

New Travelling Wave Solutions to the Perturbed Nonlinear Schrodinger's Equation

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Abstract: In this present work we applied new applications of direct algebraic method to Drinfel'd-Sokolov-Wilson system and new application of (G'/G) -expansion method to the perturbed nonlinear Schrodinger's equation. Then new types of complex solutions are obtained to the Drinfel'd-Sokolov-Wilson system. In (G'/G) -expansion method the balance number of it is not positive integer. Then new types of exact travelling wave solutions are obtained to this equation.

Key words: Direct algebraic method • (G'/G) -expansion method • Perturbed nonlinear Schrodinger • S equation • Drinfel'd-Sokolov-Wilson system

INTRODUCTION

The (G'/G) -expansion method was developed by Wang *et al.* [1]. The method is now used by many researchers in a variety of scientific fields. The method has been proved by many authors [2, 3, 4]. Recently, many powerful methods have been established and improved. Among these methods, we cite the, the hyperbolic tangent expansion method [7, 8], the trial function method [9], the homogeneous balance method [5, 6], the tanh-method [10-14], the inverse scattering transform [15], the Backlund transform [16, 17], the Hirota's bilinear method [18, 19], the motivation of the present paper is to explore the possibilities of solving such equations, the balance numbers of which are not positive integers, using the (G'/G) -expansion method. The (G'/G) -expansion method is based on the assumptions that the travelling wave solutions can be expressed by a polynomial in (G'/G) and that $G=G(\xi)$ satisfies a second order linear ordinary differential equation (ODE).

Description of the $(\frac{G'}{G})$ -expansion Method: Considering the nonlinear partial differential equation in the form

$$P(u, u_x, u_t, u_{xx}, u_{xt}, u_{xxx}, \dots) = 0$$

Where $u = u(x, t)$ an unknown function is P is a polynomial in $u = u(x, t)$ and its various partial derivatives, in which the highest order derivatives and nonlinear terms

are involved. In the following we give the main steps of the $(\frac{G'}{G})$ -expansion method.

Step 1: Combining the independent variables x and t into one variable $\xi = x - vt$, we suppose that

$$u(x, t) = u(\xi), \quad \xi = x - vt \quad (2)$$

The travelling wave variable (2) permits us to reduce Eq. (1) to an ODE for $G = G(\xi)$, namely

$$P(u, -vu', u', v^2 u'', -vu'', u'', \dots) = 0 \quad (3)$$

Setp 2: Suppose that the solution of ODE (3) can be expressed by a polynomial in $(\frac{G'}{G})$ as follows

$$u(\xi) = \alpha_m \left(\frac{G'}{G}\right) + \dots, \quad (4)$$

Where $G = G(\xi)$ satisfies the second order LODE in the form

$$G'' + \lambda G' + \mu G = 0 \quad (5)$$

α_m, \dots, λ and μ are constants to be determined later $\alpha_m \neq 0$, the unwritten part in is also a polynomial in $(\frac{G'}{G})$, but the

degree of which is generally equal to or less than $m - 1$, the positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in ODE (3).

Step 3: By substituting (4) into Eq. (3) and using the second order linear ODE (5), collecting all terms with the same order $(\frac{G'}{G})$ together, the left-hand side of Eq. (3) is converted into another polynomial in $(\frac{G'}{G})$. Equating each coefficient of this polynomial to zero yields a set of algebraic equations for α_m, \dots, λ and μ .

Step 4: Assuming that the constants α_m, \dots, λ and μ can be obtained by solving the algebraic equations in Step 3, since the general solutions of the second order LODE (5) have been well known for us, then substituting α_m, \dots, λ and the general solutions of Eq. (5) into (4) we have more travelling wave solutions of the nonlinear evolution equation (1).

