

Three-tier FMCDM Problems with Trapezoidal Fuzzy Number

A. Taleshian and S. Rezvani

Department of Mathematics, Faculty of Sciences,
 Mazandaran University, P.O. Box: 47416-1467, Babolsar, Iran

Abstract: The purpose of this paper is to further propose the representation of multiplication operation on four fuzzy numbers and then this representation proposed in this paper is applied to solve the three-tier FMCDM problem. Using this representation, the decision maker can rank quickly all alternatives and choose easily the best one under FMCDM environment.

Key words: Fuzzy logic . decision making . multiple criteria . trapezoidal fuzzy number

INTRODUCTION

The Analytic Hierarchy Process (AHP) method was first proposed by Satty. The AHP method is a popular technique often used to model subjective decision making process based on multiple criteria [1, 2]. Many researchers have used AHP method to deal with multiple criteria decision making problems. Further, the fuzzy concept has also been introduced into the AHP method [3].

The concept of fuzzy sets was introduced by Zadeh [4]. The basic arithmetical operations on one fuzzy number were developed by Mizumoto and Tanaka [5], Nahmias [6], Dubois and Prade [7], Ma *et al.* [8] and Chen [9, 10]. The basic multiplication operation on two and three fuzzy numbers was proposed by Chou [11] and then the representation of multiplication operation on two fuzzy numbers was applied to solve the two-tier FMCDM problem: the selection of account receivable collection instrument in the international trade.

The purpose of this paper is to propose a ranking method for solving Fuzzy Multiple Criteria Decision Making (FMCDM) problems. The representation of multiplication operation on fuzzy numbers is useful for the decision makers to rank all alternatives and choose the best one under the fuzzy multiple criteria decision making environment. The representation of multiplication operation on four fuzzy numbers is proposed in this paper. Finally, this representation proposed in this paper is applied to solve the three-tier fuzzy multiple criteria decision making problem.

THE GRADED MEAN INTEGRATION REPRESENTATION METHOD

Generally, a generalized fuzzy number A is described as any fuzzy subset of the real line R , whose

membership function $\mu_A(a)$ satisfies the following conditions,

- $\mu_A(a)$ is a continuous mapping from R to the closed interval $[0,1]$,
- $\mu_A(a) = 0 \quad -\infty < a \leq \bar{r}$,
- $\mu_A(a) = L(a)$ is strictly increasing on $[\bar{r}, \bar{\bar{r}}]$,
- $\mu_A(a) = w, \bar{\bar{r}} \leq a \leq \bar{\bar{\bar{r}}}$,
- $\mu_A(a) = R(a)$ is strictly decreasing on $[\bar{\bar{\bar{r}}}, \bar{\bar{\bar{\bar{r}}}}]$,
- $\mu_A(a) = 0, \bar{\bar{\bar{\bar{r}}}} \leq a < \infty$

where $0 < w \leq 1$ and $\bar{r}, \bar{\bar{r}}, \bar{\bar{\bar{r}}}$ and $\bar{\bar{\bar{\bar{r}}}}$ are real numbers.

We call this type of generalized fuzzy number a trapezoidal fuzzy number and it is denoted by

$$A = (\bar{r}, \bar{\bar{r}}, \bar{\bar{\bar{r}}}, \bar{\bar{\bar{\bar{r}}}}; w)_{LR}$$

When $w = 1$, this type of generalized fuzzy number is called normal fuzzy number and is represented by

$$A = (\bar{r}, \bar{\bar{r}}, \bar{\bar{\bar{r}}}, \bar{\bar{\bar{\bar{r}}}})_{LR}$$

Let L^{-1} and R^{-1} be the inverse function of functions L and R respectively, then the graded mean h -level value of

$$A = (\bar{r}, \bar{\bar{r}}, \bar{\bar{\bar{r}}}, \bar{\bar{\bar{\bar{r}}}})_{LR}$$

is

$$\frac{h[L^{-1}(h) + R^{-1}(h)]}{2}$$

Therefore, the graded mean integration representation of generalized trapezoidal fuzzy number A with grade w is

$$G(A) = \int_0^w h \left(\frac{L^{-1}(h) + R^{-1}(h)}{2} \right) dh / \int_0^w h dh \quad (1)$$

Where h lies between 0 and w , $0 < w \leq 1$. Here,

$$L(a) = w \left(\frac{a - \bar{r}}{\bar{r} - \bar{r}} \right), \bar{r} \leq a \leq \bar{r}$$

