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An Explicit Analytical Solution of a Slider Bearing with a Third Grade Non-Newtonian Fluid as Lubricant

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Abstract: This paper presents an analytical solution of an inclined slider bearing consisting of connected surfaces lubricated with a third grade (non-Newtonian) fluid. Dimension analysis and by order of magnitude comparison with the full Navier-Stokes equations give rise to a nonlinear lubrication equation in the film region. The homotopy analysis method (HAM) for strongly nonlinear problems is used to give explicit analytic solution of the problem. Both the velocity profile and pressure distribution are calculated approximately using this method and plotted graphically using different parameter values. The load carrying capacity of the bearing is discussed for a range of bearing parameters. The boundary layer effect developed near the lower wall is also presented.

Key words: Homotopy analysis method · Slider bearing · Third grade (non-Newtonian) fluid · Lubrication theory

INTRODUCTION

Lubrication flows are most applicable to processing of materials in liquid form, such as polymers, metals, composites and others. It plays an extremely important role in many current scientific and engineering applications. Since, the presence of fluid film greatly reduces the sliding friction between the solid objects. The enormous practical importance of this effect has stimulated a great deal of research both theoretically and experimentally. However, the problem of a slider bearing with non-Newtonian lubricants is difficult to analyze mathematically because of the nonlinear character of the governing equations of motion. Although, several numerical methods are available, but are somewhat more costly as the underlying mechanism in lubrication flows is more complex. In the recent years, there has been great advancement in iterative techniques for solving nonlinear problems. Some of them are Homotopy Perturbation Method (HPM) [1], Variational Iteration Method (VIM) [2] and Homotopy Analysis Method (HAM) [3, 4]. In this paper, we proposed a new solution method to the lubrication problem with non Newtonian third grade fluid as lubricant. We have used homotopy analysis method (HAM), which was originally developed by S. Liao [5, 6] for solving nonlinear problems. HAM is simple, powerful and efficient analytical method that remain valid even if the strong nonlinearity exist. The method was successfully applied by many authors including [7, 8] in solving different nonlinear problems of fluid mechanics. Recent studies of non-Newtonian fluids have attracted many researchers due to its practical importance and rapid development of modern industrial materials. Many solid such as polymer solution, molten plastics, ceramics, mammalian, and synovial are the common examples of non-Newtonian fluids. The non-Newtonian fluids are different from Newtonian fluids in terms of the existing complexities of the relationship between shear stress and the flow field. For this reason, several models have been proposed to predict the behaviors of various types of non-Newtonian fluids. Third grade fluid is one of the non-Newtonian models, derived with third order truncation, and which describe a special subclass of fluids. The study of such fluids is of wide interest and significance in lubrication theory and applied sciences [9-12].

The problem of Non-Newtonian lubrication in bearing has been studied for many years. For example, Harnoy and Hanin [13] have studied elastic-viscous lubricants in dynamically loaded bearing. Bourgin [14] applied the constitutive relation of second order fluid to study of non-Newtonian lubrication using the perturbation approach. Among others Rajagopal [15] carried out a study of the creeping flow in a bearing. Kacou, Rajagopal and Szeri [16] studied the flow of second and third grade

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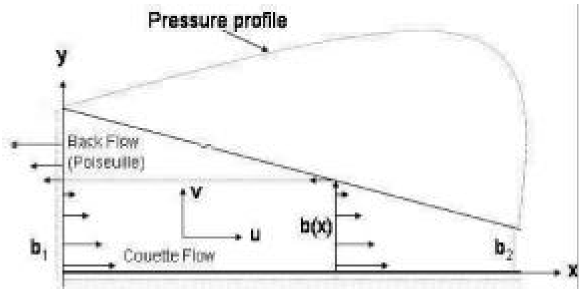

Fig. 1: Geometrical configuration of a slider bearing

fluid model in a journal bearing. Tichy [17] studied the non-Newtonian lubrication using convected Maxwell model. Yürüsoy [18] has studied the pressure distribution in a slider bearing with Powell-Eyring model and constructed a perturbation solution while the same results are recovered by Siddiqui [19] using homotopy analysis method. Buckholz [20] used a power law model as a non-Newtonian lubricant in a slider bearing. Agrawal [21] studied the magnetic fluid based porous inclined slider bearing. Rajesh [22] analyzed exponential form a slider bearing using a Ferro-fluid as lubricant.

