

## A Comparison of Hypothesis Testing Methods for the Mean of a Log-Normal Distribution

<sup>1</sup>F. Negahdari, <sup>2</sup>K. Abdollahnezhad and <sup>3</sup>A.A. Jafari

<sup>1</sup>Islamic Azad University, Neyriz Branch, Iran

<sup>2</sup>Department of Statistics, Golestan University, Gorgan, Iran

<sup>3</sup>Department of Statistics, Yazd University, Yazd, Iran

**Abstract:** This paper deals with testing the mean of a log-normal population. We apply a newly developed Computational Approach Test (CAT), which is essentially a parametric bootstrap method. An advantage of the CAT is that it does not require the explicit knowledge of the sampling distribution of the test statistic. The CAT is then compared with three accepted tests- Cox method, modified Cox method and generalized p-value method with Monte Carlo simulations. Our detailed studies indicate some interesting results including the facts that the size and power of CAT is better than other methods. Using real data, we have illustrated our method.

**Key words:** Computational Approach test • Log-normal distribution • Generalized p-value • Testing hypothesis

### INTRODUCTION

Accessibility computational resources has contributed to the fundamental researches to be carried out in many areas of mathematical sciences. Complex theoretical results can now be better realized through numerical computations and/or Monte Carlo simulations well before they can be verified analytically. Recently, [1] developed a simple computational technique, called Computational Approach Test (CAT), for hypothesis testing problems. The CAT looks similar to parametric bootstrap, but has some great differences-especially the way it exploits the nuisance parameter(s). Among the noteworthy aspects of this method it is observed that: (i) the CAT is a parametric method which does not require the knowledge of the exact sampling distributions of the parameter estimators; and as a result, (ii) the CAT can be used even for a very complicated parametric model which often relies heavily on the asymptotic results only. [2] applied the CAT for Behrens-Fisher problem and compared it with four test methods and [3] showed that for the one-way ANOVA the CAT can provide almost as much power as the classical F-test under the homoscedastic normal model.

In applied statistics classes, we sometimes come across data that need to be transformed prior to analysis.

For example, income data can often be considered to be log-normal. One way of analyzing such data is to log-transform the original variable  $X$  and to base the inference on the transformed variable  $Y = \ln(X)$ . This means the distribution from which our data emerges can be approximated with a log-normal distribution. In this paper, we have discussed the hypothesis tests of the arithmetic mean value of  $X$  in a log-normal distribution. It is true that the median is often used to describe the average of skewed distributions like income distributions. However, there are situations when the arithmetic mean is a parameter of interest.

Let  $X$  denote the random variable that follows a log-normal distribution with probability density function

$$f_X(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2\sigma^2}(\ln x - \mu)^2\right\},$$

then  $E(X) = \exp\left(\mu + \frac{1}{2}\sigma^2\right)$ . We let  $Y$  denote the log-transformed, normally distributed variable  $Y = \ln(X)$ , that has mean value  $\mu$  and variance  $\sigma^2$ .

This article is organized as follows. In Section 2 we propose a CAT for testing hypothesis for log-normal mean. In Section 3, we illustrate our approach using a real example. The size of our method is compared with other existing methods in Section 4.

**Methods for Testing Mean:** Assume that  $X_i, i = 1, \dots, n$  is a independent random sample from log-normal population, i.e.  $X_i \sim \text{log-normal}(\mu, \sigma^2)$ , where  $\mu$  and  $\sigma^2$  are unknown. Based on the above independent samples, the our interested test is

$$H_0 : M = M_0 \text{ vs. } H_A : M \neq (< \text{ or } >) M_0$$

where  $M = \exp(\mu + \frac{1}{2}\sigma^2)$ . Note that the above test is equivalent to the following test:

$$H_0^* : \eta^{(1)} = \eta_0^{(1)} \text{ vs. } H_A^* : \eta^{(1)} \neq (< \text{ or } >) \eta_0^{(1)}, \quad (1)$$

where  $\eta^{(1)} = \mu + \frac{1}{2}\sigma^2, \eta_0^{(1)} = \ln(M_0)$ . Now, set  $Y_i = \ln(X_i), i = 1, \dots, n$ . Then  $Y_i \sim N(\mu, \sigma^2)$ . Let  $\eta^{(1)}$  be a parameter of interest and let the nuisance parameter  $\eta^{(2)} = \sigma^2$

There are several methods for testing (1). The methods include a nave method based on transformation; a method proposed by Cox; a modified version of the Cox method; a method motivated by large-sample theory; and a method based on generalized p-value. According to simulation results the Cox method, modified Cox method and generalized p-value are better than other methods. Therefore, we consider these three methods and compare them with our proposed CAT.