Application to the Perturbed Nonlinear Schrodinger's Equation: The perturbed nonlinear Schrodinger's equation reads

$$iu_t + u_{xx} + \alpha |u|^2 u + i \left[\gamma_1 u_{xxx} + \gamma_2 |u|^2 u_x + \gamma_3 (|u|^2)_x u \right] = 0$$

We may choose the following travelling wave transformation:

$$u = \phi(\xi) e^{i(sx - \Omega t)}, \quad \xi = k(x - ct)$$

Where s, Ω, c are constants to be determined later. Using the traveling wave solutions (7) we have the nonlinear ordinary differential equation.

$$i\gamma_1 k^3 \phi''' + (2ski - 3i\gamma_1 s^2 k - cik)\phi' + (k^2 + 3\gamma_1 s k^2)\phi'' + (ik\gamma_2 + 2ik\gamma_3)\phi^2 \phi' + (\alpha - \gamma_2 s)\phi^3 + (\Omega - s + \gamma_1 s^3)\phi = 0$$

Suppose that the solution of ODE (8) can be expressed by a polynomial in (G'/G) as follows:

$$u(\xi) = \alpha_m \left(\frac{G'}{G}\right) + \dots,$$

Where $G = G(\xi)$ satisfies (5). Considering the homogeneous balance between ϕ''' and ϕ^3 in Eq. (7), we required that $3m = m + 3$. It should be noticed that m is not a positive integer. However, we may still choose the solution of Eq. (8) in the form

$$\phi = AF^{\frac{3}{2}}, \tag{28}$$

$$\begin{aligned} \phi' &= \frac{3}{2} A \left[-\lambda \left(\frac{G'}{G}\right)^{\frac{1}{2}} - \mu \left(\frac{G'}{G}\right)^{\frac{3}{2}} - \left(\frac{G'}{G}\right)^{\frac{5}{2}} \right], \\ \phi'' &= \frac{3}{2} A \left[\left(\frac{3}{2} \lambda^2 + 2\mu\right) \left(\frac{G'}{G}\right)^{\frac{3}{2}} + 2\lambda\mu \left(\frac{G'}{G}\right)^{\frac{1}{2}} + 4\lambda \left(\frac{G'}{G}\right)^{\frac{5}{2}} + \frac{1}{2} \mu^2 \left(\frac{G'}{G}\right)^{-\frac{1}{2}} \right], \\ \phi''' &= \frac{3}{2} A \left[\left(-\frac{9}{2} \lambda^3 - \frac{11}{2} \lambda\mu\right) \left(\frac{G'}{G}\right)^{\frac{3}{2}} + \left(-\frac{13}{4} \lambda\mu - \frac{9}{4} \mu^2\right) \left(\frac{G'}{G}\right)^{\frac{1}{2}} + \left(-\frac{9}{4} \lambda^2 - \frac{53}{4} \mu\right) \left(\frac{G'}{G}\right)^{\frac{5}{2}} - \right. \\ &\quad \left. \frac{3}{4} \lambda \mu^2 \left(\frac{G'}{G}\right)^{-\frac{1}{2}} + \frac{1}{4} \mu^3 \left(\frac{G'}{G}\right)^{-\frac{3}{2}} - \frac{35}{4} \lambda \left(\frac{G'}{G}\right)^{\frac{7}{2}} - \frac{35}{4} \left(\frac{G'}{G}\right)^{\frac{9}{2}} \right] \\ \phi^2 \phi' &= \frac{3}{2} A^3 \left[-\lambda \left(\frac{G'}{G}\right)^{\frac{9}{2}} - \mu \left(\frac{G'}{G}\right)^{\frac{7}{2}} - \left(\frac{G'}{G}\right)^{\frac{11}{2}} \right], \end{aligned}$$

