Comparison of Empirical Equations' Application in the Advection-Dispersion Equation (ADE) on Sediment Transport Modelling

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Abstract: This paper describes the effect of applying the appropriate methods for estimating some empirical coefficients in the Advection-Dispersion Equation (ADE) on the accurate suspended sediment concentration predictions. In this study three existing empirical sediment discharge equations, four particles fall velocity formulas and three dispersion equations used in the ADE were evaluated for accurate estimation of suspended sediment concentrations. The main model used was the one dimensional FASTER model in which, the Saint Venant equations were numerically solved by the finite difference central with a staggered implicit scheme and the numerical method used for the solution of the ADE was a combination of the implicit finite volume central and ULTIMATE QUICKEST schemes. Since the source of FASTER program was available, a computer model was developed and all considered experimental equations for dispersion coefficient, sediment discharges and particles fall velocity were included to the model, enabling the user to apply any combination of these equations. The model was then applied to Karoon River located in south west of Iran for two separate periods. Comparison of the predicted suspended sediment concentrations with the corresponding measured values at the survey site showed that the Van Rijn sediment discharge model, Fischer *et al* dispersion equation and Cheng settling velocity equation performed very well in comparison with all considered experimental equations. It was found that using an appropriate experimental equation for these important parameters in the ADE can significantly increase the accuracy of estimation of suspended sediment concentrations along rivers.

Key words: Sediment transport · Advection dispersion equation · Numerical modelling · Environmental management

INTRODUCTION

One of the most important problems in rivers is sediment transport processes and its impact on environment and local habitat. Deposition of fine sediments along rivers and riverine deltas causes a restriction to navigation and exacerbate flooding problems. Moreover, sedimentation in natural and artificial channels can also create adverse environmental conditions. Reducing discharge capacity, increasing undesirable vegetation along the channels, transport of contaminated particles such as heavy metals, generating many maintenance problems all are some examples produced by sedimentation. Therefore, being able to accurately model and predict sediment processes is one of the main tasks for river engineers and environmental managers.

Numerical models provide valuable tool for predicting flow, sediment and solute transport processes and are increasingly important. Over the past four decades numerical models have been widely used in all aspects of environmental science and engineering and particularly in the field of environmental water management relating to river, estuarine and coastal waters. Accurate numerical model predictions of suspended sediment fluxes in open channel flows can assist in the planning and design of effective and economic projects for environment. For dynamically modelling suspended sediment transport processes in riverine or estuarine basins the Advection-Dispersion-Equation (ADE) must be numerically solved. Because of the complexity of sediment transport processes, all existing sediment transport formulas are empirical or semi-empirical. Large discrepancies may exist among the outputs of these formulas when they are applied in real-life engineering problems. Therefore, evaluation of their performances in various situations is very important [1].

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There are a few empirical coefficients such as dispersion coefficient, particles fall velocity and sediment discharge in the ADE with accurate estimate of these coefficients being very essential for the accuracy of suspended sediment concentration predictions. There are many empirical and/or semi-empirical equations presented by scientists in the literature to calculate these coefficients.

Considering more accurate estimation of sediment transport processes in riverine basins highlights having more researches on all parameters affecting this phenomenon. Therefore, the main aim of this paper is to answer this question: "How much the empirical coefficients in the ADE are really effective on accurate predictions of the sediment transport processes in riverine basins".

Governing Equations: The one dimensional St Venant equations are generally applied for modelling flow in riverine and narrow estuarine basins. These equations for dynamically modelling of suspended sediment concentrations have to be first numerically solved. There are many commercial models in market; however in this research study the FASTER[1] model, which was first developed by Kashefipour [2], is used as the main model. In this model the St Venant equations are solved using an implicit finite difference central with staggered scheme. It is possible to solve the St. Venant equations and the advection dispersion equation in a fully coupled scheme [3].