**Cox Method and Modified Cox Method:** Denote the sample mean and sample variance of  $Y$  with  $\bar{Y}$  and  $S_u^2$ , respectively, where

$$\bar{Y} = \frac{1}{n} \sum_{i=1}^n Y_i, \quad S_u^2 = \frac{1}{n-1} \sum_{i=1}^n (Y_i - \bar{Y})^2.$$

An estimation for  $\eta^{(1)}$  is  $\tilde{\eta}^{(1)} = \bar{Y} + \frac{1}{2}S_u^2$  and an estimation for the variance of  $\tilde{\eta}^{(1)}$  is given by

$$\frac{S_u^2}{n} + \frac{S_u^4}{2(n-1)}.$$

Cox has suggested that a test statistic for hypothesis test (1) can be derive as [4, 5]

$$Z = \frac{\tilde{\eta}^{(1)} - \eta_0^{(1)}}{\sqrt{\frac{S^2}{n} + \frac{S^4}{2(n-1)}}}.$$

An approximate distribution for this statistics is standard normal distribution. Therefore, we can reject  $H_0^*$

if  $|Z| > z_{\alpha/2}$  where  $z_{\alpha/2}$  is the upper  $\frac{\alpha}{2}$ -level cut-off point

standard normal distribution. Also, we can use  $t$ -student, with  $n-1$  degrees of freedom. Therefore, we reject  $H_0^*$  if

$|Z| > t_{\alpha/2, n-1}$ , where  $t_{\alpha/2, n-1}$ , is the upper  $\frac{\alpha}{2}$ -level cut-off point

$t_{n-1}$ -distribution.

**Generalized P-value:** Generalized p-value can be used for inference about parameters when there is nuisance parameter. [6] suggested the following procedure for computing a generalized p-value for the log-normal mean; For a given dataset  $x_1, \dots, x_n$  set  $y_i = \ln(x_i), i = 1, \dots, n$  and calculate  $\bar{y}$  and  $s_u^2$  from the data.

For  $j = 1$  to  $m$ ,

- Generate  $Z \sim N(0, 1)$  and  $U^2 \sim \chi_{(n-1)}^2$ . Set

$$T_j = \bar{y} - \frac{\sqrt{n-1}}{U} \frac{s_u}{\sqrt{n}} Z + \frac{(n-1)s_u^2}{2U^2}.$$

- Let  $I_j = 1$  if  $T_j > \eta_0^{(1)}$ , otherwise  $I_j = 0$ . Set

$$p_m = \frac{1}{m} \sum_{j=1}^m I_j, \text{ then } 2 \min\{p_m, 1-p_m\} \text{ is a Monte Carlo}$$

estimate of the generalized p-value for testing (1).

**The Cat for Mean:** Let  $X_1, X_2, \dots, X_n$  is a random sample from density  $f(x, \theta)$ , where the functional form of  $f$  is assumed to be known. The parameter  $\theta$  is partitioned as  $\theta = (\theta^{(1)}, \theta^{(2)})$  where  $\theta^{(2)}$  if available, is the nuisance parameter and  $\theta^{(1)}$  is the parameter of interest. The methodology of the CAT for testing  $H_0 : \theta^{(1)} = \theta_0^{(1)}$ , at a desired level  $\alpha$ , is given through the following steps, (for more detail see [2]).

**Step 1:** Obtain  $\hat{\theta} = (\hat{\theta}^{(1)}, \hat{\theta}^{(2)})$ , the MLE of  $\theta$

**Step 2:**

- Assume that  $H_0$  is true, i.e. set  $H_0 : \theta^{(1)} = \theta_0^{(1)}$ . Then find the MLE of  $\theta^{(2)}$  from the data again. Call this as the 'restricted MLE of  $\theta^{(2)}$ ' under  $H_0$  denoted by  $\hat{\theta}_{RML}^{(2)}$ .

