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Efficiency of Different Stopping Rules for Selecting a Good System

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Abstract: We consider the problem of selecting the best simulated system from a finite and huge set of alternatives. Almomani and Abdul Rahman [1] presented a new selection approach to solve this problem. The approach was a combination between cardinal and ordinal optimizations to solve the size problem of alternatives set. Since the cardinal optimization (Ranking and Selection) procedures cannot be used with a huge number of alternatives, therefore the Ordinal Optimization procedure is used to reduce the number of elements in the alternatives set to be appropriate for Ranking and Selection procedures. In this paper, we study the efficiency of Almomani and Abdul Rahman [1] selection approach with different sets of the stopping rule changes. In particular, there are three different stopping rules; sequential, probability of good selection and expected opportunity cost. Almomani and Abdul Rahman [1] used the sequential stopping rule in their work. In our study, we change the stopping rule in their algorithm from sequential to expected opportunity cost and probability of good selection. Then we apply the selection approach with these different stopping rules on the *M/M/1* queuing systems to see the efficiency of the approach by identifying the most effective stopping rule.

Key words: Ranking and Selection · Ordinal Optimization · Optimal Computing Budget Allocation · Simulation Optimization · Stopping Rules

INTRODUCTION

In many real life applications, we need to select the best system (set of systems) from the alternatives set, based on some performance measures (mean). When the number of alternatives small, Ranking and Selection (*R&S*) procedures are used to select the best systems, see Kim and Nelson [2]. Meanwhile, when the number of alternatives large the idea of Ordinal Optimization (*OO*) procedure that proposed by Ho *et al*. [3] is used to select a good system with high probability. However, the problem arises for the large scale problems (i.e. problems with a huge set of alternatives) because it needs a huge computational time. In fact, for each sample of the performance values will require one simulation run. Therefore, for the large scale problems will need a large number of samples, which is very time-consuming and may be impossible, especially when we are dealing with a huge number of alternatives in the feasible solution set. In this situation, we would change our objective to finding good systems rather than estimating accurately the performance value for these systems.

Before year 2000, most works in selecting the best system for the large scale problems are involved with either a two stage, or three stage procedures with common known or unknown variances. See, Tamhane [4], Tamhane and Bechhofer [5], Hochberg and Marcus [6] and Santner and Behaxeteguy [7]. Later, Nelson *et al*. [8] proposed a combination approach between Subset Selection (*SS*) and Indifference-Zone (*IZ*) procedures to obtain a computationally and statistically efficient procedure for selecting the best system when the number of alternatives is large with the unknown variances. In fact, this procedure consists of two stages; in the first stage, alternatives that are not competitive are screened out and eliminated by *SS* procedure. Then from the competitive alternative systems, the best system is selected in the second stage by *IZ* procedure. Kim and Nelson [9] proposed a fully sequential procedure, to select the best system when the number of alternative systems is large. They showed that their procedure works well for up to 500 system and required unequal variances for all systems. The goal of this procedure is to eliminate, at an early stage, those simulation systems that are apparently inferior, in order to reduce the overall computational effort that required in selecting the best system. Recently, Alrefaei and Almomani [10] proposed two sequential algorithms for selecting a subset of *k* systems that contained in the set of the top *s* systems.

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In this paper, we consider the following optimization problem

$$\min_{\theta \in \Theta} f(\theta) \quad (1.1)$$

Where Θ and finite. Let f be the expected performance measure of some complex simulation system and is written as $f(\theta) = E[L(\theta, Y)]$, where θ is a vector representing the system design parameters, Y represents all the random effect of the system and L is a deterministic function that depends on θ and Y. In this paper, without loss of generality we assume that the best system is the system with the smallest mean, which is unknown and to be inferred from simulation. Therefore, our goal is selecting the system that has the smallest sample mean. Suppose that there are n systems and let Y_{ij} represents the j^{th} output from the system i, where $I = 1,2,...,n$. Let $Y_i = \{Y_{ij}, j = 1,2,...\}$ denotes the output sequence from the system i. We assume Y_{ij} are independent and identically normal distributed with unknown means $\mu_i = E(Y_i)$ and variances $\sigma_i^2 = Var(Y_{ij})$ and $Y_1, Y_2,...,Y_n$, are also mutually independent. In practice, the σ_i^2 are unknown, so we need to estimate it using the sample variances s_i^2 for Y_{ij}. However, since we assume that the smallest mean is better, therefore if the ordered μ_i-values are denoted by $\mu_{[1]} \leq \mu_{[2]} \leq ... \leq \mu_{[n]}$, then the system having mean $\mu_{[1]}$ is referred to as the best system. Actually, the Correct Selection (CS) occurs when the system selected by the selection approach is the same as the actual best system.