The ADE is applied to dynamically model the sediment and solute transport processes. The complete one dimensional form of the ADE may be written as [4]:

Table 1: Empirical equations for solving ADE and estimating suspended sediment concentrations

Parameter	Eq. No.	Equation	Reference
Dispersion coefficient	3	$D_L = 0.011\dfrac{U^2 W^2}{H U_*}$	Fischer *et al* [6]
	4	$D_L = 5.915\left[\dfrac{W}{H}\right]^{0.62}\left[\dfrac{U}{U_*}\right]^{1.428} H U_*$	Seo and Cheong [7]
	5	$D_L = \left[7.428 + 1.775\left(\dfrac{W}{H}\right)^{0.62}\left(\dfrac{U_*}{U}\right)^{0.572}\right]\left[\dfrac{U}{U_*}\right] HU$	Kashefipour and Falconer [5]
Sediment fall velocity	6	$W_S = \dfrac{D_{50}^2}{18}\left(\dfrac{S_G - 1}{\upsilon}\right) g$	Wu and Wang [9]: Stocks Equation
	7	$W_S = \dfrac{1}{18}\dfrac{(S_G - 1)g D_S^2}{\upsilon}$ for $D_{50} \langle 100 \ \mu m$ $W_S = 10\dfrac{\upsilon}{D_S}\left\{\left[1 + \dfrac{0.01(S_G - 1)g D_S^3}{\upsilon^2}\right]^{0.5} - 1\right\}$ for $100 \le D_{50} \le 1000 \ \mu m$ $W_S = 1.1\left[(S_G - 1)g D_S\right]^{0.5}$ for $D_{50} \rangle 1000 \ \mu m$	Van Rijn [10-12]
	8	$W_S = \left[\dfrac{4}{3}\dfrac{g}{C_D}(S_G - 1)D_S\right]^{0.5}$	Rubey [13]
	9	$W_S = \dfrac{\upsilon}{D_{50}}\left(\sqrt{25 + 1.2 D_*^2} - 5\right)^{1.5}$	Cheng [14]
Sediment flux	11	$q_S = 0.01\left(\dfrac{1}{S_G - 1}\right)\dfrac{\tau U}{W_S}$	Bagnold [17]
	12	$q_S = \phi_S \gamma_S D_S \sqrt{(S_G - 1)g D_S}$	Garde and Ranga Raju [18]: Samaga
Equation	13	$q_S = FUHS_a \ \ and \ \ F = \dfrac{\left[\dfrac{a}{H}\right]^{Z'} - \left[\dfrac{a}{H}\right]^{1.2}}{\left[1 - \dfrac{a}{H}\right]^{Z'}(1.2 - Z')}$	Van Rijn [10-12]

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[1]Flow And Solute Transport for Estuaries and Rivers

$$\frac{\partial CA}{\partial t} + \frac{\partial CQ}{\partial x} - \frac{\partial}{\partial x} D_L A \frac{\partial C}{\partial x} = \frac{Q_L C_L}{\delta x} + S_T \qquad (1)$$

There are many empirical equations in the literature that are able to estimate the longitudinal dispersion coefficient (D_L), which have been reviewed by Kashefipour and Falconer [5]. In current research study three of them were selected to calculate the dispersion coefficient as a part of the ADE numerical solution. These equations were proposed by Fischer *et al* [6], Seo and Cheong [7], Kashefipour and Falconer [5] and are defined in Table 1 as Equations (3) to (5).

S_T is the source term and defined as [1, 8]:

$$S_T = \gamma W W_S (C_e - C) \qquad (2)$$

There are many equations in the literature describing the sediment particles fall velocity (W_S). In current research work, four equations including: the Stocks relationship [9], Van Rijn formulas [10-12], Rubey equation [13] and Cheng equation [14] were considered to investigate the effect of particle fall velocity on suspended sediment concentration predictions and are presented in Table 1 as Equations (6) to (9), respectively. C_e in Equation (2) is the mean equilibrium concentration across the cross section and defined as [15, 16]:

$$C_e = 1.13 \frac{q_S}{q} \qquad (10)$$

Equation (2) represents the dynamic net exchange flux of suspended load (sediment deposition and re-suspension) near bed, in which the near-bed capacity formulas are applied. There are many empirical and semi-empirical equations in the literature for q_S [1]. Among them, three popular equations were selected to investigate the effect of the type of suspended sediment flux formula on the suspended sediment concentration predictions. These equations are Bagnold equation [17], Samaga method [18] and Van Rijn equation [10-12] and are presented by Equations (11) to (13) in Table 1, respectively.