On substituting (9)-(12) into (8), collecting all terms with the same powers of (G'/G) and setting each coefficient to zero, we obtain the following system of algebraic equations:

$$\begin{aligned} -\frac{9}{8}Ai\gamma_1k^3\lambda\mu^2 + \frac{3}{4}A\mu^2 &= 0, \quad A^3(\alpha - \gamma_2s) - \frac{105}{8}Ai\gamma_1k^3 - \frac{3}{2}A^3\lambda = 0, \\ -\frac{3}{2}A^3\mu(ik\gamma_2 + 2ik\gamma_3) - \frac{105}{8}Ai\gamma_1k^3\lambda &= 0, \\ -\frac{3}{2}A(2ski - 3i\gamma_1s^2k - cik) + 6\lambda A(k^2 + 3\gamma_1sk^2) + \frac{3}{2}Ai\gamma_1k^3(-\frac{9}{4}\lambda^2 - \frac{53}{4}\mu) &= 0, \quad \frac{3}{8}A\mu^3 = 0 \end{aligned} \quad (30)$$

On solving the above algebraic Eq. (14) by using the Maple, we get

$$\begin{aligned} c &= -3\gamma_1s^2 + 2s + 6ik\lambda + 12isk\lambda\gamma_1 + \frac{2}{3}\gamma_1^2k^2 - \frac{9}{4}\lambda^2i, \\ \lambda &= -\frac{2i}{\gamma_1k^3}, \quad \mu = 0 \\ A &= \pm \frac{k}{2} \sqrt{\frac{-105i\gamma_1}{2\alpha - 2\gamma_2s - 3\lambda}}, \end{aligned}$$

From (5), (7), (9) and (15), we obtain the exact travelling wave solution of (6) as follows:

$$A = \pm \frac{k}{2} \sqrt{\frac{-105i\gamma_1}{2\alpha - 2\gamma_2s - 3\lambda}} \lambda \left(\frac{c_2 e^{-\lambda\xi}}{c_1 + c_2 e^{-\lambda\xi}} \right)^{\frac{3}{2}} e^{i(sx - \Omega t)}$$

Hence

$$A = \pm \frac{k}{2} \sqrt{\frac{-105i\gamma_1}{2\alpha - 2\gamma_2s - 3\lambda}} \lambda \left(\frac{c_2 e^{\frac{2i}{\gamma_1k^3}k(x - (-3\gamma_1s^2 + 2s + 6ik\lambda + 12isk\lambda\gamma_1 + \frac{2}{3}\gamma_1^2k^2 - \frac{9}{4}\lambda^2i)t)}}{c_1 + c_2 e^{\frac{2i}{\gamma_1k^3}k(x - (-3\gamma_1s^2 + 2s + 6ik\lambda + 12isk\lambda\gamma_1 + \frac{2}{3}\gamma_1^2k^2 - \frac{9}{4}\lambda^2i)t)}}} \right)^{\frac{3}{2}} e^{i(sx - \Omega t)}$$

Where c_1, c_2 and k and s, Ω are arbitrary constants. Eq. (16) is a new type of exact travelling wave solution to the perturbed nonlinear Schrodinger's equation. Especially, if we choose $c_1 = -c_2$ in (16), we obtain the envelope solitary wave solutions of Eq. (6),

$$A = \mp \frac{i}{\gamma_1k^2} \sqrt{\frac{105i\gamma_1}{2\alpha - 2\gamma_2s + \frac{6i}{\gamma_1k^3}}} \left(\frac{1}{2} - \frac{1}{2} \tanh\left(\frac{\frac{-2i}{\gamma_1k^2}(x - (-3\gamma_1s^2 + 2s + \frac{12}{\gamma_1k^2} + \frac{24s}{k^2} + \frac{2}{3}\gamma_1^2k^2 + \frac{9}{\gamma_1k^6}i)t)}}{2}\right) \right)^{\frac{3}{2}} e^{i(sx - \Omega t)}$$

CONCLUSION

We have noted that the (G'/G) expansion method changes the given difficult problems into simple problems which can be solved easily. This paper presents a wider applicability for handling nonlinear evolution equations using the (G'/G) -expansion method. The new type of exact travelling wave solution obtained in this paper might have significant impact on future researches.

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