

Step Effects Investigation on the Flow Regime and Cavitation in Stepped Morning Glory Spillways

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Abstract: Stepped morning glory spillways are circle sections generally used for discharging floods of earth dams. These spillways passes more flow discharges through themselves in comparison with smooth spillways. Therefore, the risk of flow pressure decrease to less than fluid vapor pressure called cavitation should be prevented as far as possible. At this study, it has been tried to study cavitation from the view point of the effects of flow regime changes on spillway, changes of step dimensions and the change of number of steps. Therefore, five spillway models (one smooth spillway and four stepped spillways) have been used to assess the cavitation risk. With regard to the inlet discharge to spillway, the parameters of pressure and flow velocity on spillway surface have been measures at several points and after fulfillment of tests, the cavitation risk in spillways in comparison with the dimensionless parameter of Froude number at different points of the spillway, height to width of the step (h/b), number of steps as well as the distance from the beginning of the spillway have been measured theoretically and in the form of a diagram for all the five spillway types. Results indicated that the best type of spillway in regard to design and resistance against cavitation risk and concrete erosion is the second type (eleven-stepped spillway) with regression index over 99%. Besides, some equations have been developed for designing the steps dimensions based upon flow regime for the second type spillway using regression analysis.

Key words: Morning Glory Spillway • Cavitation • Flow Regime

INTRODUCTION

Designing stepped canals goes back to 3500 years ago and the Greek were the first people who designed them. Water flow loses a part of its kinetic energy while passing the steps and as a result, the flow velocity is decreased and aeration is increased in the spillway [1]. Energy loss in stepped spillways is a key factor for minimizing erosion potentiality of the flow in their downstream. The steps can significantly decrease the energy loss resulted from chute and eliminate the need to establishment of energy loss system in structure's downstream or decrease it significantly [1]. The flow on stepped spillways occurs in two skimming and nappe forms. In high discharges, we have skimming flow and in low and intermediate discharges nappe flow. Water flow

on a stepped or unsmooth surface in earth dam spillways is completely turbulent and makes small bubbles and their development [2]. Such flow may depreciate a major part of its energy. Therefore, the more is the lost energy the less is the risk of cavitation due to intense fall of velocity. In this study, the cavitation risk is measured in spillways in regard to three dimensionless parameters of Froude number at several points of the spillway surface, the h/b ratio for each step and number of steps for five different types of spillway and finally, the best type of spillway in regard to resistance against cavitation risk and concrete erosion on spillway surface is to be determined. Therefore, where the flow is supercritical (Froude number is greater than one) the cavitation risk is high. We try at this study to find out to what distance from the beginning of the spillway would the durance and resistance of sub

critical flow (Froude number is smaller than one) continue. Sub critical flow is considered a safe flow due to the fact that the flow velocity is low in this case and the less is the flow velocity the less would be the cavitation risk. All over the world, only one physical model of stepped morning glory spillway has been examined so far (1945). The tests conducted on this model in England have indicated that the discharge capacity of the modeled stepped morning glory spillway is more than that of ladybower smooth spillway [1]. However, no analysis or study has been performed on cavitation on such spillway model. At northeast of Australia, also, some small morning glory spillways have had a stepped level at their downstream, but no modeling test has been conducted on them [1]. Falvey (1990) offered the cavitation number as the formula below empirically for sloped steps of the chutes with $L_c/H > 5$:

$$(1)\sigma_i = 1.8 \left(\frac{L_c}{H} \right)^{-u.7} \quad (1)$$

In formula 1, L_c and H respectively indicate the horizontal distance and height of the loped step [3]. Hazzab and *et al.* (2006) have studied several hydraulic parameters including critical depth and specific energy for stepped spillways and presented equations in this regard [4]. Egemen and *et al.* (2009) have studied aeration on two types of smooth and stepped spillways and finally concluded that the mixture of water and air transferred in skimming flow regime is more in stepped spillway than in smooth spillway [5]. In another study, Barani and *et al.* (2005) studied the energy loss on stepped spillway at different slopes and indicated that the spillways with bigger steps and more discharge have more effect on energy loss [6]. Number of stepped spillways of chute type in the world is very high and many modeling studies have also been conducted on them. One of the most reliable documents presented in this case is Dr. Chanson's book entitled "Hydraulic of Stepped Chutes and Spillways". But the present study includes spillways with circle sections called morning glory spillways; therefore, the information related to stepped spillways of chute type has no application in this study in this regard and the said information is not comparable with the information resulted from the present study. The flow process in morning glory spillways is different from chutes due to the following reasons: A) chute spillways have smooth and linear concrete surface, while morning glory spillways have completely curved surfaces and the flow regime is

totally different on them, B) chute spillways are rectangular from the top view, while upper section of morning glory spillways is circle, C) because of circle section of morning glory spillways, the flow in such spillways is under more compression in comparison with chute spillways.

