q_M(x) - P_N(x) = Z(x)$:

$$\left[\sum_{i=0}^{\infty} a_i x^i\right]\left[\sum_{i=0}^{M} q_i x^i\right]-\left[\sum_{i=0}^{N} p_i x^i\right]=\left[\sum_{i=N+M+1}^{\infty} c_i x^i\right], \tag{19}$$

The lower index $j = N + M + 1$ in the summation on the right side of (19) is chosen because the first $N + M$ derivatives of $f(x)$ and $R_{N,M}(x)$ are to agree at $x = 0$.

When the left side of (19) is multiplied out and the coefficients of the powers of x^i are set equal to zero for $k = 0,1,2,\ldots,N + M$, the result is a system of $N + M + 1$ linear equations:

$$\begin{aligned}
&a_0 - p_0 = 0\\
&q_1a_0 + a_1 - p_1 = 0\\
&q_2a_0 + q_1a_1 + a_2 - p_2 = 0\\
&q_3a_0 + q_2a_1 + q_1a_2 + a_3 - p_3 = 0\\
&q_Ma_{N-M} + q_{M-1}a_{N-M+1} + a_N - p_N = 0
\end{aligned} \tag{20}$$

and

$$
\begin{aligned}
&q_M a_{N-M+1} + q_{M-1} a_{N-M+2} + \dots + q_1 a_N \quad + a_{N+2} = 0\\
&q_M a_{N-M+2} + q_{M-1} a_{N-M+3} + \dots + q_1 a_{N+1} \quad + a_{N+2} = 0\\
&. \qquad\qquad\qquad\qquad .\\
&. \qquad\qquad\qquad\qquad .\\
&. \qquad\qquad\qquad\qquad .\\
&q_M a_N + q_{M-1} a_{N+1} + \dots + q_1 a_{N+M+1} \quad + a_{N+M} = 0
\end{aligned}
\tag{21}
$$

Notice that in each equation the sum of the subscripts on the factors of each product is the same and this sum increases consecutively from 0 to $N + M$. The M equations in (21) involve only the unknowns $q_1,q_2,q_3,...,q_M$ and must be solved first. Then the equations in (20) are used successively to find $p_1,p_2,p_3,...,p_N$.

Aplications: In this section, we will apply the differential transformed method to nonlinear ordinary differential equations (3).

Differential Transformed to the Model for a Horizontal Cross Section of the Center Span: Now, the application of the differential transform to Eq. (3) give the following recurrence relations for $k \geq 0$:

$$
\begin{gathered}
(k+1)(k+2)Y(k+2) + \delta_v (k+1) Y(k+1) + \frac{2k}{m} Y(k) = \lambda_v \frac{w^k}{k!} \sin\left(\frac{\pi k}{2}\right)\\
\mathrm{Y}(0) = y_0,\, Y(1) = y_1
\end{gathered}
\tag{22}
$$

and

$$
\begin{gathered}
(k+1)(k+2)\Theta(k+2) + \delta_t (k+1)\Theta(k+1) + \frac{3k}{m} F(k) = 0\\
\Theta(0) = \theta_0,\, \Theta(1) = \theta_1
\end{gathered}
\tag{23}
$$

Here, from (11) and (12)

$$
F(k) = \begin{cases} \sin\left(2\Theta(0)\right), & k = 0,\\ 2\displaystyle\sum_{r=0}^{k-1}\left(\frac{k-r}{k}\right) G(r)\Theta(k-r), & k \geq 1. \end{cases}
\tag{24}
$$

and

$$
G(k) = \begin{cases} \cos\left(2\Theta(0)\right), & k = 0,\\ -2\displaystyle\sum_{r=0}^{k-1}\left(\frac{k-r}{k}\right) F(r)\Theta(k-r), & k \geq 1. \end{cases}
\tag{25}
$$