- Generate artificial sample  $X_1, X_2, \dots, X_n$ , i.i.d. from density  $f(x, \theta_0^{(1)}, \hat{\theta}_{RML}^{(2)})$  a large number of times (say,  $M$  times). For each of these replicated samples, recalculated the MLE of  $\theta = (\theta^{(1)}, \theta^{(2)})$  (pretending that  $\theta$  were unknown). Retain only the component that is relevant for  $\theta^{(1)}$ . Let these recalculated MLE values of  $\theta^{(1)}$  be  $\hat{\theta}_{01}^{(1)}, \hat{\theta}_{02}^{(1)}, \dots, \hat{\theta}_{0M}^{(1)}$ .
- Let  $\hat{\theta}_{0(1)}^{(1)} \leq \hat{\theta}_{0(2)}^{(1)} \leq \dots \leq \hat{\theta}_{0(M)}^{(1)}$  be the ordered values of  $\hat{\theta}_{0l}^{(1)}$ ,  $1 \leq l \leq M$ .

**Step 3:**

- For testing  $H_0: \theta^{(1)} = \theta_0^{(1)}$  against  $H_A: \theta^{(1)} < \theta_0^{(1)}$  (if such an alternative is meaningful) at level  $\alpha$  define  $\hat{\theta}_L^{(1)} = \hat{\theta}_{0(\alpha M)}^{(1)}$ . Reject  $H_0$  if  $\hat{\theta}^{(1)} < \hat{\theta}_L^{(1)}$ . Alternatively, calculate the p-value as:

$$p = \frac{\text{(number of } \hat{\theta}_{0(l)}^{(1)} \text{'s } < \hat{\theta}^{(1)})}{M} = \frac{1}{M} \sum_{l=1}^M I_{(\hat{\theta}_{0(l)}^{(1)} < \hat{\theta}^{(1)})}.$$

- For testing  $H_0: \theta^{(1)} = \theta_0^{(1)}$  against  $H_A: \theta^{(1)} > \theta_0^{(1)}$  at level  $\alpha$  define  $\hat{\theta}_U^{(1)} = \hat{\theta}_{0((1-\alpha)M)}^{(1)}$ . Reject  $H_0$  if  $\hat{\theta}_{ML}^{(1)} > \hat{\theta}_U^{(1)}$ . Alternatively, calculate the p-value as:

$$p = \frac{\text{(number of } \hat{\theta}_{0(l)}^{(1)} \text{'s } > \hat{\theta}^{(1)})}{M} = \frac{1}{M} \sum_{l=1}^M I_{(\hat{\theta}_{0(l)}^{(1)} > \hat{\theta}^{(1)})}.$$

- For testing  $H_0: \theta^{(1)} = \theta_0^{(1)}$  against  $H_0: \theta^{(1)} \neq \theta_0^{(1)}$  define the cut-off points as  $\hat{\theta}_L^{(1)} = \hat{\theta}_{0((\alpha/2)M)}^{(1)}$  and  $\hat{\theta}_U^{(1)} = \hat{\theta}_{0((1-\alpha/2)M)}^{(1)}$ . Reject  $H_0$  if  $\hat{\theta}_L^{(1)} < \hat{\theta}^{(1)} < \hat{\theta}_U^{(1)}$ . Alternatively, the p-value is computed as:  $p = 2\min(p_1, p_2)$ , where

$$p_1 = \frac{\text{(number of } \hat{\theta}_{0(l)}^{(1)} \text{'s } < \hat{\theta}^{(1)})}{M}$$

and

$$p_2 = \frac{\text{(number of } \hat{\theta}_{0(l)}^{(1)} \text{'s } > \hat{\theta}^{(1)})}{M}.$$

The following steps give the implementation of our proposed CAT for hypothesis testing of log-normal problem.

**Step 1:** Get the MLEs of the parameters as  $\hat{\eta}^{(1)} = \bar{Y} + \frac{1}{2}S_b^2, \hat{\eta}^{(2)} = S_b^2$ , where

$$\bar{Y} = \frac{1}{n} \sum_{i=1}^n Y_i, \quad S_b^2 = \frac{1}{n} \sum_{i=1}^n (Y_i - \bar{Y})^2.$$

**Step 2:**

- Assume that  $H_0^*$  is true, i.e.  $\eta^{(1)} = \eta_0^{(1)}$ . Then  $Y_i \sim N(\eta_0^{(1)} - \frac{1}{2}\eta^{(2)}, \eta^{(2)})$ , where  $\eta^{(2)}$  is unknown. The MLEs of the parameter  $\eta^{(2)} = \sigma^{(2)}$  which are called the restricted MLE' is

$$\hat{\eta}_{RML}^{(2)} = -2 + 2\sqrt{1 + \frac{1}{n} \sum_{i=1}^n (y_i - \eta_0^{(1)})^2}.$$