The remaining of the paper is organized as follows. In the next section, the governing equations of the problem accompanied with appropriate boundary conditions are given. Section 3 describes basic idea of homotopy analysis method for finding velocity and pressure distribution in a slider bearing. In section 4 the rise in pressure and appearance of boundary layer effect for different parameter values are presented and discussed. Section 5 concludes this paper.

Geometry and Governing Equation: Consider the two dimensional inclined slider bearing as shown in figure 1, in which the plane $y = 0$ moves with constant velocity in the x-direction while the top of the bearing (the slider) is fixed. Through the action of viscous shear forces, the moving wall sweeps fluid into a narrowing passage b_2. This gives rise to a local velocity profile of Couette-type. Since $b(x)$ is decreasing, the flow then sets up a pressure gradient, in order to supply a Poiseuille-type flow component that redistributes the fluid and maintains a constant flow rate. It is assumed that the fluid inertia is small, the side leakage is negligible, and the flow is incompressible and laminar. The non-dimensional basic lubrication equations for third grade fluid flow in the film region are [23, 24].

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.1}$$

$$0=-\frac{\partial p}{\partial x}+\frac{\partial^2 u}{\partial y^2}+6\beta\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2} \tag{2.2}$$

$$0=-\frac{\partial p}{\partial y} \tag{2.3}$$

Which is obtained by dimensionless analysis, and by order of magnitude comparisons with the full Navier-Stokes equation. It is followed from eq. (2.3) that.

$$p = p\,(x) \tag{2.4}$$

Which shows that the pressure does not vary across the film thickness. Also in eq. (2.3) β is the material constant. Thus, the governing equation that describe the flow simplifies as.

$$0=-\frac{\partial p}{\partial x}+\frac{\partial^2 u}{\partial y^2}+6\beta\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2} \tag{2.5}$$

A lubrication layer will generate a positive pressure, and hence, load capacity normal to the layer only when the layer is arranged so that the relative motion of the two surfaces tends to drag fluid by viscous stresses from the wider to the narrower end of the layer. The appropriate dimensionless boundary conditions of the problem are given by

$$u(0)=1,\, u(b)=0,\, v(0)=0,\, v(b)=0. \tag{2.6}$$

It is to be noted that eq. (2.1) serves only to determine the vanishing small velocity component v, given the dominant component u in eq. (2.2).

The aim of the present study is to find an analytical solution of the nonlinear eq. (2.5) subjected the boundary condition given by eq. (2.6).

Homotopy Analysis Method

Basic Idea: To explain the basic idea of homotopy analysis method, let us consider the nonlinear differential equation of the form.

$$\aleph[u(y)]=0 \tag{3.1}$$

Where $\aleph$ is a nonlinear operator and $u(y)$ is an unknown function of the independent variable. Let $u_0(y)$ denote an initial approximation of $u(y)$ and L denotes an auxiliary linear operator with the property

$$Lu = 0 \quad \text{when} \quad u = 0. \tag{3.2}$$

We then construct a family of equations, the so-called homotopy

$$\hat{H}\left[\phi(y;q);q\right] = (1-q)L\left[\phi(y;q) - u_0(y)\right] + q\aleph\left[\phi(y;q)\right] \tag{3.3}$$

Where $q \in [0, 1]$ is an embedding parameter and $\phi(y; q)$ is a function of y and q. When $q = 0$, we have

$$\hat{H}\left[\phi(y;q);q\right]_{q=0} = L\left[\phi(y;0) - u_0(y)\right]$$

and for $q = 1$

$$\hat{H}\left[\phi(y;q);q\right]_{q=1} = \aleph\left[\phi(y;1)\right] \tag{3.4}$$