And

$$R(a) = w \left(\frac{a - \bar{r}}{\bar{r} - \bar{r}} \right), \bar{r} \leq a \leq \bar{r}$$

Then

$$L^{-1}(h) = \bar{r} + (\bar{r} - \bar{r})h, 0 \leq h \leq 1$$

And

$$R^{-1}(h) = \bar{r} - (\bar{r} - \bar{r})h, 0 \leq h \leq 1$$

And

Definition 3.1

$$T_1 = (\bar{r}_1, \bar{r}_1, \bar{r}_1, \bar{r}_1), T_2 = (\bar{r}_2, \bar{r}_2, \bar{r}_2, \bar{r}_2), T_3 = (\bar{r}_3, \bar{r}_3, \bar{r}_3, \bar{r}_3)$$

and

$$T_4 = (\bar{r}_4, \bar{r}_4, \bar{r}_4, \bar{r}_4)$$

are four trapezoidal fuzzy numbers. Let $P(T_{1(h)} \otimes T_{2(h)} \otimes T_{3(h)} \otimes T_{4(h)})$ be the representation of $T_1 \otimes T_2 \otimes T_3 \otimes T_4$ at h -level.

$$\begin{aligned} P(T_{1(h)} \otimes T_{2(h)} \otimes T_{3(h)} \otimes T_{4(h)}) = & \\ & [L_{T_1}^{-1}(h) \otimes L_{T_2}^{-1}(h) \otimes L_{T_3}^{-1}(h) \otimes L_{T_4}^{-1}(h)]/10 + [R_{T_1}^{-1}(h) \otimes L_{T_2}^{-1}(h) \otimes L_{T_3}^{-1}(h) \otimes L_{T_4}^{-1}(h)]/10 \\ & [L_{T_1}^{-1}(h) \otimes R_{T_2}^{-1}(h) \otimes L_{T_3}^{-1}(h) \otimes L_{T_4}^{-1}(h)]/10 + [R_{T_1}^{-1}(h) \otimes L_{T_2}^{-1}(h) \otimes R_{T_3}^{-1}(h) \otimes L_{T_4}^{-1}(h)]/10 \\ & [L_{T_1}^{-1}(h) \otimes L_{T_2}^{-1}(h) \otimes L_{T_3}^{-1}(h) \otimes R_{T_4}^{-1}(h)]/10 + [L_{T_1}^{-1}(h) \otimes R_{T_2}^{-1}(h) \otimes R_{T_3}^{-1}(h) \otimes R_{T_4}^{-1}(h)]/10 \\ & [R_{T_1}^{-1}(h) \otimes L_{T_2}^{-1}(h) \otimes R_{T_3}^{-1}(h) \otimes R_{T_4}^{-1}(h)]/10 + [R_{T_1}^{-1}(h) \otimes R_{T_2}^{-1}(h) \otimes L_{T_3}^{-1}(h) \otimes R_{T_4}^{-1}(h)]/10 \\ & [R_{T_1}^{-1}(h) \otimes R_{T_2}^{-1}(h) \otimes R_{T_3}^{-1}(h) \otimes L_{T_4}^{-1}(h)]/10 + [R_{T_1}^{-1}(h) \otimes R_{T_2}^{-1}(h) \otimes R_{T_3}^{-1}(h) \otimes R_{T_4}^{-1}(h)]/10 \end{aligned}$$

$L_{T_1}(a)$ and $R_{T_1}(a)$ are the functions L and R of fuzzy number T_1 , respectively. $L_{T_1}^{-1}(a)$ and $R_{T_1}^{-1}(a)$ are the inverse functions of functions $L_{T_1}(a)$ and $R_{T_1}(a)$ at h -level, respectively.