MATERIALS AND METHODS

The main aim of this research is to evaluate the effects of applying empirical and/or semi-empirical equations in accurately suspended sediment concentration predictions by numerically solution of the dynamic Advection-Dispersion Equation (ADE). The main empirical and/or semi-empirical parameters used in one and/or two dimensional forms of the ADE are longitudinal dispersion coefficient (D_L), particle fall velocity (W_S) and sediment flux (q_S). There are many equations in the literature describing these parameters. In this study, three dispersion equations, four particle fall velocity equations and three suspended sediment flux equations (Table 1) were considered to evaluate the magnitude of the effectiveness of these parameters on the accuracy of suspended sediment concentration predictions. The hydrodynamic model used was FASTER and all these equations were added as a part of the ADE numerical solution. The high accurate ULTIMATE QUICKEST scheme [19, 20] was applied for the ADE numerical solution. The FASTER model was previously applied for many research studies such as faecal coliform modelling in Ribble river basin and estuary [24] and heavy metals modelling in Karoon River [21, 22].

The model was applied to Karoon River, the largest and the only navigation river, located in south west of Iran (Figure 1). A reach of 110km with 113 cross sections was considered. The maximum and minimum distances between two consecutive cross sections were about 1500 and 550 meters, respectively. In each cross section the bed elevation and the distance from the left bank of at least 50 points along the transverse direction of the flow were specified. The time series measured discharge values at the hydrometric station, Mollasani, were specified as the upper boundary and the stage-discharge relationship at the hydrometric station, Farsiat, was applied as the lower boundary for the hydrodynamic simulation. The hydrometric station, Ahwaz (Figure 1) was the only survey site along the considered reach. Like both boundaries, water elevations and discharges were measured continuously at this station. Suspended sediment concentrations were measured once within each 10 to 15 days at the survey site and boundaries. The samples were collected at three points along the cross sections and the average value of these samples were calculated and set for the suspended sediment concentration of each cross section and each time. The particle size distribution analysis of the bed material of this region showed the values of 0.002, 0.01, 0.04 and 0.05 mm for D_{16}, D_{50}, D_{84} and D_{90}, respectively.

The data including: water level, discharge and suspended sediment concentration were available for two separate periods (March-April 2003 and March-August 2004). These data shows a wide range of measured discharges (300 m³/s to 3500m³/s) and suspended sediment concentrations (90mg/lit to 1900mg/lit).

Fig. 1: Study reach with its boundaries, survey site and cross sections

Two statistical parameters including: the mean error percentage (*E*) and the coefficient *a* in Equation (14) were used for models evaluations. It is obvious a better model prediction would have less error and the coefficient *a* would be as close as possible to one with high correlation coefficient. The mean percentage error and *a* coefficient are defined as:

$$E = \frac{\sum_{i=1}^{N} \left| X_{ip} - X_{im} \right|}{\sum_{i=1}^{N} X_{im}} \times 100 \quad a = \frac{X_{ip}}{X_{im}} \qquad (14)$$

Where, X_{ip} and X_{im} are predicted and measured considered parameters respectively; *N* is number of data. The coefficient *a* would be calculated using the slope of a line drawn between X_{ip}- X_{im}.

RESULTS AND DISCUSSION

A number of model runs were carried out with the combinations of different dispersion, particles fall velocity and sediment flux formulas for the both separate periods. A total of 72 runs were done based on the different combinations made of three considered parameters. The predicted suspended sediment concentrations were then compared with the corresponding measured values using Equation 14. The comparison results are illustrated in Tables 2-4 for the first period (two months) and Tables 5-7 for the second period (six months).