Dimensional Analysis: Cavitation parameter is empirically a function of Euler dimensionless number. Where cavitation and fluid pressure enter dimensional analysis calculations, not only they make the results more complicated but also bring us far from our main objective. Therefore, the parameters effective in flow regime and energy loss of the step are to be analyzed. Significant and effective parameters may include the velocity of flow on spillway surface (v), fluid dynamic viscosity (μ), spillway diameter (D_s), the ground gravity acceleration (g), fluid density (ρ), step width (b), height of each step (h) and number of steps (N). The equation which indicates the mentioned parameters is written as below:

$$F(v, \mu, g, \rho, b, h, D_s, N) = 0 \quad (2)$$

In accordance with Buckingham method, eight variables with three dimensions M , L and T are available. If the number of variables is deducted from the number of dimensions, the number of dimensionless equations would be achieved. In this article, five dimensionless equations are developed considering the three variables v , ρ and D_s as repeated variable:

$$\Pi_1 = (v, \rho, D_s, g) = \frac{g^{?} D_s^2}{v^2} = \frac{1}{Fr^2} \quad (3)$$

$$\Pi_2 = (v, \rho, D_s, g) = \frac{\mu}{\rho v D_s} = \frac{1}{Re} \quad (4)$$

$$\Pi_3 = (v, \rho, D_s, b) = \frac{b}{D_s} \quad (5)$$

$$\Pi_3 = (v, \rho, D_s, h) = \frac{h}{D_s} \quad (6)$$

the first and second dimensionless equations are respectively inverses of Froude and Reynolds numbers. Using multiplication or division of the two dimensionless equations a new dimensionless equation can be made; therefore, by division of the third and the forth dimensionless equations we will have:

$$\Pi_5 = \Pi_4 \div \Pi_3 = \frac{h}{D_s} \div \frac{b}{D_s} = \frac{h}{b} \quad (7)$$

$$\Pi_6 = N \quad (8)$$

We have four dimensionless equations at present (equations 1, 2, 5 and 6). However, since the flow in spillways is free and the shear stress is very small near surface, the effect of dynamic viscosity is very little and ignorable ($\mu \approx 0$). In this case, the dimensionless equation number 2 is deleted and only the first, fifth and sixth dimensionless equations are used and analyzed. The sixth dimensionless equation indicated the number of steps.

MATERIALS AND METHODS

This study has been inspired by the physical model of San Luis Forebay dam spillway located at the central valley of California, America. This model, the dimensions of which have been presented in figure (1, 2), is constituted of a 1000-liter reservoir in upstream (including the body of dam, spillway and water canal), a right angle, a tunnel for transferring the spillway's water to downstream, a 2000-liter reservoir in downstream of the water transfer tunnel and a pump for water suction from the downstream reservoir to the upstream one. In this experimental model, the spillway body including five types of spillways with completely different designs is devised in the upstream reservoir (figures 3 to 7). The surface arc on two sides of the body of all spillways follows a same equation. Besides, dimensions of all spillways are the same but the internal surface of each spillway is different from the other. The first type spillway has a smooth surface and the spillways of the second to the fifth type respectively have eleven-step, six-step, four-step and three-step surfaces. The height of each step is h and the width of each step is b . For the smooth spillway it has been supposed that the height and width of each step is very small and same to each other. For the spillways of the second to the fifth type, the height of each step is continuously changing and width of each step is fixed and respectively equal to one, two, three and four centimeters for each spillway. In the eleven-step, six-step, four-step and three-step spillways, one, two, three and four holes are made respectively on a specific section on each step for calculating water height equivalent to pressure. In smooth spillway, location of holes is considered the same as that of the holes of eleven-step spillway; therefore, the sum total of holes in spillways of type one to five is respectively 11, 11, 10, 9 and 9 in this regard. Number of holes indicates the number of Froude number and h/b ratios we require in order to be able to compare the spillways' surfaces with each other.