In real world applications, the normality and independent rarely exists. Thus, we need to approximate the raw data are to be normal and independent. In fact, the non-normality and dependence usually are not major concerns in simulation experiments, because multiple independent replications are used as the basic summary measure within average replication of a large number of raw simulation outputs. Therefore, if we take the output of interest as the average data, then by Central Limit Theorem (*CLT*) we found that the replication average will be approximately normally distributed. Moreover, if each replication is independent then the replication averages will be also independent. However, there are two methods to solve the normality and independent problems. First, by making multiple replications for each alternative and secondly, by using the batch mean of many raw outputs from a single long replication as the basic observations. Unfortunately, these two methods have disadvantages. We tend to loss out the raw output that collected during the warm-up (start-up) period for each replication, where the warm-up period is the period from the beginning of the simulation until the system has reached a steady-state. Also, if we defined the stage by batch means rather raw output then the simulation effort consumed by any stage is a multiple of the batch size.

Since the simulation methods are used to indicate the performance measure for each alternative, then there is a potential for incorrect selection. Thus, we need measures to determine the quality of selection. There are two measures of selection quality; the Probability of Correct Selection (*P (CS)*) and the Expected Opportunity Cost (*E (OC)*) of a potentially incorrect selection, see He *et al.* [11]. These two measures are also can be used to decide when to stop the sampling process. In particular, Brank *et al.* [12] proposed the following stopping rules:

- Sequential (S): Repeat sampling while $\sum_{i=1}^{n} T_i < B$, for some specified total budget B and T_i is the number of samples allocated to system i, where $i = 1,2,...,n$.
- Expected opportunity cost (*E (OC)*): Repeat sampling while (*E(OC)*), for a specified expected opportunity cost target $E(OC) > \varepsilon$.
- Probability of good selection ($P(GS)_{\delta^*}$): Repeat sampling while $P(GS)_{\delta^*} < 1 - \varphi^*$, for a specified probability target $1 - \varphi^* \in [1/n, 1]$ and given $\delta^* \geq 0$.

Almomani and Abdul Rahman [1] proposed a new selection approach to select a good system by using the *OO* and *R&S* procedures. In particular, the selection approach is a combination between four procedures; *IZ*, *SS*, *OO* and Optimal Computing Budget allocation (*OCBA*). Their goal is to select a good system from a huge number of alternatives with high probability. The first step involved with the *OO* to select a subset that overlaps with the set of the actual best *m%* system. Then, *OCBA* procedure is used to allocate the available simulation samples in a way that maximize the probability of correct selection. This is followed by *SS* procedure to get a smaller subset that contains the best system among the subset that is selected before. Finally, the *IZ* procedure is used to select the best system among the survivor systems in the previous stage. In their algorithm, they used the sequential S as a stopping rule. Nevertheless, there are other stopping rules that can provide the flexibility to stop earlier if there is evidence that the *CS* is sufficiently high and to allow for additional sampling if the CS is not sufficiently high.

In this paper, we change the stopping rule in Almomani and Abdul Rahman [1] approach from a sequential S to the expected opportunity cost *E (OC)* and probability of good selection $P(GS)_{\delta^*}$, to see the efficiency of the approach with different stopping rules. We apply a numerical illustration on this approach to display the advantages and the disadvantages for each stopping rules and to determine the most effective stopping rule that works better with Almomani and Abdul Rahman [1] selection approach.

The rest of this paper is organized as follows; In Section 2, we review the *IZ*, *SS*, *OO* and *OCBA* procedures. In Section 3, we present the algorithm of Almomani and Abdul Rahman [1] with three different stopping rules. The performances of the selection approach under different stopping rules are illustrated with a series of numerical examples in Section 4 and Section 5 concludes the paper.

Background: In this section, we briefly review the procedures of Ranking and Selection (*R&S*), Ordinal Optimization (*OO*) and Optimal Computing Budget Allocation (*OCBA*).

Ranking and Selection Procedures: Selecting the system with the smallest or largest expected performance (best system) is one of the major problems that arise in simulation. When the number of alternatives n is small then we can use *R&S* procedures to select the best system or a subset that contain the best systems. Here, we review two different *R&S* procedures; Indifference-Zone (*IZ*) and Subset Selection (*SS*) procedures.