Where $Y(k)$ and $\Theta(k)$ are the differential transforms of $y(t)$ and $\theta(t)$, respectively and the transform of the initial conditions are $Y(0) = 0.3$, $Y(1) = 0.1$, $\Theta(0) = 0.3$ and $\Theta(1) = 0.1$. The length and width of the center span were about 1000 and 12 meters, respectively, thus we choose $L_B = 1000$ and $L_E = 6$. From [25], we choose $m = 2500, k = 1000, \lambda_v = 0.4, w = 1.8, \delta_t = 0.1$ and δ_v Using these recurrence relations by taking $N = 8$, we obtain a system of algebraic equations for $k = 0,...,8$. By solving this equations fort pense he values of $Y(2),Y(3),...,Y(8)$ and $\Theta(2)$, $\Theta(3),...,\Theta(8)$, by using Maple, we get

$$
\begin{aligned}
y(t) &= 0.3 + t - .125t^2 + .1108333333t^3 + .0055625t^4 - .02398458332t^5 + .000251409722t^6\\
&\quad + .001952914781t^7 - .00002800300224t^8\\
\theta(t) &= .3 + 0.1t - .3437854841t^2 - .02155390847t^3 + .05841581345t^4 - .00362636569t^5\\
&\quad + .0008919540394t^6 + .002122609583t^7 - .001904002809t^8.
\end{aligned}
\tag{26}
$$

We apply Laplace transformation to (26), which yields

$$L\left(y(s)\right)=\frac{0.3}{s}+\frac{0.1}{s^2}-\frac{0.25}{s^3}+\frac{.6649999998}{s^4}+\frac{.1335}{s^5}-\frac{2.878149998}{s^6}+\frac{.1810149998}{s^7}+\frac{9.842690496}{s^8}-\frac{1.12908105}{s^9},$$
$$L\left(\theta(s)\right)=\frac{0.3}{s}+\frac{0.1}{s^2}-\frac{.6875709682}{s^3}-\frac{.1293234508}{s^4}+\frac{1.401979523}{s^5}-\frac{.4351638828}{s^6}+\frac{.6422069084}{s^7}+\frac{10.69795230}{s^8}-\frac{76.76939326}{s^9}. \quad (27)$$

For simplicity, let $s=\frac{1}{t}$; then

$$L\left(y(t)\right)=0.3t+.1t^2-.25t^3+.6649999998t^4+.1335t^5-2.878149998t^6+.1810149998t^7+9.842690496t^8-1.12908105t^9,$$
$$L\left(\theta(t)\right)=0.3t+0.1t^2-.6875709682t^3-.1293234508t^4+1.401979523t^5-.4351638828t^6+.6422069084t^7+10.69795230t^8-76.76939326t^9. \quad (28)$$

The [4/4] Padé approximant gives

$$\left[\frac{4}{4}\right]_{y(t)}=\frac{0.3t+.1300000008t^2+.9720000004t^3+1.1412t^4}{1+.1000000025t+4.040000001t^2+.3240000028t^3+2.591999994t^4},$$
$$\left[\frac{4}{4}\right]_{\theta(t)}=\frac{0.3t+.2654018793t^2+5.811434082t^3+2.488207993t^4}{1+.5513395976t+21.47957032t^2+2.828865045t^3+43.8505469t^4}. \quad (29)$$

Recalling $s=\frac{1}{t}$, we obtain [4/4] in terms of s

$$\left[\frac{4}{4}\right]_{y(s)}=\frac{3000000001s^3+1300000008.s^2+9720000004s+.1141200000\text{e}11}{.1000000000\text{e}11s^4+1000000025s^3+.4040000001\text{e}11s^2+3240000028s+.2591999994\text{e}11},$$
$$\left[\frac{4}{4}\right]_{\theta(s)}=\frac{3000000000s^3+2654018793s^2+.5811434082\text{e}11s+.2488207993\text{e}11}{9999999999s^4+5513395976s^3+.2147957032\text{e}12s^2+.2828865045\text{e}11s+.4385054690\text{e}12} \quad (30)$$

By using the inverse Laplace transform to the [4/4] Padé approximant, we obtain the modified solution

$$y(t)=e^{-.05000000009t}\left[.3120280647\cos(.8930285535t)+.4580886004\sin(.8930285535t)\right]+e^{-.1157367189\text{e-}8t}\left[-.01202806464\cos(1.8t)-.1630471093\sin(1.8t)\right]$$
$$\theta(t)=e^{-.2292139221t}\left[-.4706939516\text{e-}3\cos(4.369691348t)-.5610019295\text{e-}3\sin(4.369691348t)\right]+e^{-.04645587676t}\left[.300470694\cos(1.512639198t)+.07688690428\sin(1.512639198t)\right].. \quad (31)$$

