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Generalized Doubt Fuzzy Ideals of BCI-algebras

Noura Omir Al-shehri

Department of Mathematics, Faculty of Sciences(Girl's)
King Abdulaziz University, P.O. Box 126238, Jeddah, Saudi Arabia
nalshehrie@kau.edu.sa

Abstract: In this paper, we introduce the concept of generalized doubt fuzzy ideals(and q-ideals) of BCI-algebras and present some fundamental properties. We characterize the generalized doubt fuzzy q-ideals of BCI-algebras by their level subsets. We also discuss doubt fuzzy q-ideals of BCI-algebras with thresholds.

Key words: Generalized doubt fuzzy ideal, $(\in, \in \vee q_m)$-doubt fuzzy ideal, $(\in, \in \vee q_m)$-doubt fuzzy q-ideal.

INTRODUCTION

Logic appears in a 'sacred' form (resp. a 'profane' form) which is dominant in proof theory (resp. model theory). The role of logic in mathematics and computer science is twofold - as a tool for applications in both areas, and a technique for laying the foundations. Non-classical logic including many-valued logic, fuzzy logic, etc., takes the advantage of the classical logic to handle information with various facets of uncertainty [1] for a generalized theory of uncertainty), such as fuzziness, randomness, and so on. Non-classical logic has become a formal and useful tool for computer science to deal with fuzzy information and uncertain information. Many-valued logic which is a great extension and development of classical logic [2] has emerged as a useful direction in non-classic logic. Among all kinds of uncertainties, incomparability is an important one which can be encountered in our life. *BCK* and *BCI*-algebras are two classes of logical algebras. They were introduced by Imai and Iseki [3, 4] and have been extensively investigated by many researchers [5, 6]. *BCI*- algebras are generalizations of *BCK*-algebras.

After the introduction of fuzzy sets by Zadeh [7], there have been a number of generalizations of this fundamental concept. A new type of fuzzy subgroup, that is, the $(\in, \in \ \vee q)$-fuzzy subgroup, was introduced in an earlier paper of Bhakat and Das [8]. In fact,the$(\in, \in \ \vee q)$-fuzzy subgroup is an important generalization of Rosenfeld's fuzzy subgroup.It is now natural to investigate similar type of generalizations of the existing fuzzy subsystems with other algebraic structures. With this objective in view, Jun [9] introduced the concept of (α, β)-fuzzy ideals of a *BCK/BCI*-algebra and investigated related results. In this paper, we introduce the concept of generalized doubt fuzzy ideals(and q-ideals) of BCI-algebras and present some fundamental properties. We characterize the generalized doubt fuzzy q-ideals of BCI-algebras by their level subsets. We also discuss doubt fuzzy q-ideals of BCI-algebras with thresholds.
The definitions and terminologies that we used in this paper are standard. For other notations, terminologies and applications, the readers are referred to [10-27].

PRELIMINARIES

In this section we cite the fundamental definitions that will be used in the sequel.

Definition 1. *An algebra $(X, *, 0)$ of type $(2, 0)$ is called a BCI- algebra if it satisfies the following axioms for all $x, y, z \in X$:*

(i) $((x * y) * (x * z)) * (z * y) = 0,$

(ii) $(x * (x * y)) * y = 0,$

(iii) $x * x = 0,$

(iv) $x * y = 0 \; and \;\; y * x = 0$ imply $x = y$

We can define a partial ordering $\leq$ on X by $x \leq y$ if and only if $x * y = 0$. If a *BCI*- algebra X satisfies $0 * x = 0$ for all $x \in X$, then we say that X is a *BCK*-algebra. In what follows, let X denote a *BCI*-algebra unless otherwise specified. A non-empty subset I of X is called an ideal of X if it satisfies (I_1) $0 \in I$, (I_2) $x * y \in I \; and \; y \in I \; imply \; x \in I$. A non-empty subset I of X is called a q-ideal of X if it satisfies $(I_1)$and (I_3) $(x * (y * z)) \in I \; and \; y \in I \;\; imply \; x * z \in I$ for all $x, y, z \in X$.

A mapping $\mu : X \rightarrow [0, 1]$ is called a *fuzzy set* in a BCI-algebra X. For any fuzzy set μ in X and any $t \in [0, 1]$, we define set

$$U(\mu; t) = \mu^t = \{x \in X \mid \mu(x) \geq t\},$$

which is called *upper t-level cut* of μ.