From eq. (3.3), it follows that

$$\phi(y;\, 0) = u_0(y) \tag{3.5}$$

is the solution of the equation

$$\hat{H}\left[\phi(y;q);q\right]_{q=0} = 0 \tag{3.6}$$

and

$$\phi(y;\, 1) = u(y) \tag{3.7}$$

is therefore the solution of the equation

$$\hat{H}\left[\phi(y;q);q\right]_{q=1} = 0. \tag{3.8}$$

Thus when the embedding parameter q increases from 0 to 1, the solution $\phi(y; q)$ of the equation

$$\hat{H}\left[\phi(y;q);q\right] = 0 \tag{3.9}$$

depends upon the embedding parameter q and varies from initial approximation $u_0(y)$ to the solution $u(y)$ of eq. (3.1). This kind of continuous variation is called deformation [1].

Velocity Profile: The velocity profile of the flow can be found by solving eq. (2.2). For this purpose, we define the nonlinear operator $\aleph\left[\tilde{u}(y,q)\right]$ as,

$$\aleph\left[\tilde{u}(y,q)\right] = \frac{\partial^2 \tilde{u}(y,q)}{\partial y^2} + 6\beta\left(\frac{\partial \tilde{u}(y,q)}{\partial y}\right)^2 \frac{\partial^2 \tilde{u}(y,q)}{\partial y^2} - \frac{dp}{dx} \tag{3.10}$$

Further more, we construct the zero-order deformation equation as

$$(1-q)L\left[\tilde{u}(y,q) - u_0(y)\right] = -q\aleph\left[\tilde{u}(y,q)\right] \tag{3.11}$$

subject to the boundary conditions

$$\left.\begin{aligned} \tilde{u}(y,q) &= 1, \quad at \quad y = 0 \\ \tilde{u}(y,q) &= 0, \quad at \quad y = b \end{aligned}\right\} \tag{3.12}$$

Where $u_0(y)$ is an initial guess approximation and q is an embedding parameter such that $q \in [0, 1]$. We choose the auxiliary linear operator L, (which is the linear part of eq. (2.5)).

$$L = \frac{d^2}{dy^2} \tag{3.13}$$

and an initial guess approximation

$$u_0(y) = \frac{dp}{dx}\left(\frac{y^2}{2} - \frac{yb}{2}\right) + \left(1 - \frac{y}{b}\right) \tag{3.14}$$

Which can be obtained by solving eq. (2.5) with $\beta = 0$ subject to the boundary conditions given in eq. (2.6). Obviously, when $q = 0$ and $q = 1$, we have

$$\tilde{u}(y,0) = u_0(y), \;\; y > 0 \tag{3.15}$$

and

$$\tilde{u}(y,1) = u(y) \tag{3.16}$$

respectively. Therefore, according to eq. (3.15) and eq. (3.16) the variation of q from 0 to 1 is just the continuous variation $u(y,q)$ from the initial guess approximation $u_0(y)$ to the unknown solution $u(y)$ of the original eq. (2.5). Furthermore, Assume that the deformation $u(y,q)$ governed by eqs. (3.10 – 3.16) is smooth enough so that

$$u_0^{(k)}(y) = \left.\frac{\partial^k \tilde{u}(y,q)}{\partial q^k}\right|_{q=0} \qquad k \geq 1 \tag{3.17}$$

namely the k-th order deformation derivative exists. Then, in view of eq. (3.16) and Taylor's formula, we expand $u(y, q)$ in the power series

$$\tilde{u}(y,q) = u_0(y) + \sum_{k=1}^{\infty}\left[\frac{u_0^{(k)}(y)}{k!}\right]q^k \tag{3.18}$$

We note that the convergence region of the above infinite series is dependent upon $h(\neq 0)$. We define

$$u_k(y) = \frac{u_0^{(k)}(y)}{k!}, \; k \geq 1 \tag{3.19}$$

Using eqs. (3.16), (3.18) and (3.19), we get at $q = 1$, the important relationship of the form

$$u(y) = \sum_{k=0}^{\infty} u_k(y) \tag{3.20}$$

between the initial guess approximation $u_0(y)$ and the unknown solution $u(y)$. Now differentiating the zero-order deformation eq. (3.10) and eq. (3.11) k-times with respect to q and then setting $q = 0$, we obtained for $k \geq 1$, the k-th order deformation equation