According to Tables 2-7 a matrix with 72 error values due to using different formulas for dispersion coefficient, sediment flux and particles fall velocity generated and it would be possible to calculate the average error along any direction. For example, comparison of the average of all error values in each table estates the effect of dispersion

Table 2: Statistical results of different empirical formulas in the ADE for Fischer dispersion model (first period)

	Fischer *et al* Equation(1979)											
Dispersion Coefficient Fall Velocity	Stocks			Rubey			Van Rijn			Cheng		
Sediment Eq	R^2	a	%E	R^2	A	%E	R^2	a	%E	R^2	a	%E
Van Rijn	0.932	0.946	11.21	0.646	1.42	54.27	0.852	1.079	18.23	0.804	1.14	14.71
Bagnold	0.984	0.946	12.94	0.622	1.46	58.06	0.456	1.23	41.65	0.984	0.94	31.04
Samaga	0.986	0.981	11.87	0.646	1.42	54.25	0.506	1.19	38.88	0.986	0.98	27.21

Table 3: Statistical results of different empirical formulas in the ADE for Seo and Cheong dispersion model (first period)

	Seo and Cheong Equation (1998)											
Dispersion Coefficient Fall Velocity	Stocks			Rubey			Van Rijn			Cheng		
Sediment Eq	R^2	a	%E	R^2	a	%E	R^2	a	%E	R^2	a	%E
Van Rijn	0.826	0.328	66.46	0.438	0.381	60.75	0.444	1.13	42.65	0.586	1.47	61.37
Bagnold	0.190	1.04	71.33	1.45	1.56	118.53	0.357	0.974	76.76	0.490	1.40	56.45
Samaga	0.441	0.343	65.88	0.442	0.342	65.88	0.100	0.921	58.95	0.444	1.12	61.37

Table 4: Statistical results of different empirical formulas in the ADE for Kashefipour and Falconer dispersion model (first period)

	Kashefipour and Falconer Equation (2002)											
Dispersion Coefficient Fall Velocity	Stocks			Rubey			Van Rijn			Cheng		
Sediment Eq	R^2	a	%E	R^2	a	%E	R^2	a	%E	R^2	a	%E
Van Rijn	0.769	0.322	67.17	0.396	0.362	62.62	0.298	1.061	49.41	0.553	1.54	67.04
Bagnold	0.184	1.098	77.22	1.345	1.65	131.38	0.486	0.90	66.93	0.287	0.83	69.65
Samaga	0.399	0.321	68.15	0.399	0.321	68.15	0.564	1.203	37.48	0.554	1.53	67.04

Table 5: Statistical results of different empirical formulas in the ADE for Fischer dispersion model   (second period)

	Fischer *et al* Equation (1979)											
Dispersion Coefficient Fall Velocity	Stocks			Rubey			Van Rijn			Cheng		
Sediment Eq	R^2	a	%E	R^2	A	%E	R^2	a	%E	R^2	a	%E
Van Rijn	0.948	0.753	17.57	0.960	0.809	13.09	0.961	0.833	13.12	0.986	0.993	12.86
Bagnold	0.954	1.021	25.79	0.986	0.994	37.21	0.932	1.014	30.36	0.980	1.030	87.09
Samaga	0.975	0.984	15.36	0.975	0.984	15.36	0.975	0.984	15.42	0.985	0.981	20.72

Table 6: Statistical results of different empirical formulas in the ADE for Seo and Cheong dispersion model (second period)

	Seo and Cheong Equation (1998)											
Dispersion Coefficient Fall Velocity	Stocks			Rubey			Van Rijn			Cheng		
Sediment Eq	R^2	a	%E	R^2	a	%E	R^2	a	%E	R^2	a	%E
Van Rijn	0.293	0.086	76.86	0.293	0.086	76.86	0.612	0.106	75.95	0.974	0.990	15.31
Bagnold	0.184	0.877	366.6	0.184	0.626	266.1	0.206	0.367	163.2	0.048	0.246	250.3
Samaga	0.928	0.014	99.09	0.928	0.014	99.09	0.931	0.036	97.70	0.974	0.990	15.13