Definition 2. *A fuzzy set μ of X is called a fuzzy ideal of X if*

$(F_1)\, \mu(0) \geq \mu(x), \forall x \in X,$

$(F_2)\, \mu(x) \geq \min\ \{\mu(x * y), \mu(y)\}, \forall x, y \in X.$

Definition 3. *A fuzzy set μ of X is called a fuzzy q-ideal of X if it satisfies (F_1) and*

$(F_3)\ \ \mu(x * z) \geq \min\ \{\mu((x * (y * z))), \mu(y)\}$*for all* $x, y, z \in X$.

Definition 4. *A fuzzy set μ of X is called a doubt fuzzy ideal of X if*

$(DF_1)\, \mu(0) \leq \mu(x), \forall x \in X,$

$(DF_2)\, \mu(x) \leq\ \max\{A(x * y), A(y)\}, \forall x, y \in X.$

Definition 5. *A fuzzy set μ in a set X of the form*

$$\mu(y) = \begin{cases} t \in (0, 1], & \text{if } y = x \\ 0, & y \neq x \end{cases}$$

is said to be a fuzzy point with support x and value t and is denoted by x_t.

We say that a fuzzy point x_t *belong to* a fuzzy set μ and write $x_t \in \mu$, if $\mu(x) \geqslant t$. A fuzzy point x_t is *quasicoincident* with a fuzzy set μ, if $\mu(x) + t > 1$. In this case we write $x_t q\mu$.

- $x_t \in \vee q\mu$ means that $x_t \in \mu$ or $x_t q\mu$,
- $x_t \in \wedge q\mu$ means that $x_t \in \mu$ and $x_t q\mu$.

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Let m be an element of [0, 1) unless otherwise specified. By $x_t q_m \mu$, we mean $\mu(x) + t + m > 1,\ t \in (0, \frac{1-m}{2}]$. The notation $x_t \in \vee q_m \mu$ means that $x_t \in \mu$ or $x_t q_m \mu$.

Definition 6. *A fuzzy set μ of a BCI-algebra X is called an $(\in, \in \vee q_m)$- doubt fuzzy ideal of X if*

(1) $\mu(0) \leq \max\{\mu(x), \frac{1-m}{2}\}$,

(2) $\mu(x) \leq max\left\{\mu(x * y), \mu(y), \frac{1-m}{2}\right\}$

hold for all $x, y \in X$.

Example 7. *Let $X = \{0, 1, 2\}$ be a BCI-algebra with the following Cayley's table*

$*$	0	1	2
0	0	2	1
1	1	0	2
2	2	1	0

By routine computations, we see the following fuzzy doubt fuzzy ideals of BCI-algebras:

(i) *Define a fuzzy set μ in X defined by*

$$\mu(x) = \begin{cases} 0.7, & \text{if } x = 0, \\ 0.8, & \text{if } x = 1, \\ 0.4, & \text{if } x = 2, \end{cases}$$

If $m = 0.2$, then $U(\mu; t) = X$ for all $t \in (0, 0.4]$. Hence μ is an $(\in, \in \vee q_{0.2})$-doubt fuzzy ideals of BCI-algebras.

(ii) *We define a fuzzy set μ in X defined by*

$$\mu(x) = \begin{cases} 0.45, & \text{if } x = 0, \\ 0.41, & \text{if } x = 1, \\ 0.41, & \text{if } x = 2, \end{cases}$$

If $m = 0.04$, then

$$U(\mu; t) = \begin{cases} X & \text{if } t \in (0,\ 0.4], \\ \{0, 2\} & \text{if } t \in (0.4,\ 0.45] \\ \{2\} & \text{if } t \in (0.45,\ 0.48] \end{cases}$$

Since $\{2\}$ is not ideal of X, so $U(\mu; t)$ is not ideal for $t \in (0.45, 0.48]$. Hence μ is not an $(\in, \in \vee q_{0.04})$-doubt fuzzy ideals of BCI-algebras.

Proposition 8. *A fuzzy set μ in X is an $(\in, \in \vee q_m)$-doubt fuzzy ideal of X if and only if the set $U(\mu; t)$ is an ideal of X for all $\frac{1-m}{2} < t \leq 1$.*

Proof: Let μ be an $(\in, \in \vee q_m)$-doubt fuzzy ideal of X, then $\mu(0) \leq \max\{\mu(x), \frac{1-m}{2}\}$. For all $x \in U(\mu; t)$, we have $\mu(x) \leq t$, thus $\mu(0) \leq \max\{t, \frac{1-m}{2}\} = t$ and so $0 \in U(\mu; t)$. Let $x, y \in X$ be such that $x * y \in U(\mu; t)$ and $y \in U(\mu; t)$, hence $\mu(x * y) \leq t$ and $\mu(y) \leq t$.

$$\mu(x) \leq \max\left\{\mu(x * y), \mu(y), \frac{1-m}{2}\right\}$$
$$\leq \max\left\{t, \frac{1-m}{2}\right\} = t.$$

Therefore, $x \in U(\mu; t)$ which implies that $U(\mu; t)$ is an ideal of X .