$$L[u_k(y) - \chi_k u_{k-1}(y) = -R_k(y) \tag{3.21}$$

with the following boundary conditions

$$u_k(0) = u_k(b) = 0 \tag{3.22}$$

in which

$$R_k(y) = \frac{1}{(k-1)!} \frac{\partial^{k-1} \aleph[\tilde{u}(y,q)]}{\partial q^{k-1}} \tag{3.23}$$

and

$$\chi_k = \begin{cases} 0, & k \leq 1 \\ 1, & k \geq 2 \end{cases} \tag{3.24}$$

By putting $k = 1$ in eqs. (3.21-3.23), we obtained the first order solution. In particular, differentiating eq. (3.10) with respect to q, we obtain

$$\begin{aligned} &(1-q)L\left[\frac{\partial \tilde{u}(y,q)}{\partial q} - 0\right] - [\tilde{u}(y,q) - u_o(y)] = \\ &-\left[\frac{\partial^2 \tilde{u}(y,q)}{\partial y^2} + 6\beta\left(\frac{\partial \tilde{u}(y,q)}{\partial y}\right)^2 \frac{\partial^2 \tilde{u}(y,q)}{\partial y^2} - \frac{dp}{dx}\right] \\ &-q\left[\frac{\partial^3 \tilde{u}(y,q)}{\partial y^2 \partial q} + 6\beta\left\{2\frac{\partial \tilde{u}(y,q)}{\partial y}\frac{\partial^2 \tilde{u}(y,q)}{\partial y \partial q}\frac{\partial^2 \tilde{u}(y,q)}{\partial y^2} + \left(\frac{\partial \tilde{u}(y,q)}{\partial y}\right)^2 \frac{\partial^3 \tilde{u}(y,q)}{\partial y^2 \partial q}\right\}\right] \end{aligned} \tag{3.25}$$

making use of eq. (3.17) and setting $q = 0$, we have

$$L\left\{u_0^{(1)}\right\} = -\left[\frac{\partial^2 u_0}{\partial y^2}\right] + 6\beta\left(\frac{\partial u_0}{\partial y}\right)^2 \frac{\partial^2 u_0}{\partial y^2} - \frac{dp}{dx} \tag{3.26}$$

making use of eq. (3.13), we have

$$\frac{d^2 u_0^{(1)}}{dy^2} = -6\beta\left\{\left(\frac{dp}{dx}\right)^3\left(y^2 + \frac{b^2}{4} - by\right) + \frac{1}{b^2}\left(\frac{dp}{dx}\right) - \left(\frac{2y}{b} - 1\right)\left(\frac{dp}{dx}\right)^2\right\} \tag{3.27}$$

Now integrating eq. (3.27) twice with respect to y, and using the boundary conditions eq. (3.22), we have

$$u_0^{(1)} = -6\beta\left\{\left(\frac{dp}{dx}\right)^3\left(\frac{y^4}{12} + \frac{b^2 y^2}{8} - \frac{by^3}{6} - \frac{b^3 y}{24}\right) + \left(\frac{dp}{dx}\right)\left(\frac{y^2}{2b^2} - \frac{y}{2b}\right) + \left(\frac{dp}{dx}\right)^2\left(\frac{y^2}{2} - \frac{y^3}{3b} - \frac{y}{6}\right)\right\} \tag{3.28}$$

Summing up the results, we write

$$\begin{aligned} u = u_0 + u_1 = u_0 + \frac{u_0^{(1)}}{1!} + \ldots = \frac{dp}{dx}\left(\frac{y^2}{2} - \frac{yb}{2}\right) + \left(1 - \frac{y}{b}\right) \\ -6\beta\left\{\left(\frac{dp}{dx}\right)^3\left(\frac{y^4}{12} + \frac{b^2 y^2}{8} - \frac{by^3}{6} - \frac{b^3 y}{24}\right) + \left(\frac{dp}{dx}\right)\left(\frac{y^2}{2b^2} - \frac{y}{2b}\right) + \left(\frac{dp}{dx}\right)^2\left(\frac{y^2}{2} - \frac{y^3}{3b} - \frac{y}{6}\right)\right\} + \ldots \end{aligned} \tag{3.29}$$

Finally, eq. (3.29) is the analytical solution of the problem of order β.