$L_{T_2}(a)$ and $R_{T_2}(a)$ are the functions L and R of fuzzy number T_2 , respectively. $L_{T_2}^{-1}(a)$ and $R_{T_2}^{-1}(a)$ are the inverse functions of functions $L_{T_2}(a)$ and $R_{T_2}(a)$ at h -level, respectively.

$$\frac{h[L^{-1}(h) + R^{-1}(h)]}{2} = \frac{(\bar{r} + \bar{r}) + (\bar{r} - \bar{r} - \bar{r} + \bar{r})h}{2}$$

Now, using the (1) the graded mean integration representation of A is

$$\begin{aligned} G(A) = & \int_0^1 h \left(\frac{(\bar{r} + \bar{r}) + (\bar{r} - \bar{r} - \bar{r} + \bar{r})h}{2} \right) dh / \int_0^1 h dh \\ = & \int_0^1 \left(\frac{(\bar{r} + \bar{r})h}{2} + \frac{(\bar{r} - \bar{r} - \bar{r} + \bar{r})h^2}{2} \right) dh / \int_0^1 h dh \\ = & \frac{(\bar{r} + \bar{r})}{4} + \frac{(\bar{r} - \bar{r} - \bar{r} + \bar{r})}{6} / \frac{1}{2} \\ = & \frac{(\bar{r} + \bar{r})}{2} + \frac{(\bar{r} - \bar{r} - \bar{r} + \bar{r})}{3} = \frac{\bar{r} + 2\bar{r} + 2\bar{r} + \bar{r}}{6}. \end{aligned}$$

REPRESENTATION OF MULTIPLICATION ON FOUR FUZZY NUMBERS

This paper further proposes the representation of multiplication operation on four fuzzy numbers as follows.

$L_{T_3}(a)$ and $R_{T_3}(a)$ are the functions L and R of fuzzy number T_3 , respectively. $L_{T_3}^{-1}(a)$ and $R_{T_3}^{-1}(a)$ are the inverse functions of functions $L_{T_3}(a)$ and $R_{T_3}(a)$ at h -level, respectively.

$L_{T_4}(a)$ and $R_{T_4}(a)$ are the functions L and R of fuzzy number T_4 , respectively. $L_{T_4}^{-1}(a)$ and $R_{T_4}^{-1}(a)$ are the inverse functions of functions $L_{T_4}(a)$ and $R_{T_4}(a)$ at h -level, respectively.

Definition 3.2: Let $P(T_1 \otimes T_2 \otimes T_3 \otimes T_4)$ be the representation of $T_1 \otimes T_2 \otimes T_3 \otimes T_4$.

$$\begin{aligned} P(T_1 \otimes T_2 \otimes T_3 \otimes T_4) = & \int_0^1 h \left\{ \left[L_{T_1}^{-1}(h) \otimes L_{T_2}^{-1}(h) \otimes L_{T_3}^{-1}(h) \otimes L_{T_4}^{-1}(h) \right] / 10 + \left[R_{T_1}^{-1}(h) \otimes L_{T_2}^{-1}(h) \otimes L_{T_3}^{-1}(h) \otimes L_{T_4}^{-1}(h) \right] / 10 \right. \\ & \left[L_{T_1}^{-1}(h) \otimes R_{T_2}^{-1}(h) \otimes L_{T_3}^{-1}(h) \otimes L_{T_4}^{-1}(h) \right] / 10 + \left[R_{T_1}^{-1}(h) \otimes L_{T_2}^{-1}(h) \otimes R_{T_3}^{-1}(h) \otimes L_{T_4}^{-1}(h) \right] / 10 \\ & \left[L_{T_1}^{-1}(h) \otimes L_{T_2}^{-1}(h) \otimes L_{T_3}^{-1}(h) \otimes R_{T_4}^{-1}(h) \right] / 10 + \left[L_{T_1}^{-1}(h) \otimes R_{T_2}^{-1}(h) \otimes R_{T_3}^{-1}(h) \otimes R_{T_4}^{-1}(h) \right] / 10 \\ & \left[R_{T_1}^{-1}(h) \otimes L_{T_2}^{-1}(h) \otimes R_{T_3}^{-1}(h) \otimes R_{T_4}^{-1}(h) \right] / 10 + \left[R_{T_1}^{-1}(h) \otimes R_{T_2}^{-1}(h) \otimes L_{T_3}^{-1}(h) \otimes R_{T_4}^{-1}(h) \right] / 10 \\ & \left. \left[R_{T_1}^{-1}(h) \otimes R_{T_2}^{-1}(h) \otimes R_{T_3}^{-1}(h) \otimes L_{T_4}^{-1}(h) \right] / 10 + \left[R_{T_1}^{-1}(h) \otimes R_{T_2}^{-1}(h) \otimes R_{T_3}^{-1}(h) \otimes R_{T_4}^{-1}(h) \right] / 10 \right\} dh / \int_0^1 h \, dh \end{aligned}$$