Conversely, let $U(\mu; t)$ be an ideal of X, and there exists $x_0 \in X$ such that $\mu(0) > \max\left\{\mu(x_0), \frac{1-m}{2}\right\}$. Hence $\mu(0) > \mu(x_0)$. Putting $t_0 = \frac{1}{2}\{\mu(0) + \mu(x_0)\}$ then $0 \leq \mu(x_0) < t_0 < \mu(0) \leq 1$. It follows that $x_0 \in U(\mu; t_0)$, and $U(\mu; t_0) \neq \phi$. Since $U(\mu; t_0)$ is an ideal of X , thus $0 \in U(\mu; t_0)$ and so $\mu(0) \leq t_0$, a contradiction. Hence $\mu(0) \leq \max\left\{\mu(x), \frac{1-m}{2}\right\}$ for all $x \in X$. Now let there exists $x_0, y_0 \in X$ such that $\mu(x_0) > \max\left\{\mu(x_0 * y_0), \mu(y_0), \frac{1-m}{2}\right\}$. Then we have $\mu(x_0) > \max\ \{\mu(x_0 * y_0), \mu(y_0)\}$. Putting $t_0 = \frac{1}{2}\{\mu(x_0) + \max\{\mu(x_0 * y_0), \mu(y_0)\}\}$, then $t_0 < \mu(x_0)$ and $0 \leq \max\{\mu(x_0 * y_0), \mu(y_0)\} < t_0 \leq 1$. Thus $t_0 > \mu(x_0 * y_0)$ and $t_0 > \mu(y_0)$ which imply that $x_0 * y_0 \in U(\mu; t_0)$and $y_0 \in U(\mu; t_0)$. As $U(\mu; t_0)$ is an ideal of X , it follows that $x_0 \in U(\mu; t_0)$.This is a contradiction . □

Definition 9. *An $(\in, \in \vee q_m)$-doubt fuzzy ideal μ of X is called an $(\in, \in \vee q_m)$-doubt fuzzy q-ideal of X if it satisfies:*

(3) $\mu(x * z) \leq \max\left\{\mu(x * (y * z)), \mu(y), \frac{1-m}{2}\right\}$ *for all* $x, y, z \in X$.

Example 10. *Let $X = \{0, a, b, c\}$ in which is defined by*

$*$	0	a	b	c
0	0	0	0	c
a	a	0	0	c
b	b	b	0	c
c	c	c	c	0

Then $(X, *, 0)$ is a BCI-algebra. Define $\mu : X \to [0,1]$ by $\mu(0) = 0.3, \mu(a) = \mu(b) = \mu(c) = 0.6$. Routine calculations give that μ is an $(\in, \in \vee q_m)$ -doubt fuzzy q-ideal of X for $m = 0$.

The following Proposition is obvious and we omit the details.

Proposition 11. *Every doubt fuzzy q-ideal of X is an $(\in, \in \vee q_m)$-doubt fuzzy q-ideal of X .*

Remark 12. *The converse of Proposition 11 is not true in general as shown in the following example.*

Example 13. *Let $X = \{0, 1, 2, 3\}$ be a BCI-a lgebra with the following Cayley table:*

$*$	0	1	2	3
0	0	3	0	1
1	1	0	1	3
2	2	3	0	1
3	3	1	3	0

Define a fuzzy set μ in X by $\mu(0) = 0.3, \mu(1) = \mu(2) = \mu(3) = 0.4$. For m=0, by routine calculation we know that μ is an $(\in, \in \vee q_m)$-doubt fuzzy q-ideal of X, but it is not a doubt fuzzy q-ideal of X because

$$\mu(1 * 3) = 0.4 \nleq \max\{\mu(1 * (0 * 3)), \mu(0)\}$$

Theorem 14. *Let μ be an $(\in, \in \vee q_m)$-doubt fuzzy $q-$ideal of X . Then for all $\frac{1-m}{2} \leq t < 1, U(\mu; t)$ is an empty set or a q-ideal of X. Conversely, if μ is a fuzzy set of X such that $(U(\mu; t) \neq \phi)$ is a q-ideal of X for all $\frac{1-m}{2} \leq t < 1$,then μ is an $(\in, \in \vee q_m)$-doubt fuzzy q-ideal of X.*