Indifference-Zone Procedure: The goal of *IZ* procedure is selecting the best system among n systems when the number of alternatives less than or equal 20. Suppose we have n alternative systems that are normally distributed with unknown means $\mu_1, \mu_2, \ldots, \mu_n$, and suppose that these means are ordered as $\mu_{[1]}, \leq \mu_{[2]} \leq \ldots \leq \mu_{[n]}$, We want to select the system that has the best minimum mean $\mu_{[1]}$. The *IZ* is defined to be the interval $[\mu_{[1]}, \mu_{[2]} + \delta^*]$, where δ^* is a predetermined small positive real number, called indifference zone. We are interested in selecting an alternative i^* such that $\mu_{i^*} \in [\mu_{[1]}, \mu_{[1]} + \delta^*]$. Let *CS* be selecting an alternative whose mean belongs to the indifference zone. We prefer the *CS* to take place with high probability, say with a probability not smaller than P^* where $1/n \leq P^* \leq 1$.

The *IZ* procedure consists of two stages. In the first stage, all systems are sampled using initial simulation runs to get an initial estimate of the expected performance measure with their variances. Next, depending on the information obtained in the first stage, how many more samples are needed in the second stage for each system in order to guarantee that $P(CS) \geq P^*$ is computed. Rinott [13] has presented a procedure that is applicable when the data are normally distributed with all systems are simulated independently of each others. This procedure consists of two stages for the case when variances are completely unknown. On the other hand, Tamhane and Bechhofer [14] have presented a simple procedure that is valid when variances may not be equal.

Subset Selection Procedure: SS procedure screens out the feasible solution set, eliminates non-competitive systems and then constructs a subset that contains the best system with high probability. This procedure is suitable when the number of alternatives is relatively large and it is used to select a random size subset that contains the actual best system. The *SS* procedure required that $P(CS) \geq P^*$, where the *CS* is selecting a subset that contains the actual best system and P^* is a predetermined probability.

The *SS* procedure dating back to Gupta [15], who presented a single stage procedure in producing a subset of the best system with a specified probability. Extensions of this work which is relevant to the simulation setting includes Sullivan and Wilson [16] who derived a two stage *SS* procedure to determine a subset of maximum size m that, with a specified probability will contain systems that are all within a pre-specified amount of the optimum.

Another comprehensive review of *R&S* procedures can be found in Bechhofer *et al.* [17], Goldsman and Nelson [18] and Kim and Nelson [2, 19, 20].

Ordinal Optimization Procedure: The *OO* focuses on isolating a subset of good systems with high probability and reducing a required simulation time for discrete event simulation. The goal of *OO* procedure is to find a good enough system, rather than to estimate accurately the performance value of these systems. This procedure has been proposed by Ho *et al.* [3].

Consider the optimization problem given in equation (1.1). If we simulate the system to estimate $E[L(\theta, Y)]$, then the confidence interval of this estimator cannot be improved faster than $1/\sqrt{k}$ where k is the number of replications that been used to get estimates of $f(\theta)$, see Chen *et al.* [21]. This is good when the number of alternatives is small; however it is not good enough for a complex simulation problem with a large number of alternatives. Actually, for each sample of $L(\theta, Y)$ will require one simulation run. So we will need a large number of samples, when we are dealing with a huge number of alternative systems in the feasible solution set, which is very hard and maybe impossible. In this situation, one could relax the objective to get a good enough solution to avoid doing the extensive simulation.

Let the *CS* here is to select a subset *G* of *g* systems from the feasible solution set Θ that contains at least one of the top *m %* best systems. Since Θ is very huge then the probability of *CS* is given by $P(CS) \approx (1-(1-\frac{m}{100})^g)$. Furthermore, suppose that the *CS* is to select a subset *G* of *g* systems that contains at least *r* of the best *s* systems. If we assume *S* be the subset that contains the actual best *s* systems, then the probability of *CS* can be obtained using the hyper geometric distribution as.

$$P(CS) = P(|G \cap S| \geq r) = \sum_{i=r}^{g} \binom{s}{i}\binom{n-s}{g-i} \Big/ \binom{n}{g}$$

However, since the number of alternatives is very large, the *P (CS)* can be approximated by the binomial random variable, as:

$$P(CS) \approx \sum_{i=r}^{g} \binom{g}{i} \left(\frac{m}{100}\right)^{i} \left(1-\frac{m}{100}\right)^{g-i}$$

Another comprehensive review of *OO* procedure can be found in Deng *et al.* [22], Dai [23], Xiaolan [24], Deng and Ho [25], Lee *et al.* [26], Li *et al.* [27], Zhao *et al.* [28] and Ho *et al.* [29].

Optimal Computing Budget allocation Procedure: The *OCBA* is used to determine the best simulation lengths from all simulation systems in order to reduce the total computation time. In fact, this procedure is proposed to improve the performance of *OO* by determining the optimal numbers of simulation samples for each system, instead of simulating equally all systems. The goal of *OCBA* is to allocate the total simulation samples from all systems in a way that maximizes the probability of selecting the best system within a given computing budget, see Chen *et al.* [21], Chen [30], Chen *et al.* [31], Chen *et al.* [32], Banks [33] and Chen [34].