- Generate artificial sample  $Y_1, \dots, Y_n$  i.i.d. from  $N(\eta_0^{(1)} - \frac{1}{2}\hat{\eta}_{RML}^{(2)}, \hat{\eta}_{RML}^{(2)})$  a large number of times (say,  $M$  times). For each of these replicated samples, recalculated the MLE of  $\eta^{(1)}$ . Let these recalculated MLE values of  $\eta^{(1)}$  be  $\hat{\eta}_{01}^{(1)}, \hat{\eta}_{02}^{(1)}, \dots, \hat{\eta}_{0M}^{(1)}$  ( $\hat{\eta}_{0l}^{(1)} = \bar{Y}_l + \frac{1}{2}S_{bl}^2$ ).
- Let  $\hat{\eta}_{0(1)}^{(1)} \leq \hat{\eta}_{0(2)}^{(1)} \leq \dots \leq \hat{\eta}_{0(M)}^{(1)}$  be the ordered values of  $\hat{\eta}_{0l}^{(1)}$ ,  $1 \leq l \leq M$ .

**Step 3:** The same as Step 3 above, with the exception that  $\hat{\eta}_{0(l)}^{(1)}$ 's are used instead of  $\hat{\theta}_{0(l)}^{(1)}$  and  $\hat{\eta}^{(1)}$  is used in place of  $\hat{\theta}^{(1)}$ .

The size and power computation for log-normal is done through the following stages.

- For fixed  $n$ ,  $\eta^{(1)}$  and  $\eta^{(2)}$ , generate iid observations of size from  $N(\eta^{(1)} - \frac{1}{2}\eta^{(2)}, \eta^{(2)})$ , where  $\eta^{(1)} = \mu + \frac{1}{2}\sigma^2, \eta^{(2)} = \sigma^2$ .
- Get  $\hat{\eta}^{(1)}$  and  $\hat{\eta}^{(2)}$ .
- Set  $\eta^{(1)} = \eta_0^{(1)}$  ( $H_0^*$  value) and get the restricted MLE of  $\eta_0^{(2)}$  as  $\hat{\eta}_{RML}^{(2)}$ .

Table 1: Carbon monoxide levels at an oil refinery in California.

Date	9/11/1990	10/4/1990	12/3/1991	12/3/1991	12/1/1991
CO level	12.5	20	4	20	25
Date	8/6/1992	9/10/1992	9/22/1992	3/30/1993	
CO level	170	15	20	15	

Table 2: The actual size of tests when the nominal level is

$\mu$	Test	$n$			
		5	7	10	15
0	Cox	0.173	0.132	0.113	0.09
	Modified Cox	0.116	0.095	0.088	0.076
	generalized	56	0.051	0.052	0.048
	CAT	0.025	0.033	0.041	0.042
0.5	Cox	0.169	0.129	0.107	0.089
	Modified Cox	0.112	0.091	0.083	0.074
	generalized	0.058	0.054	0.054	0.054
	CAT	0.025	0.033	0.041	0.047
1	Cox	161	122	0.103	86
	Modified Cox	0.104	0.087	0.079	0.071
	generalized	0.058	0.053	0.051	0.047
	CAT	0.025	0.031	0.038	0.04
2	Cox	131	108	0.094	0.08
	Modified Cox	0.074	0.071	0.067	0.066
	generalized	0.057	0.052	0.052	0.05
	CAT	0.019	0.029	0.039	0.041
$\mu$	Test	20	25	30	50
0	Cox	79	77	0.067	0.06
	Modified Cox	0.067	0.066	0.059	0.054
	generalized	0.05	0.052	0.049	0.045
	CAT	0.045	0.049	0.046	0.044
0.5	Cox	79	0.078	0.069	0.06
	Modified Cox	0.069	0.067	0.061	0.055
	generalized	0.051	0.053	0.053	0.049
	CAT	0.045	0.05	0.05	0.047
1	Cox	0.076	0.072	65	0.059
	Modified Cox	0.063	0.062	0.058	0.053
	generalized	0.052	0.053	0.048	0.048
	CAT	0.047	0.049	0.046	0.045
2	Cox	66	0.065	65	0.053
	Modified Cox	0.055	0.055	0.056	0.049
	generalized	0.054	0.051	0.051	0.052
	CAT	0.048	0.045	0.047	0.05