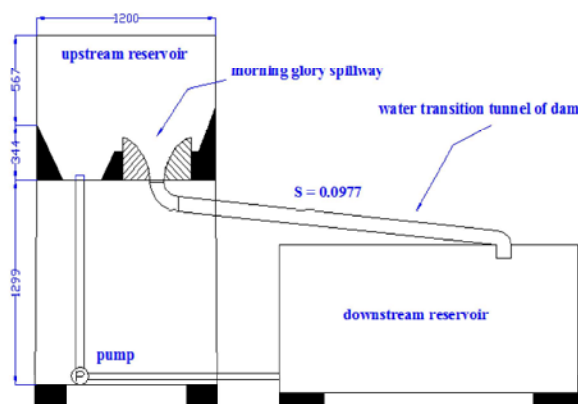


Fig. 1: Cross section of the physical model (dimensions based on millimeter)

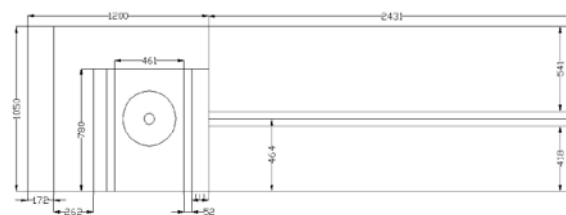


Fig. 2: Top view of the physical model (dimensions based on millimeter)

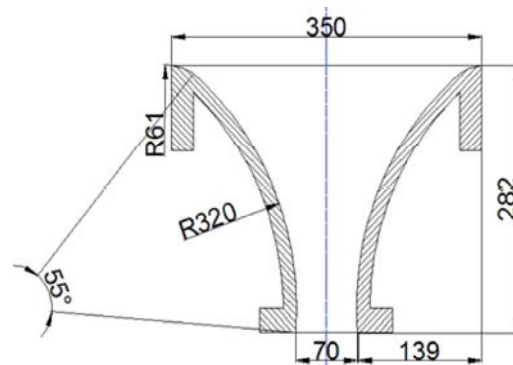


Fig. 3: model of smooth spillway

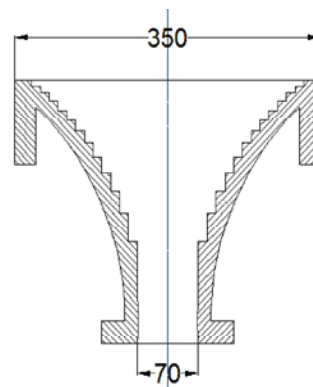


Fig. 4: model of eleven-step spillway

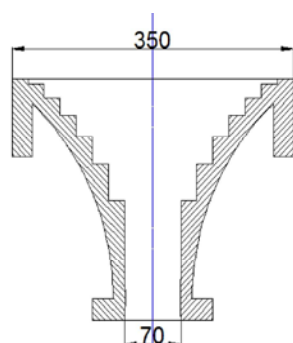


Fig. 5: model of six-step spillway

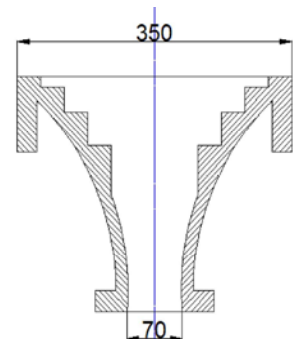


Fig. 6: model of four-step spillway

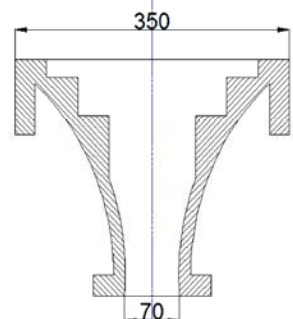


Fig. 7: model of three-step spillway
(Dimensions of all spillways are based on millimeter)

It is true that the number of holes of all of the spillways should be equal to each other, but due to long distance of the route, the CNC machine can not make holes in the ending steps from the third spillway on; therefore, only the information related to the available points are compared with each other at this experiment.