Let *B* be the total sample that available for solving the optimization problem given in (1.1). Our target is to allocate these computing simulated samples such that to maximize the *P (CS)*. In mathematical notation.

$$\max_{T_1,\ldots,T_n} P(CS)$$
$$s.t. \sum_{i=1}^{n} T_i = B$$
$$T_i \in \mathbb{N} \qquad i = 1,2,\ldots,n$$

Where ℕ is the set of non-negative integers, T_i is the number of samples allocated to system *i* and $\sum_{i=1}^{n} T_i$ denotes the total computational samples and assume that the simulation times for different systems are roughly the same. To solve this problem, Chen *et al.* [21] proposed the following theorem.

Theorem 2.1: *Given a total number of simulated samples B to be allocated to n competing systems whose performance is depicted by random variables with means* $f(\theta_1), f(\theta_2), \ldots, f(\theta_n)$, *and finite variances* $\sigma_1^2, \sigma_2^2, \ldots, \sigma_n^2$ *respectively, as* $B \to \infty$, *the approximate probability of correct selection can be asymptotically maximized when*

1. $\frac{T_i}{T_j} = \left(\frac{\sigma_i/\delta_{b,i}}{\sigma_j/\delta_{b,j}}\right)^2$ *; where* $I, j \in \{1,2,\ldots,n\}$ *and* $i \neq j \neq b$.
2. $T_b = \sigma_b \sqrt{\sum_{i=1, i\neq b}^{n} \frac{T_i^2}{\sigma_i^2}}$

where $\delta_{b,i}$ *the estimated difference between the performance of the two systems* $(\delta_{b,i} = \bar{f}_b - \bar{f}_i)$ *and* $\bar{f}_b \leq \min_i \bar{f}_i$ *for all i. Here* $\bar{f}_i = \frac{1}{T_i}\sum_{j=1}^{T_i} Y_{ij}$ *, where* Y_{ij} *is a sample from* Y_i *for* $j = 1,\ldots,T_i$.

Proof: *See Chen et al [21].*

Algorithms of Selection Approach: In this section, we present the algorithm of Almomani and Abdul Rahman [1] selection approach with different stopping rules. In fact, the *Stopping Rule* is the only step that changes whereas the remainder steps are the same.

Algorithm with a Sequential Stopping Rule: Almomani and Abdul Rahman [1] proposed a selection approach that used the sequential *S* as a stopping rule. This approach consists four stages. In the first stage, using the *OO* procedure, a subset *G* is selected randomly from the feasible solution set that intersects with the set *m%* of actual best systems with high probability (1 –*α*). In the second stage, *OCBA* procedure is used to allocate the available computing budget. It is followed by the *SS* procedure to get a smaller subset *I* that contains the best system among the subset that is selected before with high probability (1–α), where |*I*|≤ 20. Finally, using the *IZ* procedure to select the best system from set *I* with high probability (1–α).

In this approach, Almomani and Abdul Rahman [1] used the sequential S as the stopping rule step in their algorithm. The sequential condition was stated as if $\sum_{i=1}^{g} T_i^l \geq B$, then the algorithm stop. In fact, most tradition selection approach used the same stopping rule. Since the target was selecting the best system with minimum elapsed time, this was controlled by increasing or decreasing the total budget B.

The algorithm of Almomani and Abdul Rahman [1] is given as follows:

Algorithm

Setup: Specify g where $|G| = g$, k where $|G'| = k$, the number of initial simulation samples $t_0 \geq 2$, the indifference zone λ^* and $t = t_{(1-\alpha_2/2)^{\frac{1}{g-1}}, t_0 - 1}$ from the t-distribution. Let $T_1^l = T_2^l = \ldots = T_g^l = t_0$ and determine the total computing budget B. Here, G is the selected subset from Θ, that satisfies P (G contains at least one of the best $m\%$ systems) $\geq 1 - \alpha_1$, whereas G' is the selected subset from G, where $g \geq k$. The iteration number is represented by l.

Select a subset G of size g randomly from Θ and also take a random samples t_0 observations y_{ij} ($j = 1,\ldots,t_0$) for each system i in G, where $i = 1,\ldots,g$.

Initialization: Calculate the sample mean and variances $\bar{y}_i^{(1)}$ and s_i^2, where $\bar{y}_i^{(1)} = \sum_{j=1}^{T_i^l} y_{ij} \Big/ T_i^l$ and $s_i^2 = \sum_{j=1}^{T_i^l} \left(y_{ij} - \bar{y}_i^{(1)} \right)^2 \Big/ T_i^l - 1$, for all $i = 1,\ldots,g$.