- Now generate  $Y^{(1)} = (Y_1^{(1)}, \dots, Y_I^{(1)})$  i.i.d. from  $N(\eta_0^{(1)} - \frac{1}{2}\hat{\eta}_{RML}^{(2)}, \hat{\eta}_{RML}^{(2)})$ ,  $I = 1, \dots, M$ . Retain only the MLE values of  $\eta^{(1)}$  be  $\hat{\eta}_{01}^{(1)}, \hat{\eta}_{02}^{(1)}, \dots, \hat{\eta}_{0M}^{(1)}$ . Order these MLE values of  $\eta^{(1)}$  as  $\hat{\eta}_{0(1)}^{(1)}, \hat{\eta}_{0(2)}^{(1)}, \dots, \hat{\eta}_{0(M)}^{(1)}$ . Get  $\hat{\eta}_L^{(1)} = \hat{\eta}_{0(\alpha M)}^{(1)}$  and  $\hat{\eta}_U^{(1)} = \hat{\eta}_{0((1-\alpha)M)}^{(1)}$  (these are the lower and upper  $\alpha$  % cut-off points).
- Now bring the  $\hat{\eta}_0^{(1)}$  from the above Step-2 and get  $I = 1 - I(\hat{\eta}_L^{(1)} \leq \hat{\eta}_0^{(1)} \leq \hat{\eta}_U^{(1)})$ .
- Repeat the above Step-1 through Step-5 a large number of times (say, N times) and get the  $I$  values as  $I_1, I_2, \dots, I_N$ . Finally, the power is approximated by

Table 3: the power of tests when the nominal level is with.

$\mu$	Test	$n$			
		5	7	10	15
0	Cox	0.141	0.098	74	0.052
	Modified Cox	0.093	0.07	0.057	0.043
	generalized	0.076	0.072	0.074	0.077
	CAT	0.049	0.055	0.063	0.069
0.5	Cox	0.1	0.09	0.069	0.052
	Modified Cox	0.075	0.062	0.053	0.042
	generalized	0.079	0.077	0.079	0.085
	CAT	0.05	0.058	0.064	0.075
1	Cox	0.112	0.084	61	44
	Modified Cox	0.071	0.06	0.046	0.035
	generalized	0.089	0.089	0.094	0.103
	CAT	0.057	0.065	0.078	0.09
2	Cox	94	0.067	0.046	0.043
	Modified Cox	0.05	0.04	0.028	0.025
	generalized	0.124	0.13	0.137	0.161
	CAT	0.075	0.095	0.111	0.143
$\mu$	Test	20	25	30	50
0	Cox	0.045	0.041	0.04	0.048
	Modified Cox	0.037	0.034	0.032	0.04
	generalized	0.087	0.096	0.098	0.123
	CAT	0.08	0.089	0.09	0.118
0.5	Cox	0.039	0.038	0.041	0.059
	Modified Cox	0.032	0.03	0.033	0.051
	generalized	0.097	0.104	0.107	0.142
	CAT	0.089	0.096	0.1	0.137
1	Cox	0.038	0.039	0.047	0.076
	Modified Cox	0.029	0.03	0.035	0.066
	generalized	0.111	0.123	0.135	0.175
	CAT	0.1	0.114	0.126	0.168
2	Cox	0.058	0.082	101	0.196
	Modified Cox	0.038	0.062	0.079	0.176
	generalized	0.19	0.218	0.233	0.337
	CAT	0.168	0.201	0.216	0.32

$$\beta_{CAT} = \frac{1}{N} \sum_{i=1}^N I_i$$

**Real Example:** The data in Table 1 are nine measurements of carbon monoxide levels in the air. The measurements were made close to a California oil refinery in 1990 - 1993. We will use these data to tests the mean carbon monoxide level. Initial investigations of these data and of other similar datasets, indicate that a log-normal model may be appropriate. The data are posted at [lib.stat.cmu.edu/DASL/](http://lib.stat.cmu.edu/DASL/).