Table 7: Statistical results of different empirical formulas in the ADE for Kashefipour and Falconer dispersion model (second period)

Dispersion Coefficient	Kashefipour and Falconer Equation (2002)											
	Stocks			Rubey			Van Rijn			Cheng		
Fall Velocity												
Sediment Eq	R^2	a	%E	R^2	a	%E	R^2	a	%E	R^2	a	%E
Van Rijn	0.039	0.091	75.26	0.039	0.091	75.26	0.269	0.103	74.82	0.974	0.992	15.45
Bagnold	0.232	0.892	417.46	0.232	0.178	99.150	0.233	0.327	163.0	0.068	0.138	152.5
Samaga	0.925	0.007	99.53	0.925	0.007	99.53	0.925	0.024	98.52	0.974	0.992	15.60

Table 8: Average percentage error of two periods for all models considered

Dispersion Model	Ficher *et al*	Seo and Cheong	Kashefipour and Falconer	
	28.2	100.3	92.4	
Settling Velocity Model	Stocks	Rubey	VanRijn	Cheng
	91.4	80.9	64.6	57.6
Sediment Flux Model	Van Rijn	Bagnold	Samaga	
	46.6	119.6	54.8	

formula used in the ADE on suspended sediment concentration model predictions. The average values in this direction are calculated as (31.19%, 67.2% and 69.3%) for the first period and (25.28%, 133.51% and 115.5%) for the second period and for the Fischer *et al* [6], Seo and Cheong [7] and Kashefipour and Falconer [5] formulas respectively.

If the particles fall velocity formula is considered as another parameter, the average error values in this direction would be calculated as (50.3%, 74.9%, 47.9% and 50.7%) for the first period and (132.6%, 86.9%, 81.3% and 65.0%) for the second period and for the Stocks, Rubey, Van Rijn and Cheng settling velocity formulas respectively.

For sediment flux model, another parameter considered in this research study, the averages of the error values in Tables 2 to 7 were calculated for each model and for two periods. These average error percentage values were (48%, 67.7% and 52.1%) for the first period and (45.2%, 171.6% and 57.6%) for the second period and for Van Rijn, Bagnold and Samaga sediment flux models respectively.

The average errors of the considered empirical parameter when are used in the ADE for predicting suspended sediment concentrations were calculated and summarised in Table 8. As can be seen using an appropriate model for dispersion equation may reduce the error estimate in the suspended sediment concentration values significantly. Since the sediment transport is a very complicated process and many variables affect this phenomenon and also all the empirical equations are usually derived under specific conditions, this wide range of error estimate occurs in suspended sediment concentration predictions. Fischer *et al* [6] model was found to be the best formula for estimating dispersion coefficient for Karoon River and for modelling suspended sediment concentrations with an average error of 28%. This table shows that the Cheng particles fall velocity model performed better than the other models considered and could increase the accuracy of suspended sediment concentration predictions of about 34%.

A considerable difference in accuracy of the suspended sediment concentration values is seen when different flux models are used as a part of the ADE solution. The Van Rijn sediment flux model was found to have better performance than the other two considered methods (Table 8).