Order the systems in G according to their sample averages; $\bar{y}_{[1]}^{(1)} \leq \bar{y}_{[2]}^{(1)} \leq \ldots \leq \bar{y}_{[g]}^{(1)}$. Then select the best k systems from the set G and represent this subset as G'.

Stopping Rule: If $\sum_{i=1}^{g} T_i^l \geq B$, then stop. Otherwise, randomly select a subset G'' of the g-k alternatives from $\Theta - G'$, let ($G = G' \cup G''$).

Simulation Budget Allocation: Increase the computing budget by Δ and compute the new budget allocation, $T_1^{l+1}, T_2^{l+1}, \ldots, T_g^{l+1}$ by using Theorem 2.1.

Perform additional $\max\{0, T_i^{l+1} - T_i^l\}$ simulations for each system i, $i = 1,\ldots,g$ let l. Go to *Initialization*.

Screening: Set $I = \{i : 1 \leq i \leq \text{and} \bar{y}_i^{(1)} \geq \bar{y}_j^{(1)} - [W_{ij} - \delta^*]^-, \forall i \neq j\}$, where $W_{ij} = t\left(\frac{s_i^2}{T_i} + \frac{s_j^2}{T_j} \right)^{1/2}$ for all $i \neq$ j and $[x]^- = x$ if $x < 0$ and $[x]^- = 0$ otherwise.

If I contain a single index, then this system is the best system. Otherwise, for all $i \in I$ compute the second sample size $N_i = \max\left\{ T_i, \left\lceil \left(\frac{h s_i}{\delta^*} \right)^2 \right\rceil \right\}$, where $h = h(1 - \alpha_3 / 2, t_0, |I|)$ be the Rinott [13] constant and can be obtained from tables of [35].

Take additional $N_i - T_i$ random samples of y_{ij} for each system $i \in I$ and compute the overall sample means for $i \in I$ as $\bar{y}_i^{(2)} = \sum_{j=1}^{N_i} y_{ij} \Big/ N_i$.

Select system $i \in I$ with the smallest $\bar{y}_i^{(1)}$ as the best.

Algorithm with Expected Opportunity Cost as a Stopping Rule: In this new algorithm, we change the stopping rule from sequential S to expected opportunity cost E *(OC)*. Actually, this stopping rule (E *(OC)*) is effective when the goal is selecting the best system with the minimum E *(OC)*, like in business applications. Using the same algorithm from Almomani and Abdul Rahman [1], we change the *Stopping Rule* as:

Stopping Rule: If $E(OC) \leq \varepsilon$, for a specified expected opportunity cost target $\varepsilon > 0$, then the algorithm is stop. Otherwise, select randomly a subset G'' of the g-k alternatives from $\Theta - G'$, where we let $G = G' \cup G''$.

Algorithm with Probability of Good Selection as a Stopping Rule: A selected system within δ^* from the best system is called the "good" system. However, in this paper our focus is not on the difference between the concepts of "good" or "best" system but to select a system from a very large alternatives set. Therefore, if we can get a good system, we can call it a best system. Anyway, the P *(CS)* can be equal to the $P(GS)_{\delta^*}$ when δ^* = 0. Clearly, the *IZ* procedure requires that $\delta^* = 0$ and the *OCBA* permits $\delta^* = 0$, see Brank *et al.* [12]. Again, using the same algorithm from Almomani and Abdul Rahman [1], we use the probability of good selection $P(GS)\delta^*$ as a stopping rule as follows:

Stopping Rule: If $P(GS)\delta^* \geq 1 - \varphi^*$, for a specified probability target $1 - \varphi^* \in [1/n,1]$ and given $\delta^* \geq 0$, then the algorithm is stop. Otherwise, select randomly a subset G'' of the g-k alternatives from $\Theta - G'$, let ($G = G' \cup G''$).

Empirical Illustration: Empirical illustrations for both stopping rules (the *E (OC)* and the $P(GS)_{\delta^*}$), are presented using example of *M/M/1* queuing systems, where the inter arrival times and the service times are exponentially distributed and the system has one server, see Ross [36]. Our goal is selecting one of the best *m%* systems that have the minimum average waiting time per customer from *n M/M/1* queuing systems. We use the Probability of Correct Selection (*P (CS)*) and the Expected Opportunity Cost (*E (OC)*) of a potentially incorrect selection as a measure of selection quality, where the Opportunity Cost (*OC*) is defined as the difference between unknown means of the selected best system and the actual best system.

We apply the algorithm of Almomani and Abdul Rahman [1] selection approach in this example with different stopping rules under some assumptions. We assume that the arrival rate λ is a fix number, $\lambda = 1$ and the service rate μ is belong to the interval $[a, b]$, such that $\mu \in [7, 8]$. Suppose that we have 3000 of *M/M/1* queuing systems and we discretize the problem by assuming that $\mu \in \Theta = 7 + i / 3000$, where $i = 0,1,\ldots,3000$. Therefore, the best queuing system that has a minimum average waiting time would be the 3000^{th} queuing system with $\mu_{3000} = 8.0$.