To determine the flow regime (Froude number) at surface of each spillway some holes are made in the spillway body with specific distances from the beginning of each spillway. The role of each hole is to measure the water height equivalent to fluid pressure at that specific point using Piezometer pipe. (It is to be mentioned that Piezometer pipe is the most accurate fluid pressure

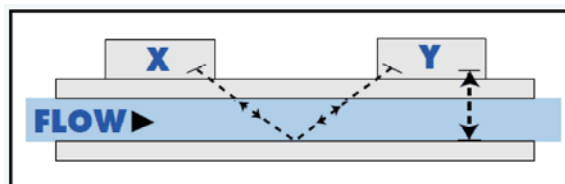


Fig. 8: Location of PF300 sensors

measurement instrument). Afterwards, energy equation is established between every two points on spillway surface according to Bernoulli principle regardless of friction loss (equation 9).

Supposing that the flow velocity on the first step is equal to the velocity of the flow entering the spillway, the flow velocity can be calculated from step two on having available the difference of height equivalent to fluid pressure. If the flow velocity at each point is specified, the Froude number related to that point can be calculated using formula 10. Besides, to measure the velocity and inlet flow discharge for each spillway, PF300 flow analyzer has been used. This system called portable ultrasonic flow meter is capable of measuring 50000 samples in every five seconds. The measurement error accuracy of this flow meter is between ± 1 and ± 3 percent. Figure (8) indicates the location of the sensors of this tool on the pipe. By transmission of supersonic wave in the pipe and contact of two waves with each other, the flow velocity can be achieved using wave velocity equations and then it is multiplied in the area of the completely filled pipe and the flow discharge is achieved hence. In this study, from among the twelve measured inlet discharges of the spillway, three discharges have been selected within the limits of 26-440 L/min as representatives of low, intermediate and high flows which are respectively equal to 172.2, 196.2 and 331.2 liter/minute.

$$\frac{v_1^2}{2g} + \frac{P_1}{\gamma} + z_1 = \frac{v_2^2}{2g} + \frac{P_2}{\gamma} + z_2 + h_f \quad (9)$$

$$Fr = \frac{v^2}{gD} \quad (10)$$

$$D = 4R = \frac{4A}{P} = D_s \text{ Spillway bigger diameter} \quad (11)$$

RESULTS AND DISCUSSION

Tables 1 to 3 represents the experimental results of five physical models of morning glory spillways. At this table, spillway type (ST), step number (S.N), distance from the beginning of the spillway in meter (D), the fixed value

Table 1: Information on location of flow regime changes on spillways ($Q_2=172.2$)

$Q_1=172.2$					
ST	S.N	D	h/b	FR	ROF
1	9	0.08	1	1.17	SC
2	9	0.08	1.93	1.02	SC
3	4	0.05	1.242	1.14	SC
4	4	0.067	1.315	1.02	SC
5	2	0.042	0.550	1.07	SC

Table 2: Information on location of flow regime changes on spillways ($Q_2=196.2$)

$Q_2=196.2$					
ST	S.N	D	h/b	FR	ROF
1	9	0.08	1	1.20	SC
2	9	0.08	1.93	1.06	SC
3	5	0.08	1.593	1.19	SC
4	3	0.045	0.984	1.11	SC
5	2	0.042	0.550	1.64	SC

Table 3: Information on location of flow regime changes on spillways ($Q_3=331.2$)

$Q_3=331.2$					
ST	S.N	D	h/b	FR	ROF
1	6	0.05	1	1.02	SC
2	7	0.06	1.461	1.02	SC
3	4	0.06	1.242	1.11	SC
4	3	0.037	0.984	1.02	SC
5	2	0.042	0.550	1.16	SC

of step height/step width (h/b), Froude number (FR) and supercritical regime of flow (SC) are respectively given from left to right for three types of discharge 172.2, 196.2 and 331.2 L/min. According to the information relate to flow velocity gained from the physical models, the limitations of regime of flow in discharges $Q_1 = 172.2$, $Q_2 = 196.2$ and $Q_3 = 331.2$ on each step is specified base on Froude number bigger or smaller than one for all five types of spillway. If the Froude number is smaller than one, the flow is sub critical and if it is bigger than one, the flow is supercritical. Due to significance of the flow regime changes from sub critical to supercritical in dams spillways, the flow regime changes on the considered step of all spillways in all three discharges are according to the data given in tables 1 to 3.