Proof: Let μ be an $(\in, \in \vee q_m)$-doubt fuzzy q-ideal of X and $0.5 \leq t < 1$.Hence μ is an $(\in, \in \vee q_m)$-doubt fuzzy ideal of X . Then by Proposition 11 we have that $U(\mu; t)$ is an ideal of X . Let $(x * (y * z)) \in \mu^t$ and $y \in \mu_t$, then $\mu(x * (y * z)) \leq t$ and $\mu(y) \leq t$. thus $\mu(x * z) \leq \max\left\{\mu(x * (y * z)), \mu(y), \frac{1-m}{2}\right\} \leq \max\left\{t, \frac{1-m}{2}\right\} = t$, which implies that $x * z \in \mu$ and so $U(\mu; t)$ is a fuzzy q-ideal of X. Conversely, let μ be a fuzzy set of X such that $U(\mu; t)$ $(\neq \phi)$ is a q-ideal of X for all $\frac{1-m}{2} \leq t < 1$. Then from Proposition 11, we have that μ is an $(\in, \in \vee q_m)$-doubt fuzzy ideal of X. We can write

$$\begin{aligned} &\mu(x * (y * z)) \leq \max\left\{\mu(x * (y * z)), \mu(y,) \tfrac{1-m}{2}\right\} \\ &= t_0 \\ &\mu(y) \leq \max\left\{\mu(x * (y * z)), \mu(y), \tfrac{1-m}{2}\right\} \\ &= t_0 \end{aligned}$$

Hence, $x * (y * z) \in U(\mu; t_0), y \in U(\mu; t_0)$,which implies $x * z \in U(\mu; t_0)$,and so

$$\mu(x * z) \leq t_0 = \max\left\{\mu(x * (y * z)), \mu(y), \frac{1-m}{2}\right\}.$$

Then, μ is an $(\in, \in \vee q_m)$-doubt fuzzy q-ideal of X. □

Definition 15. *Let μ and ν be fuzzy subsets of a set S. The doubt Cartesian product of μ and ν is defined by*

$$(\mu \otimes \nu)(x, y) = \max\left\{\mu(x), \nu(y), \frac{1-m}{2}\right\}, \forall x, y \in S.$$

Theorem 16. *Let μ and ν be $(\in, \in \vee q_m)-$ doubt fuzzy q-ideal of a BCI-algebra X. Then $(\mu \otimes \nu)$ is an $(\in, \in \vee q_m)$-doubt fuzzy q-ideal of $X \times X$.*

Proof: Let $(x, y) \in X \times X$. Then

$$\begin{aligned} &(\mu \otimes \nu)(0,0) = \max\left\{\mu(0), \nu(0), \tfrac{1-m}{2}\right\} \\ &\leq \max\left\{\max\left\{\mu(x), \tfrac{1-m}{2}\right\}, \max\left\{\nu(y), \tfrac{1-m}{2}\right\}, \tfrac{1-m}{2}\right\} \\ &= \max\left\{\mu(x), \nu(x) \tfrac{1-m}{2}\right\} = (\mu \otimes \nu)(x * y). \end{aligned}$$

Now let $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in X \times X$. Then:

$$\begin{aligned} &\max\{(\mu \otimes \nu)((x_1, x_2) * ((y_1, y_2) * (z_1, z_2))) \\ &, (\mu \otimes \nu)(y_1, y_2), \tfrac{1-m}{2}\} \\ &= \max\{(\mu \otimes \nu)(((x_1 * (y_1 * z_1)), (x_2 * (y_2 * z_2))), \\ &(\mu \otimes \nu)(y_1, y_2), \tfrac{1-m}{2}\} \\ &= \max\{\max\{\mu((x_1 * (y_1 * z_1)), \nu(x_2 * (y_2 * z_2))), \tfrac{1-m}{2}\}, \\ &\max\{\mu(y_1), \nu(y_2), \tfrac{1-m}{2}\}, \tfrac{1-m}{2}\} \\ &= \max\{\max\{\mu(x_1 * (y_1 * z_1)), \mu(y_1), \tfrac{1-m}{2}\}, \\ &\max\{\nu(x_2 * (y_2 * z_2)), \nu(y_2), \tfrac{1-m}{2}\}, \tfrac{1-m}{2}\} \\ &\geq \max\{\mu(x_1 * z_1), \nu(x_2 * z_2), \tfrac{1-m}{2}\} \\ &= (\mu \otimes \nu)((x_1 * z_1), (x_2 * z_2)) \\ &= (\mu \otimes V)((x_1, x_2) * (z_1 * z_2)). \end{aligned}$$