Pressure Distribution: Using the continuity equation (2.1) together with the derived velocity profile, one may find the ordinary differential equation for the pressure distribution. Integrating the continuity equation along the y-coordinate with the boundary condition $v(0) = v(b) = 0$

$$\int_0^b \frac{\partial u}{\partial x} dy = -\int_0^b \frac{\partial v}{\partial y} dy = v(0) - v(b) = 0 \tag{3.30}$$

one has

$$\int_0^b \frac{\partial u}{\partial x} dy = 0 \tag{3.31}$$

Substituting equation eq. (3.29) into equation eq. (2.3) and integrating, we get

$$\frac{d}{dx}\left[-\frac{b^3}{12}\frac{dp}{dx} + \frac{b}{2} + 6\beta \left\{ \frac{b^5}{240}\left(\frac{dp}{dx}\right)^3 + \frac{b}{12}\left(\frac{dp}{dx}\right) \right\} \right] = 0 \tag{3.32}$$

An approximate solution will be searched for the above equation since it is variable coefficient and highly nonlinear differential in p. The associated boundary conditions are

$$P(0) = p(1) = 0 \tag{3.33}$$

Integrating (3.32) with respect to *x*

$$\left[-\frac{b^3}{12}\frac{dp}{dx} + \frac{b}{2} + 6\beta \left\{ \frac{b^5}{240}\left(\frac{dp}{dx}\right)^3 + \frac{b}{12}\left(\frac{dp}{dx}\right) \right\} \right] = C \tag{3.34}$$

Where C is a constant of integration.
After some simplification, we can write eq. (3.34) as

$$\frac{dp}{dx} - \frac{6}{b^2} + \frac{12C}{b^3} - 6\beta \left\{ \frac{b^2}{20}\left(\frac{dp}{dx}\right)^3 + \frac{1}{b^2}\left(\frac{dp}{dx}\right) \right\} = 0 \tag{3.35}$$

Again we wish to solve (3.35) for p by using HAM. We construct the zero-order deformation equation as in we did in eq. (3.10),

$$(1-q)Ł[\tilde{p}(x,q) - p_0(x)] = -q\left[\frac{d\tilde{p}(x,q)}{dx} - \frac{6}{b^2} + \frac{12C}{b^3} - 6\beta \left\{ \frac{b^2}{20}\left(\frac{d\tilde{p}(x,q)}{dx}\right)^3 + \frac{1}{b^2}\left(\frac{d\tilde{p}(x,q)}{dx}\right) \right\} \right] \tag{3.36}$$

subject to the boundary conditions

$$\begin{aligned} \tilde{p}(x,q) &= 0, \quad \text{at} \quad x = 0 \\ \tilde{p}(x,q) &= 0, \quad \text{at} \quad x = 1 \end{aligned} \tag{3.37}$$

taking the initial gauss approximation as

$$p_0(x) = \frac{6x(b-r)}{b^2(1+r)} = \left(\frac{6x(1-x+rx-r)}{(1-x+rx)^2(1+r)} \right) \tag{3.38}$$

Where

$$b(x) = (1-x+rx), \quad r = \frac{b_2}{b_1} \tag{3.39}$$

is the inclined slider in which b_1 is the maximum and b_2 is the minimum value of b.
Defining the linear operator as

$$\mathcal{L} = \frac{d}{dx} \tag{3.40}$$

and an embedding parameter q such that $q \in [0, 1]$.
Setting $q = 0$ and $q = 1$ in eq. (3.36) respectively, we get

$$\tilde{p}(x,0) = p_0(x) \qquad x > 0 \tag{3.41}$$

and

$$\tilde{p}(x,1) = p(x) \tag{3.42}$$

Therefore, according to eq. (3.41) and eq. (3.42), the variation of q from 0 to 1 is just the continuous variation $\tilde{p}(x,q)$ from the initial guess approximation $p_0(x)$ to the unknown solution $p(x)$ of eq. (3.36). Assume that the deformation $\tilde{p}(x,q)$ governed by eq. (3.36) and eq. (3.42) is smooth enough so that

$$p_0^{(k)}(x) = \left.\frac{\partial^k \tilde{p}(x,q)}{\partial q^k}\right|_{q=0} \qquad k \geq 1 \tag{3.43}$$

namely the k-th order deformation derivative exists. Then, according to eq. (3.41) and Taylor's formula, we have

$$\tilde{p}(x,q) = p_0(x) + \sum_{k=1}^{\infty} \left[\frac{p_0^{(k)}(y)}{k!} \right] q^k \tag{3.44}$$