According to the above results it seems that the best combination of the empirical and/or semi-empirical equations used in the ADE for Karoon River is Fischer formula (D_L), Cheng method (W_S) and Van Rijn model (q_S). Using this combination leads to an average error percentage about 13.8% for two periods, which is the lowest value between all combinations of these methods. For this combination the a value (Equation 14) in Tables 2 and 7 is very close to unity with high correlation coefficients for both considered periods. Comparisons of the predicted suspended sediment concentrations with the corresponding measured values for this combination during two considered periods are shown in Figures 2 and 3. The measured and predicted values for this combination of empirical coefficients in the ADE are also
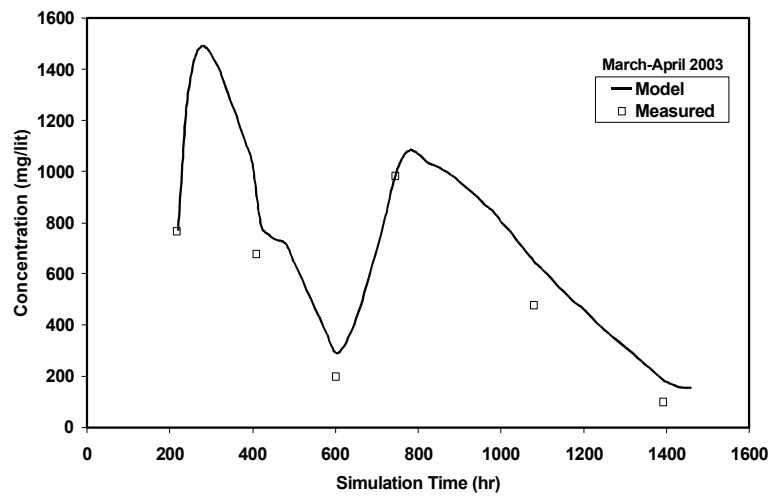
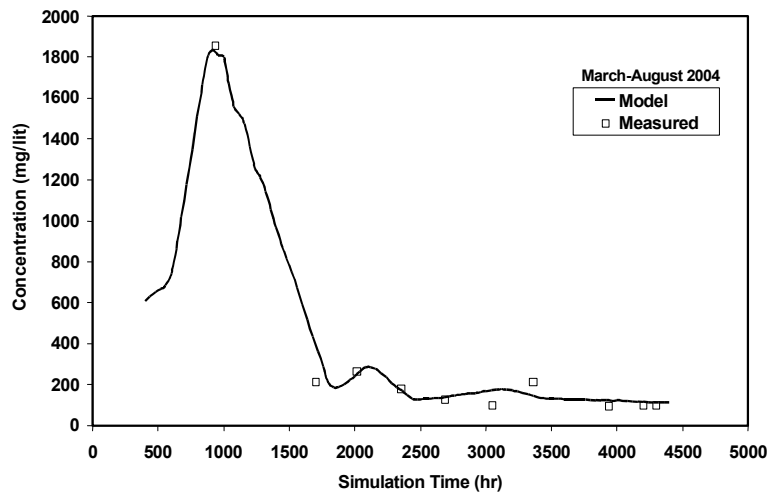Fig. 2: Predicted and measured suspended sediment concentrations for the first period

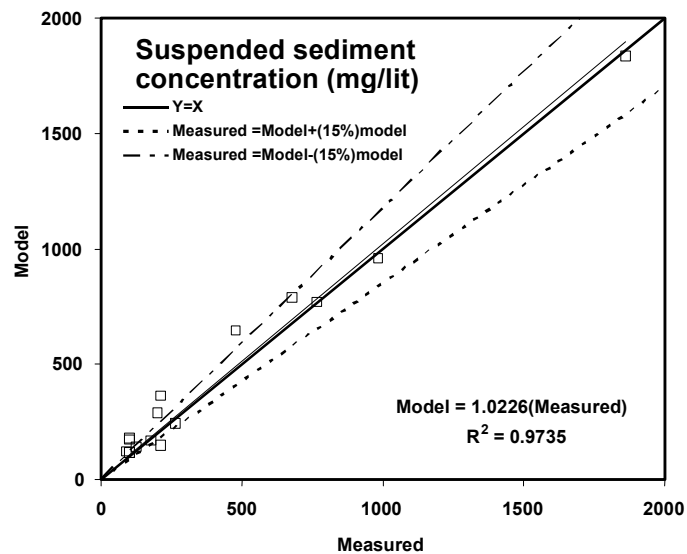Fig. 3: Predicted and measured suspended sediment concentrations for the second period

Fig. 4: Comparison of the measured and predicted suspended sediment concentrations

compared together in Figure 4 for all existing data during both periods. As can be seen from this figure a good agreement was obtained between both the measured and predicted suspended sediment concentration values with the *a* value being about 1.023 with a correlation coefficient of 0.974.