Discussion on Smooth Spillway: in such a spillway, the flow in low and intermediate discharges is sub critical and has nappe regime to the 8th step and from the ninth point on, the flow regime is smooth and supercritical in a 0.08

meter (8 centimeters) distance from the beginning of the spillway. Observations indicate that the cavitation risk does not threaten the spillway until the 8th point and the spillway is exposed to cavitation risk from the ninth point on. Due to the fact that roughness of smooth spillways is very little, the values of h and b are very small and considered to be almost equal to each other. For the same purpose, their ratio for the points on which the flow is supercritical and which are presented in tables (1 to 3) is always fixed and equal to one. Since the effect of step is a direct function of h/b ratio, the flow regime (the dimensionless Froude number) of the spillway can be put against the dimensionless h/b ratio (in comparison with different points of spillway at same distances from each other) which is shown in figure 9. In high discharge, the regime is sub critical and the flow is nappe (therefore, the spillway is not exposed to any threat) up to the sixth step and from the seventh step on, the flow is supercritical and smooth at a 0.05 meter (5 centimeter) distance from the beginning of the spillway (therefore, the spillway is exposed to cavitation risk).

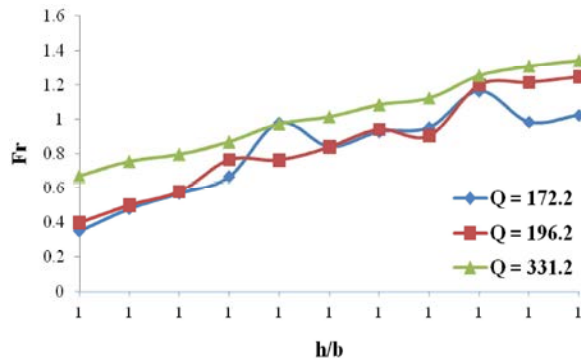


Fig. 9: Effect of step on flow regime in a smooth spillway

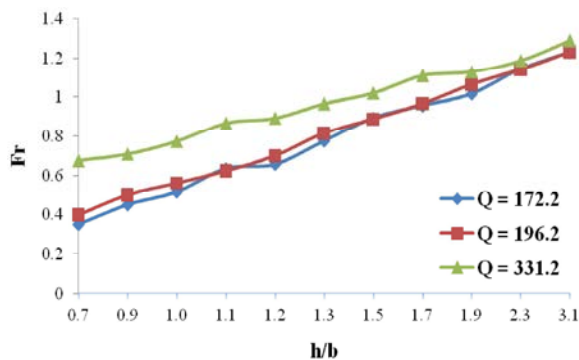


Fig. 10: Effect of step on flow regime in an eleven-step spillway

Discussion on Eleven-Step Spillway: in such a spillway, the flow in low and intermediate discharges is sub critical and has nappe regime to the 8th step and from the ninth point on, the flow regime is smooth and supercritical in a 0.08 meter (8 centimeters) distance from the beginning of the spillway. Observations indicate that the cavitation risk does not threaten the spillway until the 8th point and the spillway is exposed to cavitation risk from the ninth point on. In this spillway, h and b do not have equal values; therefore, their ratio is not the same for each step and is greater than one. Since the effect of step is a direct function of h/b ratio, the flow regime (the dimensionless Froude number) of the spillway can be put against the dimensionless h/b ratio (in comparison with different points of spillway at same distances from each other) which is shown in figure 10. In high discharge, the regime is sub critical and the flow is nappe (therefore, the spillway is not exposed to any threat) up to the sixth step and from the seventh step on, the flow is supercritical and smooth at a 0.06 meter (6 centimeter) distance from the beginning of the spillway (therefore, the spillway is exposed to cavitation risk).