Defining

$$p_k(x) = \frac{p_0^{(k)}(x)}{k!} \tag{3.45}$$

Using eq. (3.42), eq. (3.44) and eq. (3.45), we get at $q = 1$, the important relationship

$$p(x) = \sum_{k=0}^{\infty} p_k(x) \tag{3.46}$$

between the initial guess approximation $p_0(x)$ and the unknown solution $p(x)$.
Setting $p(x)$ in eq. (3.36), gives

$$\tilde{p}(x,0) = p_0(x) \tag{3.47}$$

In particular, differentiating (3.36) with respect to q, making use of (3.43) and setting $q = 0$, we have

$$\mathcal{L}\left\{p_0^{(1)}\right\}\left[\frac{dp_0}{dx} - \frac{6}{b^2} + \frac{12C}{b^3} - 6\beta\left\{\frac{b^2}{20}\left(\frac{dp_0}{dx}\right)^3 + \frac{1}{b^2}\left(\frac{dp_0}{dx}\right)\right\}\right] \tag{3.48}$$

making use of (3.38)

$$\begin{aligned}\frac{dp_0^{(1)}}{dx} &= \frac{12r}{(r+1)(rx-x+1)^3} - \frac{12C}{(1-x+rx)^3} \\ &+6\beta\left\{\frac{54(r-1)^3(x-rx-1)^3}{5(r+1)^3(rx-x+1)^7} + \frac{6(r-1)(x-rx-1)}{(r+1)(rx-x+1)^5}\right\}\end{aligned} \tag{3.49}$$

Integrating eq. (3.38) with respect to x, gives

$$p_0^{(1)} = \frac{6C}{(r-1)(rx-x+1)^2} + D - \frac{6r}{(r-1)(r+1)(1-x+rx)^2} - \frac{24\beta\begin{pmatrix} 30rx-105x-15r+27r^2-13r^3+105x^2-35x^3 \\ -15rx^2+78r^2x-30r^3x+27r^4x-210r^2x^2+105r^2x^3 \\ +30r^3x^2+105r^4x^2-105r^4x^3-15r^5x^2+35r^6x^3+35 \end{pmatrix}}{25(r-1)(r+1)^3(rx-x+1)^6} \tag{3.50}$$

Where D is a constant of integration. Now using the boundary condition (3.37), yields

$$\begin{aligned} p_0^{(1)} = {} & \frac{\lambda_3(1152r+1872r^2+1872)}{300r^4-300r^2-150r+150r^5} - \frac{6r}{(r-1)(r+1)(1-x+rx)^2} \\ & - \frac{\begin{pmatrix} 1872\beta-3888r\beta-900r^2-1800r^3-900r^4 \\ +4032r^2\beta-3888r^3\beta+1872r^4\beta \end{pmatrix}}{150r(r-1)(r+1)^3(rx-x+1)^2} \\ & - \frac{24\beta\begin{pmatrix} 30rx-105x-15r+27r^2-13r^3+105x^2-35x^3 \\ -15rx^2+78r^2x-30r^3x+27r^4x-210r^2x^2+105r^2x^3 \\ +30r^3x^2+105r^4x^2-105r^4x^3-15r^5x^2+35r^6x^3+35 \end{pmatrix}}{25(r-1)(r+1)^3(rx-x+1)^6} \end{aligned} \tag{3.51}$$