The p-values for Cox method, Modified Cox method, generalized p-value method and CAT, for testing  $H_0 : \eta^{(1)} = 3$  vs  $H_1 : \eta^{(1)} \neq 3$  are 0.2762, 0.3079, 0.1685 and 0.1878, respectively. Therefore, The four methods do not reject  $H_0$ .

**Simulation Study:** A simulation study is performed to compare size and power test of four methods; i) the Cox method, ii) the modified Cox method, iii) the generalized p-value method and iv) the CAT.

Table 4: The power of tests when the nominal level is with

$\mu$	Test	$n$			
		5	7	10	15
0	Cox	0.118	0.075	0.052	0.034
	Modified Cox	0.076	0.055	0.039	0.024
	generalized	0.104	0.107	0.116	0.133
	CAT	0.073	0.086	0.101	0.122
0.5	Cox	0.108	0.069	0.045	0.03
	Modified Cox	0.068	0.048	0.034	0.02
	generalized	0.116	0.125	0.134	0.154
	CAT	0.084	0.099	0.116	0.142
1	Cox	0.097	0.06	0.039	0.031
	Modified Cox	0.058	0.042	0.027	0.018
	generalized	0.133	0.145	0.158	0.189
	CAT	0.098	0.119	0.139	0.172
2	Cox	0.07	0.049	0.047	0.085
	Modified Cox	0.036	0.028	0.021	0.044
	generalized	0.188	0.215	0.258	0.332
	CAT	0.139	0.176	0.224	0.304
$\mu$	Test	20	25	30	50
0	Cox	0.035	0.045	0.053	0.122
	Modified Cox	0.026	0.031	0.039	0.107
	generalized	0.154	0.179	0.195	0.27
	CAT	0.145	0.168	0.185	0.263
0.5	Cox	0.039	0.056	0.07	0.163
	Modified Cox	0.025	0.039	0.051	0.145
	generalized	0.183	0.217	0.233	0.327
	CAT	0.171	0.203	0.223	0.317
1	Cox	0.048	0.077	0.1	0.231
	Modified Cox	0.029	0.053	0.078	0.208
	generalized	0.226	0.266	0.29	0.413
	CAT	0.209	0.251	0.276	0.401
2	Cox	0.139	0.216	0.283	0.521
	Modified Cox	0.094	0.166	0.242	0.494
	generalized	0.386	0.456	0.508	0.692
	CAT	0.358	0.433	0.488	0.679

For this propose we generated samples with sizes  $n=5, 7, 10, 15, 20, 25, 30, 50$  from a log-normal distribution with parameters  $\mu = 0, 0.5, 1, 2$  and  $\sigma^2 = 2(\eta^{(1)} - \mu)$  5,000 replication were used.

We consider the test  $H_0^* : \eta^{(1)} = 3$  vs.  $H_A^* : \eta^{(1)} \neq 3$ . The actual size of tests are given in Table 2 and the power of tests for  $\eta^{(1)} = 3.5, 4$  are given in Tables 3 and 4, respectively.

It can be observed from the below tables that the actual size of test of CAT method is always less than the nominal level, however, this can not be happen in the other methods. In the generalized p-value method the actual size is as good as the CAT method, but its actual size is often more than the nominal level. The other two methods are very liberal. Moreover, in all methods, the actual size will be close to the nominal level, as the sample size increases, (Table 2).

## REFERENCES

1. Pal, N., W.K. Lim and C.H. Ling, 2007. A computational approach to statistical inferences, J. Appl. Probab. Statist. 2(1): 13-35.
2. Chang, C.H. and N. Pal, 2008. A Revisit to the Behrens-Fisher Problem: Comparison of Five Test Methods, Communications in statistics-Simulation and Computation, 37: 1064-1085.
3. Chang, C.H., N. Pal, W. Lim and J.J. Lin, 2010. Comparing several population means: a parametric bootstrap method and its comparison with usual ANOVA F test as well as ANOM, Computational Statistics, 25(1): 71-95.
4. Zhou, X.H. and S. Gao, 1997. Confidence intervals for the log-normal mean, Statistics in Med., 16: 783-790.
5. Zhou, X.H., S. Gao and S.L. Hui, 1997. Methods for comparing the means of two independent log-normal samples, Biometrics, 53: 1129-1135.
6. Krishnamoorthy, K. and T. Mathew, 2003. Inferences on the means of lognormal distributions using generalized p-values and generalized confidence intervals, J. Statistical Planning and Inference, 115: 103-121.