The main point of this research study was to show how the empirical and/or semi-empirical equations used in the ADE are important in the accuracy prediction of suspended sediment concentrations and the measured data in Karoon River were presented here only as an example. As mentioned above the error estimate can be significantly reduced when the appropriate equations are used to estimate dispersion coefficient, particles fall velocity and sediment flux, which are the main parameters in the numerical solution of the ADE. This significant effect of some parameters on accuracy prediction of numerical solution of the ADE brings in mind that this amount of effectiveness would be even more than what usually occurs for the type of applied numerical scheme. Many partial differential equations have one or more empirical coefficients and generally the more attention is focused on the numerical scheme to increase the accuracy predictions and less to the other parameters involved in numerical solutions. Therefore, choosing a proper equation (s) to describe such parameters could significantly increase the accuracy prediction of any dependent variable. For example, a precise inspection of Tables 2-8 shows that the Bagnold method has had the maximum errors for almost all cases and this result confirms that this sediment flux equation would not be a suitable method for applying to such large rivers like Karoon River. Almost the same comparison but in another way was carried out by Alonso [23] and he has reported the same conclusion that the Bagnold sediment transport formula performed very weak for the field data. According to the literature the Bagnold equation was derived using the laboratory measurements and might not be very effective for large rivers.

CONCLUSIONS

The main aim of this paper was to evaluate the effectiveness of applying different existing empirical and/or semi-empirical equations describing the coefficients in the ADE, such as dispersion coefficient, settling velocity and sediment flux. Three, four and three different existing popular methods and formulas were used to estimate dispersion coefficient, particles fall velocity and sediment flux, respectively. The existing measured suspended sediment concentration data for two separate periods in Karoon River, the largest river in Iran, were used as a case study for this evaluation. Totally 72 runs using different combinations from the considered empirical and semi-empirical equations were carried out. The main conclusions drawn from this research study are:

- It was found that using the appropriate empirical equations to estimate the most important parameters in the ADE including: dispersion coefficient, particles fall velocity and sediment flux, significantly increases the accuracy prediction of suspended sediment concentrations.
- The best equations to predict suspended sediment concentrations in Karoon River were Fischer *et al*, Cheng and van Rijn equations for dispersion coefficient, settling velocity and sediment flux respectively, with the average error in concentration predictions being calculated about 13.5%, which shows a very good agreement between the model results and measured data.
- Comparison of the predicted suspended sediment concentrations with the corresponding measured values showed that the most predicted values were obtained within a range of measured values ± %15 (Figure 4).

ACKNOWLEDGEMENTS

LIST OF SYMBOLS

C=suspended sediment concentration;
A=flow cross section;
Q=discharge;
Q_L= lateral inflow or outflow discharge;
C_L=lateral suspended sediment concentration;
D_L=longitudinal dispersion coefficient;
x=exponent x in flow direction;
t= time;
S_T=source term;
U=cross sectional average velocity;
H=flow depth;
W=channel width;
U_*=bed shear velocity;

γ=adaptation coefficient;

C_e=mean equilibrium concentration across the cross section;

W_S=sediment particles fall velocity;

D_{50}=a diameter with %50 of particles finer;

S_G=specific gravity $(1-\gamma_S/\gamma)$;

υ= kinematics viscosity;

g= acceleration due to gravity;

D_S= representative particle diameter of the suspended sediment particles;

C_D=drag coefficient;

R_e=Reynolds Number;

D_*=dimensionless particle parameter;

q= discharge per unit width;

q_s= suspended sediment flux per unit width and ($q_S = \int_a^H s u dz$);

a=reference depth;

s, u=sediment particle and flow velocity distributions across the flow depth respectively;

τ= shear stress;

$\phi_S K_S L_S = f(\tau_*)$ and $\tau_* = \tau / (\gamma_S - \gamma) D_S$;

K_S, L_S= two coefficients;

S_a= sediment concentration at the reference depth a;

$z\grave{}$= modified form of Rouse parameter;

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