Therefore, the final pressure distribution is of the form

$$\begin{aligned} p = {} & p_0 + \frac{p_0^{(1)}}{1!} + \ldots = p_0 + p_1 + \ldots = \frac{6x(1-x+rx-r)}{(1-x+rx)^2(1+r)} \\ & + \frac{\beta(1152r+1872r^2+1872)}{300r^4-300r^2-150r+150r^5} - \frac{6r}{(r-1)(r+1)(1-x+rx)^2} \\ & - \frac{\begin{pmatrix} 1872\beta-3888r\beta-900r^2-1800r^3-900r^4 \\ +4032r^2\beta-3888r^3\beta+1872r^4\beta \end{pmatrix}}{150r(r-1)(r+1)^3(rx-x+1)^2} \\ & - \frac{24\beta\begin{pmatrix} 30rx-105x-15r+27r^2-13r^3+105x^2-35x^3 \\ -15rx^2+78r^2x-30r^3x+27r^4x-210r^2x^2+105r^2x^3 \\ +30r^3x^2+105r^4x^2-105r^4x^3-15r^5x^2+35r^6x^3+35 \end{pmatrix}}{25(r-1)(r+1)^3(rx-x+1)^6} \end{aligned} \tag{3.52}$$

DISCUSSION

In this section, the pressure distribution and velocity profile in the bearing is determined for various values of the parameter β and clearance ratio r. Since $b(x)$ is decreasing from left to right, the flow then set up a pressure gradient that redistributes the fluid and maintains a constant flow rate. Figure 2 indicates the variation of the pressure with respect to x when r is held fixed and β is varied. It is seen that the pressure increases

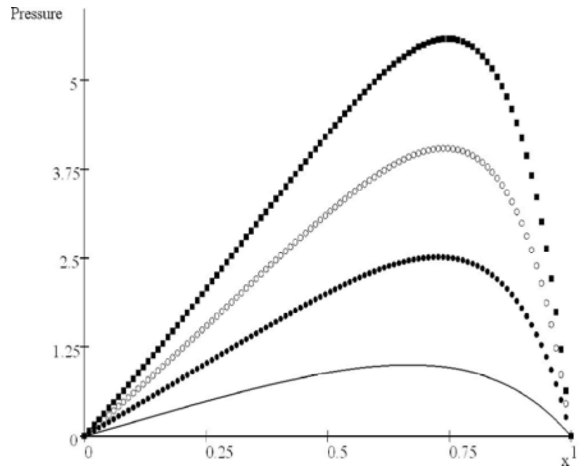

Fig. 2: r = 0.5, (ß = 0-line, ß = 0.1-dots, ß = 0.2-cicle, ß = 0.3-cross)

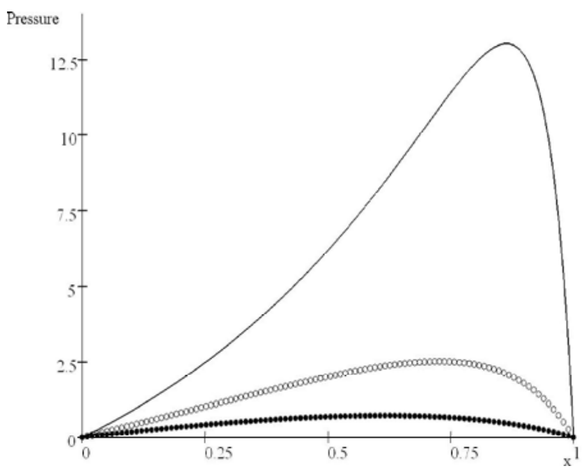

Fig. 3: ß = 0.1, (r = 0.3-line, r = 0.5-circle, r = 0.7-box)

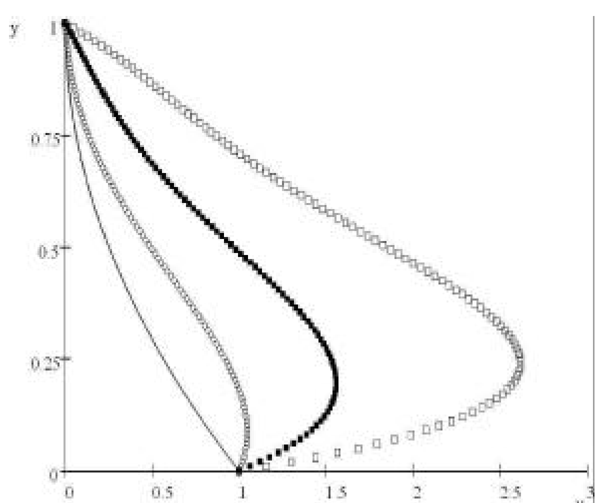

Fig. 4: r = 0.5, x = 0.0 (ß = 0-line, 0.1-circle, 0.2-cross, 0.3-box)