In the first experiment, we consider a sequential *S* stopping rule. Assume that, n=3000, g =200, $\alpha_1 = \alpha_2 = 0.005$, $\delta^* = 0.05$, k=20,Δ =50, $t_0 = 20$ and total budget B=10000 (these settings are chosen arbitrarily). Suppose we want to select one of the best (1%) systems, then our target is the systems from 2971 to 3000. The *CS* would be, selecting the system that belongs to {2971, 2972,…, 3000} and the analytical probability of the correct selection here can be calculated as $P(CS) \geq 1 - \left(\left(1 - \frac{1}{100}\right)^{200} + 0.005 + 0.005\right) \geq 0.85$. Furthermore, to achieve the normality assumption in this experiment we use multiple replications method, where the number of multiple replications *M* for each alternative equal 1000.

In the second experiment, we consider the expected opportunity cost as a stopping rule with the same parameter settings as in the first experiment. The total budget condition is removed and replaced with the expected opportunity cost condition such that $E(OC) \leq 0.01$ (i.e. $\varepsilon = 0.01$).

In order to improve the efficiency of Almomani and Abdul Rahman [1] selection approach (with expected opportunity cost as a stopping rule), in the next experiment, the value of ε is reduced from 0.01 to 0.001. Thus, the expected opportunity cost condition will be $E(OC) \leq 0.001$.. Unfortunately, with this condition the algorithm does not work, unless we increase the number of multiple replications *M* for each system from 1000 to 10000. Since we want to make a comparison with the Almomani and Abdul Rahman [1], we also rerun their algorithm (used sequential *S* as a stopping rule) with *M*=10000.

Table 1: The performance of the selection approach under different stopping rules for n=3000, g=200, k=20, Δ =50, t_0 = 20, $m\%$=1% over 100 replications

M	1000	10000	1000	10000
$\overline{T}$	16160.54	153645.38	16474.99	163473.01
$\overline{\sum_{i=1}^{g} T_i}$	20793	21592	21865	23882
$\overline{\sum_{i\in I} N_i}$	4372	5050	5354	7124
Our approach *P(CS)*	83%	81%	82%	84%
Analytical *P(CS)*	85%	85%	85%	85%
Our approach $\overline{E(OC)}$	0.00181892	0.00048408	0.00181381	0.00057008
Analytical $\overline{E(OC)}$	0.00180099	0.0014456	0.00192674	0.00120067

Table 1 contains the results of all the above experiments, over 100 replications for selecting one of the best (1%) systems. From the table, $\overline{T}$ is the average of the elapsed (execution) time, $\overline{\sum_{i=1}^{g} T_i}$ is the average number of the total sample sizes that are computed in *Simulation Budget allocation* step in the algorithm and $\overline{\sum_{i\in I} N_i}$ is the average number of the total sample size in ***Screening*** step in the algorithm and $\overline{E(OC)}$ is the average number of the Expected Opportunity Cost. We calculate the "Our approach of $\overline{E(OC)}$" by taking the absolute value of the difference sample means between the best system that selected by Almomani and Abdul Rahman [1] approach and the actual best system.

We discuss the results in Table 1 from two points. Firstly, we are comparing between two stopping rules; the sequential *S* and the expected opportunity cost *E (OC)*, with the same number of multiple replications *M* for each system. Secondly, we want to find answer to the question of what would happen when we increase the number of multiple replications *M*?

We find that the performance of Almomani and Abdul Rahman [1] selection approach under the two stopping rules; sequential *S* and the expected opportunity cost *E (OC)*, are almost the same. Clearly, the two stopping rules with the same number of multiple replications *M* have almost the same elapsed time, sample size, *P (CS)* and *E (OC)*. Thus, this implies that the Almomani and Abdul Rahman [1] selection approach is effective with both stopping rules. Nevertheless, if we want to improve the efficiency of Almomani and Abdul Rahman [1] selection approach in context of *E (OC)* as a measure of selection quality, we should increase the number of multiple replications *M* for each system. In particular, in order to reduce the value of ε from 0.01 to 0.001 in the expected opportunity cost condition, we need to increase the number of multiple replications *M* from 1000 to 10000. In fact, with this changes it just will increase the elapsed time and of course decrease the *E (OC)*.

