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Modified Decomposition Method for E-W Equation

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Abstract: A relatively new algorithm which is called the modified decomposition method (MDM) is applied to find an analytical solution for Equal-width wave (E-W) equation which play an important role in fluid flows, plasma physics, electrostatic potential and applied sciences. Numerical results explicitly reveal the complete reliability and efficiency of the proposed algorithm (MDM).

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INTRODUCTION

The equal-width wave equation is of great significance in the diversified nonlinear physical problems related to physics, astrophysics, fluid flows, plasma physics, electrostatic potential and applied sciences [1-17]. Several techniques including fourth-order Runge-Kutta, least square finite element, Sink-Galerkin and Variational iteration have been employed to solve such equations analytically and numerically, see [1-3, 5-12, 17] and the reference therein. Most of these used schemes are coupled with the inbuilt deficiencies. Recently, Geijji and Jafari [4] introduced modified decomposition method (MDM) to solve a wide class [4, 13-16] of initial and boundary value problems. In the present study, we apply this reliable algorithm (MDM) to find an analytical solution of equal-width wave (E-W) equation. It is shown that the proposed method (MDM) provides the solution in a rapid convergent series with easily computable components. The proposed modified decomposition method (MDM) [4] is applied without any discretization, perturbation, transformation or restrictive assumptions and is free from round off errors, calculation of the so-called Adomian's polynomials and the identification of Lagrange multipliers. Numerical results show the complete reliability and efficiency of the proposed technique. Moreover, the proposed MDM is easier to implement and reduces the computational work while still maintaining a higher level of accuracy.

Modified Decomposition Method (MDM): Consider the following general functional equations:

$$f(x) = 0, \tag{1}$$

To convey the idea of the modified decomposition method [4], we rewrite the above equation as:

$$y = N(y) + c, \tag{2}$$

Where N is a nonlinear operator from a banach space $B \to B$ and f is a known function. We are looking for a solution of equation (1) having the series form:

$$y = \sum_{i=0}^{\infty} y_i. \tag{3}$$

The nonlinear operator N can be decomposed as

$$N\left(\sum_{i=0}^{\infty} y_i\right) = N(y_0) + \sum_{i=0}^{\infty}\left\{N\left(\sum_{j=0}^{i} y_j\right) - N\left(\sum_{j=0}^{i-1} y_j\right)\right\}. \tag{4}$$

From equations (3) and (4), equation (2) is equivalent to

$$\sum_{i=0}^{\infty} y_i = c + N(y_0) + \sum_{i=0}^{\infty}\left\{N\left(\sum_{j=0}^{i} y_j\right) - N\left(\sum_{j=0}^{i-1} y_j\right)\right\}. \tag{5}$$

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We define the following recurrence relation:

$$\begin{cases} y_0 = c, \\ y_1 = N(y_0), \\ y_{m+1} = N(y_0 + ... + y_m) - N(y_0 + ... + y_{m-1}), \quad m = 1,2,3,..., \end{cases} \tag{6}$$

then

$$(y_1 + ... + y_{m+1}) = N(y_0 + ... + y_m), \; m = 1,2,3,...,$$

and

$$y = f + \sum_{i=1}^{\infty} y_i,$$

if N is a contraction, i.e. $\|N(x) - N(y)\| \le \|x - y\|$, $0<K<1$, then

$$\|y_{m+1}\| = \|N(y_0 + ... + y_m) - N(y_0 + ... + y_{m-1}),\| \le K\|y_m\| \le K^m\|y_0\|, \quad m = 0,1,2,3,...,$$

and the series $\sum_{i=1}^{\infty} = y_i$ absolutely and uniformly converges to a solution of equation (1) [4, 13-16], which is unique, in view of the Banach fixed-point theorem.

Solution Procedure: Consider the following equal-width wave (E-W) equation

$$u_t + uu_x - u_{xxx} = 0,$$

with initial condition

$$u(x,0) = 3\sec h^2\left(\frac{x-15}{2}\right).$$

The exact solution is given by

$$u(x,t) = 3\sec h^2\left(\frac{x-15-t}{2}\right).$$

Applying the modified decomposition method (MDM), we get

$$u_{n+1}(x,t) = u_n - \int_0^t \left(u_n \frac{\partial u_n}{\partial x} - \frac{\partial^3 u_n}{\partial x^3}\right) dx.$$

Consequently, following approximants are obtained

$$u_0(x,t) = c,$$

$$u_0(x,t) = 3\sec h^2\left(\frac{x-15}{2}\right),$$

$$u_1(x,t) = Nu_0(x,t),$$

$$u_1(x,t) = 3\sec h^2\left(\frac{x-15}{2}\right) + \left(9\sec h^4\left(\frac{x-15}{2}\right)\tan h\left(\frac{x-15}{2}\right)\right)t,$$

$$u_2(x,t) = N(u_0(x,t) + u_1(x,t)) - Nu_0(x,t),$$

$$\begin{aligned} u_2(x,t) = 3\sec h^2\left(\frac{x-15}{2}\right) &+ \left(\sec h^4\left(\frac{x-15}{2}\right)\frac{135}{2}\tan h^3\left(\frac{x-15}{2}\right) - \frac{45}{2}\tan\left(\frac{x-15}{2}\right)\right)t + \\ &\sec h^6\left(\frac{x-15}{2}\right)\left(\frac{189}{4}\tan h^2\left(\frac{x-15}{2}\right) + \frac{27}{4}\right)t^2 + \\ &\sec h^8\left(\frac{x-15}{2}\right)\left(\frac{135}{2}\tan h^3\left(\frac{x-15}{2}\right) - \frac{27}{2}\tan h\left(\frac{x-15}{2}\right)\right)t^3, \end{aligned}$$

⋮.

The series solution is given by

$$\begin{aligned} u(x,t) = 3\sec h^2\left(\frac{x-15}{2}\right) &+ \left(\sec h^4\left(\frac{x-15}{2}\right)\frac{135}{2}\tan h^3\left(\frac{x-15}{2}\right) - \frac{45}{2}\tan\left(\frac{x-15}{2}\right)\right)t + \\ &\sec h^6\left(\frac{x-15}{2}\right)\left(\frac{189}{4}\tan h^2\left(\frac{x-15}{2}\right) + \frac{27}{4}\right)t^2 + \\ &\sec h^8\left(\frac{x-15}{2}\right)\left(\frac{135}{2}\tan h^3\left(\frac{x-15}{2}\right) - \frac{27}{2}\tan h\left(\frac{x-15}{2}\right)\right)t^3 + \cdots. \end{aligned}$$

Table 1: (Error Estimates)

x = 0		x = 5	x = 10	x = 15	x = 20
t	MDM	MDM	MDM	MDM	MDM
0.001	3.668 E-9	5.382 E-7	3.611 E-5	5.137 E-5	3.605 E-5
0.002	7.333 E-9	1.075 E-6	7.228 E-5	2.055 E-5	7.205 E-5
0.003	1.099 E-8	1.612 E-6	1.085 E-4	4.623 E-5	1.080 E-4
0.004	1.465 E-8	2.149 E-6	1.448 E-4	8.220 E-5	1.438 E-4
0.005	1.830 E-8	2.685 E-6	1.811 E-4	1.284 E-4	1.797 E-4
0.01	3.652 E-8	5.357 E-6	3.637 E-4	5.137 E-4	3.580 E-4

**Error = Exact solution – Series solution*

CONCLUSION

In this paper, we applied modified decomposition method (MDM) to solve equal-width wave equation. Numerical results clearly reveal the complete reliability and efficiency of the proposed algorithm (MDM).

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