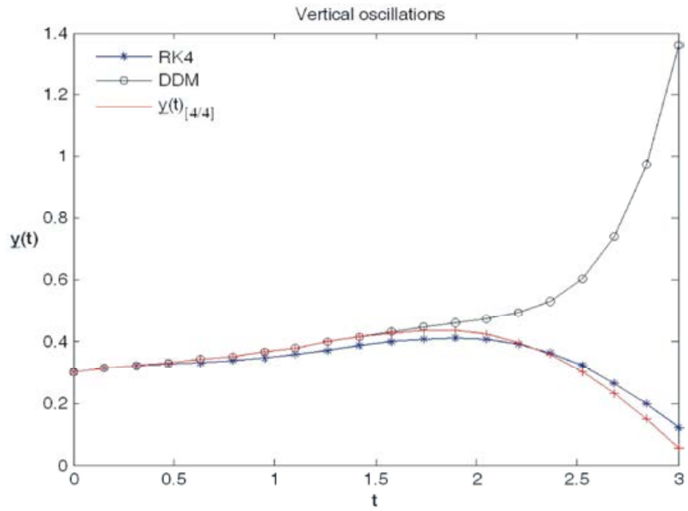

Fig. 2: The comparison of the results of the three methods for Eq.(3), at *N=8*.

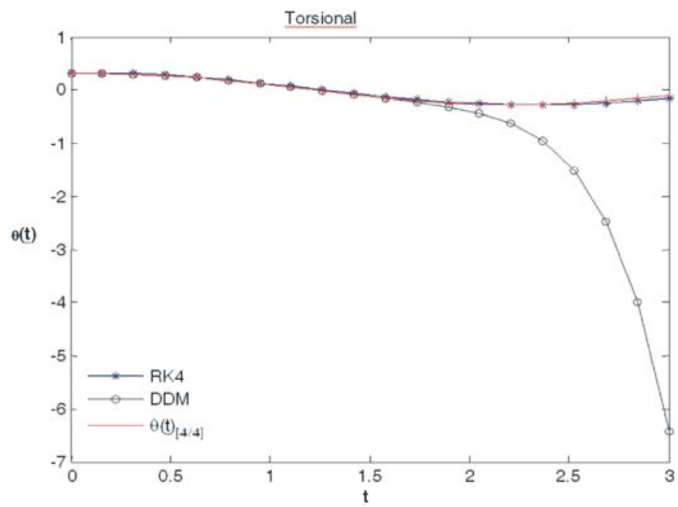

Fig. 3: The comparison of the results of the three methods for Eq.(3), at *N=8*.

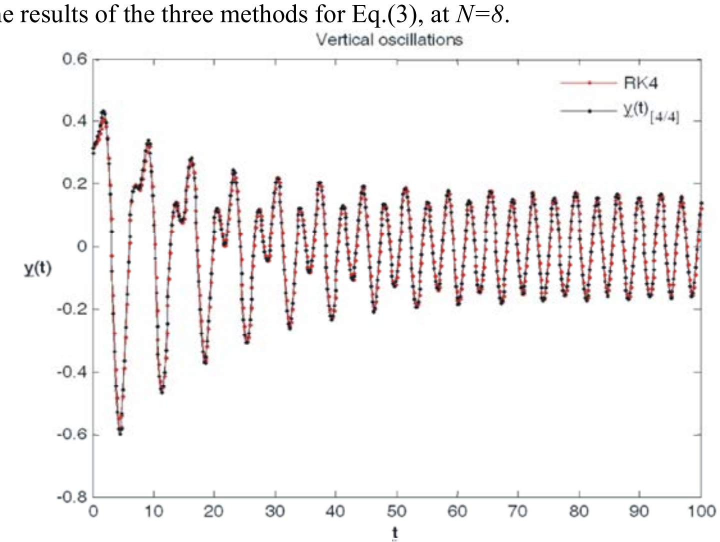

Fig. 4: The comparison of the results of the two methods for Eq.(3), at *N=8*.