This complete the proof. □

Definition 17. *Let $\alpha, \beta \in [0,1]$ and $\alpha < \beta$. Then a fuzzy set μ of X is called a doubt fuzzy q-ideal with thresholds$(\alpha, \beta)$of X if it satisfies:*

(4) $\min\{\mu(0), \beta\} \leq \max\{\mu(x), \alpha\}$

(5)

$$\min\{\mu(x * z), \beta\} \leq \max\{\mu(x * (y * z)), \mu(y), \alpha\}$$

for all $x, y, z \in X$.

Example 18. *Let $X = \{0, 1, 2, 3, 4\}$ be a BCI -algebra with the following Cayley table:*

$*$	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	0
2	2	2	0	0	0
3	3	3	3	0	0
4	4	3	4	0	0

Define a fuzzy set μ *in* X *by* $\mu(0) = 0.3$,$\mu(1) = 0, \mu(2) = 1$ *and* $\mu(3) = \mu(4) = 0.6$. *By routine calculations, we can see that* μ *is a doubt fuzzy q-ideal with thresholds* $(0.3, 0.6)$ *of* X . *But* μ *is neither a fuzzy q-ideal of* X *nor an* $(\in, \in \vee q)-$*doubt fuzzy q-ideal of* X.

Theorem 19. *A fuzzy set* μ *of* X *is a doubt fuzzy q-ideal with thresholds* (α, β) *of* X *if and only if* $U(\mu; t)$ $(\neq \phi)$ *is a fuzzy q- ideal of* X *for all* $\alpha \leq t < \beta$.

Proof: Similar to the proof of Theorem 9. □

Theorem 20. *Let* $f : X \rightarrow X'$ *be an epimomorphism of BCI -algebra and let* μ *and* ν *be doubt fuzzy q-ideal of* X *and* X' *respectively. Then* $f(\mu)$ *defined by*

$$f(\mu)(y) = \inf\left\{\mu(x)|f(x) = y \quad for\ all\ y \in X'\right\}$$

and $f^{-1}(\nu)$ *defined by*

$$f^{-1}(\nu)(x) = \nu(f(x))\ for\ all\ x \in X$$

are doubt fuzzy q-ideals of X' *and* X *,respectively. Moreover, if* μ *and* ν *are with thresholds* (α, β) *, then also* $f(\mu)$ *and* $f^{-1}(\nu)$ *are with thresholds* (α, β).

Proof: Let μ is a doubt fuzzy q-ideal with thresholds(α, β), and let $x', y', z' \in X'$.Since f is an epimomorphism thus $x' = f(x), y' = f(y)$ and $z' = f(z)$, for some $x, y, z \in X$,Then

$$\begin{aligned}
&\min\{f(\mu)(x' * z'), \beta\} \\
&= \min\{\inf\{\mu(x * z)\} \Big| f(x * z) = x' * z' , \beta\} \\
&= \inf\{\min\{\mu(x * z), \beta\} \Big| f(x * z) = x' * z' \} \\
&\leq \inf\{\max\{\mu(x * (y * z)), \mu(y), \alpha\} | f(x * (y * z)) \\
&= x' * (y' * z'), f(y) = y' \} \\
&= \max\{\inf\{\mu(x * (y * z)) | f(x * (y * z)) \\
&= x' * (y' * z')\} \\
&, \inf\{\mu(y) | f(y) = y'\}, \alpha\} \\
&= \max\{f(\mu)(x' * (y' * z')), f(\mu)(y'), \alpha\} .
\end{aligned}$$

Hence $f(\mu)$ is a doubt fuzzy q-ideal with thresholds (α, β). Similarly, if ν is a doubt fuzzy q-ideal with thresholds(α, β), then for any $x, y, z \in X$.

$$\begin{aligned}
&\min\{f^{-1}(\nu)(x' * z'), \beta\} = \min(\nu(f(x' * z')), \beta\} \\
&= \min\{\nu(f(x') * f(z')), \beta\} \\
&\leq \max\{\nu(f(x') * (f(y') * f(z'))), \nu(f(y')), \beta\} \\
&= \max\{\nu(f(x' * (y' * z'))), \nu(f(y')), \beta\} \\
&= \max\{f^{-1}(\nu)(x * (y * z)), f^{-1}(\nu)(y), \beta\}.
\end{aligned}$$

This completes the proof. □

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