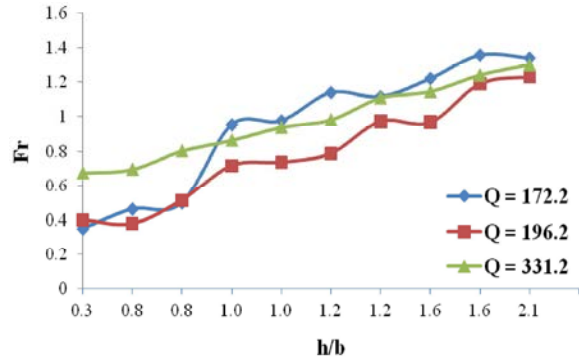


Fig. 11: Effect of step on flow regime in a six-step spillway

Discussion on Six-step Spillway: in such a spillway, the flow in low and intermediate discharges is sub critical and has nappe regime respectively at a 5 and 8 centimeters distance from the beginning of the spillway until the third and forth steps and nothing threatens the spillway. From the forth and fifth steps on, the flow regime is supercritical and smooth and the spillway is threatened by cavitation risk. In this spillway, h and b do not have equal values; therefore, their ratio is not the same for each step and is greater than one. Since effect of step is a direct function of h/b ratio, the flow regime (the dimensionless Froude number) of the spillway can be put against the dimensionless h/b ratio (in comparison with different points of spillway at same distances from each other) which is shown in figure 11. In high discharge, the regime is sub critical and the flow is nappe (therefore, the spillway is not exposed to any threat) up to the third step and from the forth step on, the flow is supercritical and smooth at a 0.06 meter (6 centimeter) distance from the beginning of the spillway (therefore, the spillway is exposed to cavitation risk).

Discussion on Four-step Spillway: in such a spillway, the flow in low, intermediate and high discharges is sub critical and has nappe regime respectively at a 6.75, 4.5 and 3.75 centimeters distance from the beginning of the spillway until the third, second and second steps (therefore, nothing threatens the spillway). Respectively from the forth, third and third steps on, the flow regime is supercritical and smooth and the spillway is threatened by cavitation risk. In this spillway, h and b do not have equal values; therefore, their ratio is not the same for each step and is greater or smaller than one. Since effect of step is a direct function of h/b ratio, the flow regime (the dimensionless Froude number) of the spillway can be put

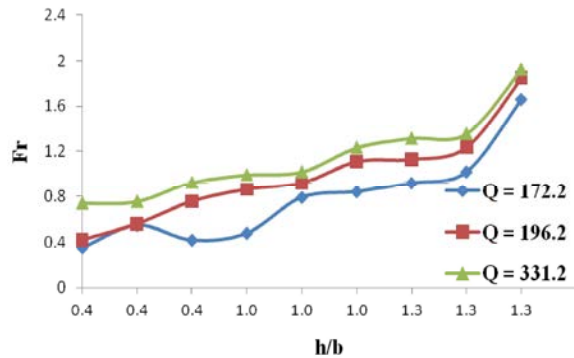


Fig. 12: Effect of step on flow regime in an four-step spillway

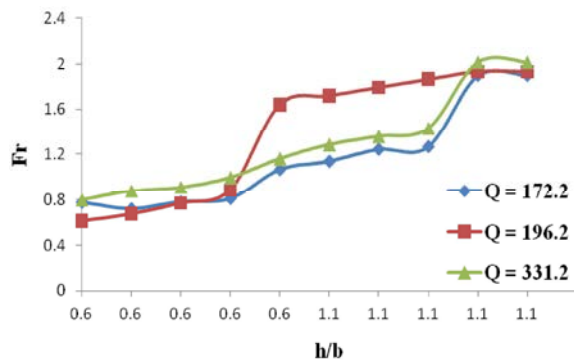


Fig. 13: Effect of step on flow regime in an three-step spillway

against the dimensionless h/b ratio (in comparison with different points of spillway at same distances from each other) which is shown in figure 12.

Discussion on Three-step Spillway: in such a spillway, the flow in low, intermediate and high discharges is sub critical and has nappe regime respectively at a 4.25 centimeters distance from the beginning of the spillway until the first step (therefore, nothing threatens the spillway). Respectively from the second step on, the flow regime is supercritical and smooth and the spillway is threatened by cavitation risk. In this spillway, h and b have equal values; therefore, their ratio is the same for each step and is smaller than one. Since effect of step is a direct function of h/b ratio, the flow regime (the dimensionless Froude number) of the spillway can be put against the dimensionless h/b ratio (in comparison with different points of spillway at same distances from each other) which is shown in figure 13.