We suggest that, if the experiment's target is selecting the best system with high *P (CS)* and minimum elapsed time then the Almomani and Abdul Rahman [1] selection approach with the sequential *S* as a stopping rule is the best way to achieve that. On the other hand, if the experiment's target is selecting the best system with minimum *E (OC)*, then we suggest using the Almomani and Abdul Rahman [1] selection approach but with the expected opportunity cost *E (OC)* as a stopping rule, provided that the number of multiple replications *M* is increased, such that it will increase the elapsed time. However, it is clear that when the value of ε is decreasing then the *E (OC)* will decrease but the optimal value for the *E (OC)* will be zero when the $\varepsilon = 0$. In this case the mean of the selected system will be equal to the mean of the actual best system. Nevertheless, we cannot take too small value of ε since this will require that the number of multiple replications *M* to increase and of course this will lead to a huge elapsed time.

Now, we consider the probability of good selection $P(GS)_{\delta^*}$ as a stopping rule. Using the same example of *M/M/1* queuing systems, we apply the Almomani and Abdul Rahman [1] selection approach with $P(GS)_{\delta^*}$ as a stopping rule. We assume that n=3000, g =200, $\alpha_1 = \alpha_2 = 0.005$, k=20, Δ = 50, t_0 = 20 and the number of multiple replications M=1000 for each system. The *CS* is selecting one of the best (1%) systems, which is the system that belongs to {2971, 2972,…, 3000}.

In this experiment, we also consider different values of δ^*; δ^* = 0.001,0.01,0.05,0.5 and for each δ^* we test with different values of φ^*; φ^* = 0.01,0.05,0.1,0.15,0.2. Figure 1 shows the efficiency curves in $\left(\overline{\sum_{i=1}^{g} T_i}, \varphi^*\right)$ plane for Almomani and Abdul Rahman [1] selection approach with $P(GS)\delta^*$ as a stopping rule, for all values of δ^*, where $\overline{\sum_{i=1}^{g} T_i}$ is defined as the average of the total simulation samples size T_i.

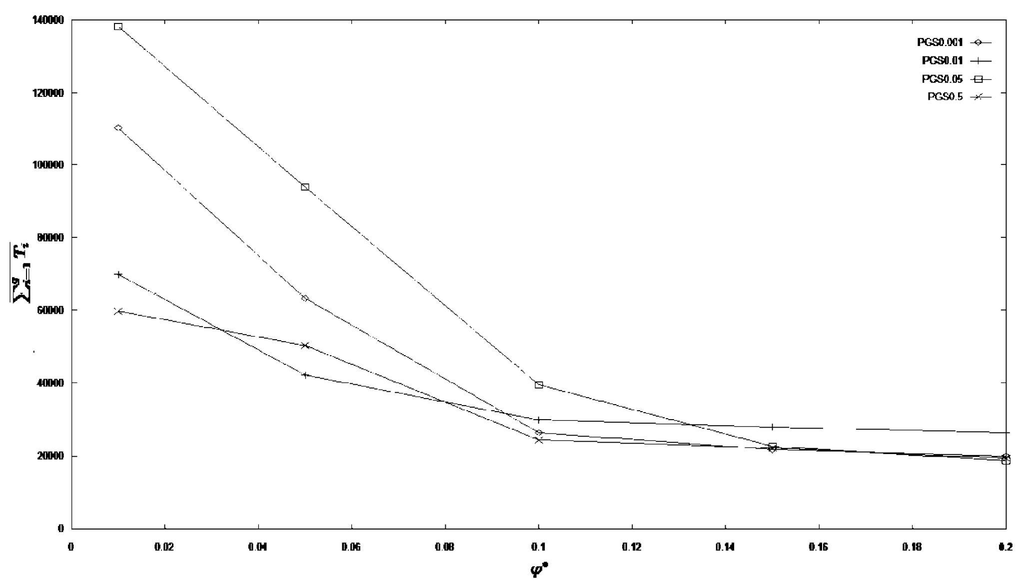

Fig. 1: The efficiency curves between the average of total simulation samples size $\overline{\sum_{i=1}^{g} T_i}$ and φ^* for different values of δ^*.