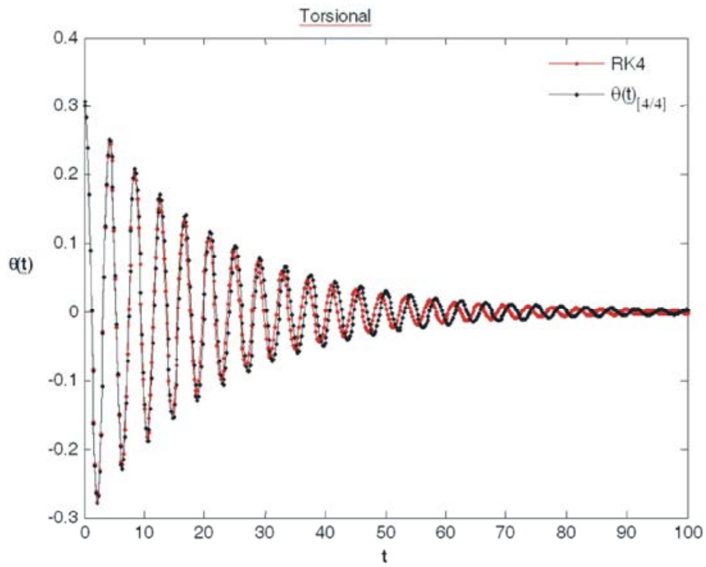

Fig. 5: The comparison of the results of the two methods for Eq.(3), at $N=8$.

Table 2. Displays the values of $y(t)$ for $N = 8$ for $t = 0.0$ to 10.0

	$y(t_i)$		
t_i	*RK*4	$y_{[2/2]}$	$y_{[4/4]}$
0	0.30000000000000	0.30000000000000	0.30000000006000
1	0.28656455615509	0.33888868518004	0.36950237829753
2	0.25159958156418	0.34728811929504	0.42756008749797
3	0.14511885254060	0.35365384009666	0.05393941846376
4	-0.21926272599785	0.35998031168674	-0.52540337973351
5	-0.41338749093111	0.36640904244183	-0.46161875746108
6	-0.16713598402269	0.37295181681137	0.03036826544790
7	0.06790068192464	0.37961136855656	0.19194314387689
8	0.10414193891887	0.38638983172595	0.21489144141070
9	0.22633618661972	0.39328933305994	0.33856623555846
10	0.27229644819814	0.40031203409445	0.07978118808778

Table 3. Displays the values of $\theta(t)$ for $N = 8$ for $t = 0.0$ to 10.0

	$\theta(t_i)$		
t_i	*RK*4	$\theta_{[2/2]}$	$\theta_{[4/4]}$
0	0.30000000000000	0.30000000000000	0.30000000000000
1	0.13167964927717	0.09572417325330	0.09049046313068
2	-0.20152265610648	-0.24801561903124	-0.26382392138971
3	-0.19029444249469	-0.11747800778000	-0.11159009453780
4	0.09794592177270	0.19976187847517	0.22825893099310
5	0.19820967062728	0.12977603604960	0.12680454008444
6	-0.00944573529767	-0.15605767367260	-0.19396300392096
7	-0.17089449189593	-0.13446768854013	-0.13692707756505
8	-0.05409082499452	0.11736140882256	0.16149558962120
9	0.12456245605187	0.13322624307296	0.14263760233941
10	0.09034747096136	-0.08384187688566	-0.13126517283032

The following comparison of the results obtained from three methods are given.

Figs. 2 and 3 show the results from the DTM and the modified DTM solutions $y(t)$ and $\theta(t)$ compared with the fourth-order Runge-Kutta (RK4) method solution $y(t)$ and $\theta(t)$, respectively, for $N=8$ and $0 \le t \le 3$.

Comparison of the modified approximate solution (31) and the solutions obtained by the fourth-order Runge-Kutta method in Fig. 4 and Fig. 5 show that the modified DTM greatly improves the differential transform truncated series (26) in the convergence rate and the accuracy. Amplitude of torsional oscillation is nearly reduced to zero. Amplitude of vertical oscillation reduced after a certain time the balance sits.

CONCLUSIONS

In this article, the application of differential transform method was extended to obtain approximate analytical and numerical solutions of linear and nonlinear oscillations. The differential transform method produces the Taylor series of the exact solution. For the oscillatory systems, Laplace transformation of the differential transform series solution has some specific properties, so we applied Laplace transformation and Pade´ approximant to obtain an analytic solution and to develop the accuracy of differential transform method. The modified DTM is an efficient method for calculating periodic solutions for non-linear oscillatory systems. It is seen from the results of the modified DTM and the results of the fourth-order Runge-Kutta(RK4) solution that rate of convergence and accuracy of the modified DTM is very good.

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