To determine the rate of changes and fluctuations of flow regime in all five spillways in regard to h/b parameter,

Table 4: Regression between Froude Number (Fr) and (h/b) ratio for determination of flow regime fluctuations

	Q_1	Q_2	Q_3
Spillway Type	R^2	R^2	R^2
1	0.9109	0.9617	0.9929
2	0.9952	0.9982	0.9929
3	0.9521	0.9694	0.9934
4	0.8735	0.8865	0.9003
5	0.8898	0.8877	0.9178

Table 5: Design equations of step dimensions based on flow regime in different discharges in the eleven stepped spillway

Stepped Eleven Spillway	Equations
Q_1	$+ 0.2779\left(\frac{h}{b}\right) + 0.08\left(\frac{h}{b}\right)^2 = 0.0006Fr$
Q_2	$+ 0.3323\left(\frac{h}{b}\right) + 0.0732\left(\frac{h}{b}\right)^2 = 0.0008Fr$
Q_3	$+ 0.6072\left(\frac{h}{b}\right) + 0.0579\left(\frac{h}{b}\right)^2 = 0.0002Fr$

the results of regression analysis have been presented for each diagram at table 4. The results indicated that the eleven-step spillway has the least changes and fluctuations in the flow regime in all flow discharge limitations with regression index higher than 99%. The less is the rate of changes and fluctuations of flow regime (Froude number), the flow on the spillway surface remains the same for a longer time; therefore, the flow remains sub critical for a longer period and it leads to delay of the change from sub critical to supercritical and as a result the cavitation occurs in a longer distance from the beginning of the spillway. In the same regard, the step design equations have been presented at the table 5 for eleven-step spillway based on flow regime using regression analysis and they can be used in designing stepped spillways.

CONCLUSION

From the discussions 1 to 5 it can be concluded that the more is the distance of the steps on which the flow is supercritical from the beginning of the spillway in low, intermediate and high discharges, the spillway is more resistant against cavitation risk. In fact, continuation of sub critical flow until the end of spillway indicates the low velocity of the flow in the spillway and also the low

velocity of flow on spillway surface guarantees the security of the spillway surface against concrete erosion; therefore, second type spillway is the most secure one from among the five types of spillway in regard to surface resistance against cavitation risk because its flow becomes supercritical in different discharges at further distances from the beginning of the spillway in comparison with other types of spillways. Actually, it indicates the fact that the number of steps and h/b dimensionless ratio are the most effective factors in flow regime changes in spillways. This study indicates that the more is the number of steps in morning glory spillways, the less is the cavitation risk. Besides, limitation of h/b in low (Q_1), intermediate (Q_2) and high discharges (Q_3) should be selected to be greater than 1.93. Such data can be used in designing stepped morning glory spillways. Results indicate that the higher is the flow discharge, the lower is h/b ratio and the sooner the flow becomes supercritical. Furthermore, the supercritical flow goes up to the higher steps, meaning the beginning steps of the spillway. Here, the probability of cavitation risk is increased; therefore, it is recommended that h/b ratio should be selected as high as possible and for a high h/b ratio the number of steps should also be increased as far as possible. Correct selection of the said factors minimizes the risk for stepped spillways.

In low and intermediate discharges, meaning where we have a flow lower than the orifice range (equivalent to $Q_0 = 264.65$ L/min), the first and second types of spillways have the best flow regime, while for high discharges, meaning where the flow is greater than the orifice range, the second and third types of spillways have the best flow regime. In general, the second type of spillways is selected as the best one due to its priority over other spillways in limitations of low, intermediate and high discharges and since this priority has been proved by regression analysis. The regression index over 99% represented in table 4 for eleven-step spillway indicates very little fluctuation changes of flow regime (Froude number) in this spillway in comparison with step effect (h/b parameter) at equivalent points of each hole on spillway surface. The lower is the rate of changes and fluctuations of flow regime (Froude number), the flow on spillway surface remains the same for a longer time; therefore, the flow remains sub critical for a longer period

of time and this leads to delay of the change from sub critical to supercritical and as a result the cavitation occurs in a longer distance from the beginning of the spillway. Therefore, the information related to eleven-step spillway can be used in patterning and primary design of stepped spillways in earth dams and preventing cavitation risks. At the end, it is recommended that the following three factors should be met for designing stepped morning glory spillways: A) the average value of h/b should be greater than 1.93, B) number of steps should be more than eleven and C) optimum design of step dimensions should be done using the formulae offered at table 5. the three factors above are met only if the equation of the curve of morning glory spillway wall is optimized according to these three recommendations.

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