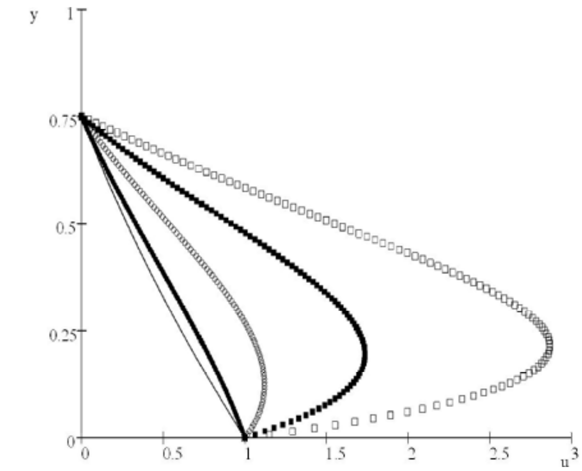

Fig. 5: r = 0.5, x = 0.5 (ß = 0-line, 0.1-dot, 0.2-circle, 0.3-cross, 0.4-box)

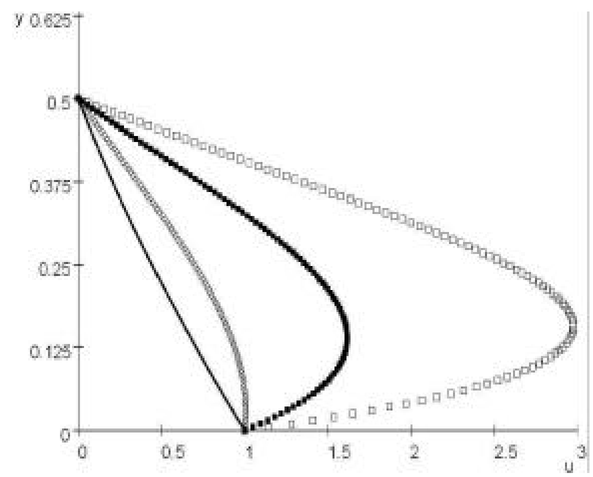

Fig. 6: r = 0.5, x = 1.0 (ß = 0-line, 0.05-circle, 0.1-cross, 0.2-box)

with increasing β, which mean higher load capacity for the bearing due to non-Newtonian effects. In figure 3, $\beta = 0.1$ is held fixed and the dimensionless length versus dimensionless pressure is plotted for different clearance ratios r. It is seen that pressure build up for lower clearance ratios. In Figure 4 , at the left corner ($x = 0.0$, $b = 1.0$) of the bearing, the Newtonian and non-Newtonian velocity profile is plotted for the clearance ratio ($r = 0.5$).

Through the action of viscous shear forces, flow component are distributed into Couette and Poiseuille-type flow. It is also observed that, as the non-Newtonian effects increases, boundary layer develop near the lower boundary of the bearing. Similar flow behavior is also observed in Figure 5 ($x = 0.5$, $b = .75$) and Figure 6 ($x = 1.0$, $b = 0.5$).

CONCLUSION

In this paper, the homotopy analysis method is successfully applied in finding an explicit analytical solution of the inclined shape slider bearing with non-Newtonian third grade fluid as lubricant. The velocity profile and pressure distribution in the bearing are calculated and are analyzed graphically. The success of HAM for solving this problem verifies that it is indeed a useful iterative method to solve nonlinear problems. It is to be noted that our results are in a very good agreement in comparison with the perturbation solution provided by Yürüsoy [23, 24]. The proposed solution may be useful for engineers in designing bearing systems with maximum load carrying capacity using non-Newtonian fluid.

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