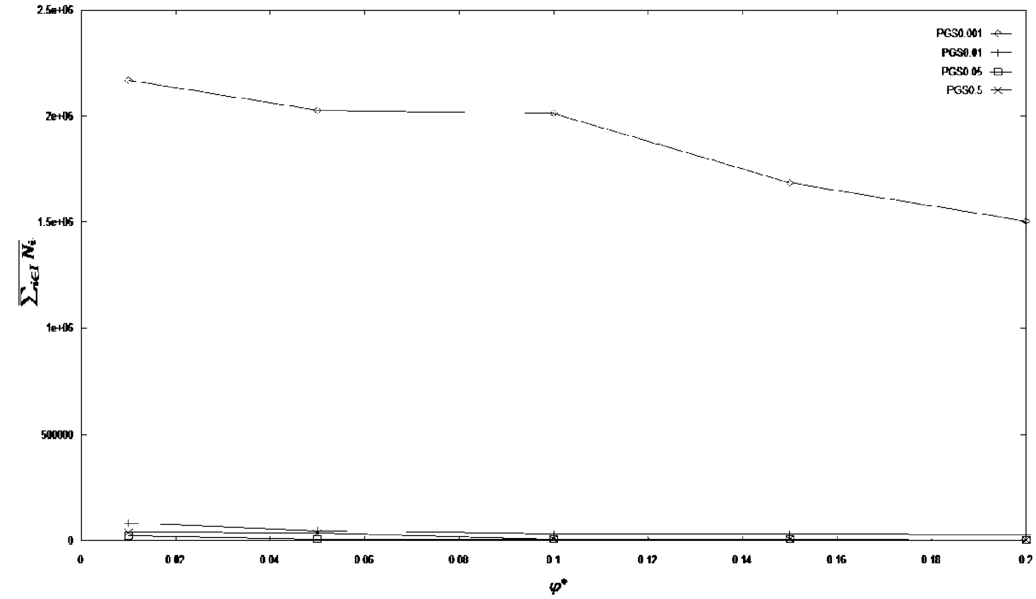

Fig. 2: The efficiency curves between the average of total simulation samples size $\overline{\sum_{i\in I} N_i}$ and φ^* for different values of δ^*.

Figure 1 shows clearly that the average of the total simulation samples size $\overline{\sum_{i=1}^{g} T_i}$ that are computed in *Simulation Budget Allocation* step in the algorithm of Almomani and Abdul Rahman [1], are decreasing when the values of φ^* are increase for each of δ^*. In fact, this is expected since, when the φ^* increases then $1 - \delta^*$ will decrease and will imply that the $P(GS)\delta^*$ is decreasing. When the probability of good selection becomes small, it means that we can get this probability with a small number of simulation samples. Moreover, we also note that the largest simulation samples size $\overline{\sum_{i=1}^{g} T_i}$ for this experiment occurs when the value of $\delta^* = 0.05$.

Figure 2 show the efficiency curves in $\left(\overline{\sum_{i\in I} N_i}, \varphi^*\right)$ plane for $P(GS)\delta^*$ with all values of δ^*, where $\overline{\sum_{i\in I} N_i}$ is the average of the total simulation samples size N_i.

From Figure 2 we note that the average of the total simulation samples size $\overline{\sum_{i\in I} N_i}$ that are used in *Screening* step in Almomani and Abdul Rahman [1] algorithm, are also decreasing when the value of φ^* increases for all values of δ^*. However, the decreasing here is not very fast compared to Figure 1. Obviously, note that the average of the total simulation samples size $\overline{\sum_{i\in I} N_i}$ when the $\delta^* = 0.001$ is very large compared to the other values of δ^*.

CONCLUSION

This paper discusses the effect of different stopping rules on the performance of selection approach that has been proposed by Almomani and Abdul Rahman [1]. The selection approach consists four stages. Initially, using *OO* procedure, a subset *G* is randomly selected form a feasible solution set that intersects with the set that contains the actual best *m%* systems with high probability. Then *OCBA* procedure is used to allocate the available computing budget. This is follows with *SS* procedure to get a smaller subset *I* with high probability, that contains the best system among the previous selected subset. Finally, *IZ* procedure is applied to select the best system from that set *I*. We apply Almomani and Abdul Rahman [1] algorithm on example of *M/M/1* queuing system under some parameter settings, with different stopping rules; sequential *S*, expected opportunity cost *E (OC)* and probability of good selection $P(GS)_{\delta^*}$. In the first two experiments, using the sequential *S* and the expected opportunity cost *E (OC)*, we found that the performance of the selection approach are almost the same. However, for the *E (OC)* in order to improve the efficiency of the Almomani and

Abdul Rahman [1] approach, we need to increase the number of multiple replications M or to increase the batch size for each system. Unfortunately, this will cause an increase in the elapsed time. Thus our suggestion is, if the goal of experiment is selecting the best system with high P *(CS)* and minimum elapsed time then the algorithm should use the sequential S as a stopping rule instead. On the other hand, if the goal is selecting the best system with minimum E *(OC)*, the algorithm should be used together with E *(OC)* as a stopping rule. In final experiment, we consider the probability of good selection $P(GS)_{\delta^*}$ as a stopping rule. Numerical results clearly show that the average of the total simulation samples are decreasing when the value of φ^* increase for all δ^*. Finally, we conclude that the selection approach as proposed in Almomani and Abdul Rahman [1] is effective if applied with these three stopping rules. However, we should be careful in choosing the right stopping rule to comply with the target of the experiment.

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