By the above definition 3.2,

$$\begin{aligned} & \int_0^1 h \left\{ \left[L_{T_1}^{-1}(h) \otimes L_{T_2}^{-1}(h) \otimes L_{T_3}^{-1}(h) \otimes L_{T_4}^{-1}(h) \right] / 10 + \left[R_{T_1}^{-1}(h) \otimes L_{T_2}^{-1}(h) \otimes L_{T_3}^{-1}(h) \otimes L_{T_4}^{-1}(h) \right] / 10 \right. \\ & \left[L_{T_1}^{-1}(h) \otimes R_{T_2}^{-1}(h) \otimes L_{T_3}^{-1}(h) \otimes L_{T_4}^{-1}(h) \right] / 10 + \left[R_{T_1}^{-1}(h) \otimes L_{T_2}^{-1}(h) \otimes R_{T_3}^{-1}(h) \otimes L_{T_4}^{-1}(h) \right] / 10 \\ & \left[L_{T_1}^{-1}(h) \otimes L_{T_2}^{-1}(h) \otimes L_{T_3}^{-1}(h) \otimes R_{T_4}^{-1}(h) \right] / 10 + \left[L_{T_1}^{-1}(h) \otimes R_{T_2}^{-1}(h) \otimes R_{T_3}^{-1}(h) \otimes R_{T_4}^{-1}(h) \right] / 10 \\ & \left[R_{T_1}^{-1}(h) \otimes L_{T_2}^{-1}(h) \otimes R_{T_3}^{-1}(h) \otimes R_{T_4}^{-1}(h) \right] / 10 + \left[R_{T_1}^{-1}(h) \otimes R_{T_2}^{-1}(h) \otimes L_{T_3}^{-1}(h) \otimes R_{T_4}^{-1}(h) \right] / 10 \\ & \left. \left[R_{T_1}^{-1}(h) \otimes R_{T_2}^{-1}(h) \otimes R_{T_3}^{-1}(h) \otimes L_{T_4}^{-1}(h) \right] / 10 + \left[R_{T_1}^{-1}(h) \otimes R_{T_2}^{-1}(h) \otimes R_{T_3}^{-1}(h) \otimes R_{T_4}^{-1}(h) \right] / 10 \right\} dh / \int_0^1 h \, dh \\ & = \frac{1}{10} \int_0^1 h \left\{ \left[\bar{r}_1 + (\bar{r}_1 - \bar{r}_1)h \right] \times \left[\bar{r}_2 + (\bar{r}_2 - \bar{r}_2)h \right] \times \left[\bar{r}_3 + (\bar{r}_3 - \bar{r}_3)h \right] \times \left[\bar{r}_4 + (\bar{r}_4 - \bar{r}_4)h \right] \right. \\ & + \left[\bar{\bar{r}}_1 - (\bar{\bar{r}}_1 - \bar{\bar{r}}_1)h \right] \times \left[\bar{r}_2 + (\bar{r}_2 - \bar{r}_2)h \right] \times \left[\bar{r}_3 + (\bar{r}_3 - \bar{r}_3)h \right] \times \left[\bar{r}_4 + (\bar{r}_4 - \bar{r}_4)h \right] \\ & + \left[\bar{r}_1 + (\bar{r}_1 - \bar{r}_1)h \right] \times \left[\bar{\bar{r}}_2 - (\bar{\bar{r}}_2 - \bar{\bar{r}}_2)h \right] \times \left[\bar{r}_3 + (\bar{r}_3 - \bar{r}_3)h \right] \times \left[\bar{r}_4 + (\bar{r}_4 - \bar{r}_4)h \right] \\ & + \left[\bar{r}_1 + (\bar{r}_1 - \bar{r}_1)h \right] \times \left[\bar{\bar{r}}_2 - (\bar{\bar{r}}_2 - \bar{\bar{r}}_2)h \right] \times \left[\bar{\bar{r}}_3 - (\bar{\bar{r}}_3 - \bar{\bar{r}}_3)h \right] \times \left[\bar{r}_4 + (\bar{r}_4 - \bar{r}_4)h \right] \\ & + \left[\bar{r}_1 + (\bar{r}_1 - \bar{r}_1)h \right] \times \left[\bar{r}_2 + (\bar{r}_2 - \bar{r}_2)h \right] \times \left[\bar{r}_3 + (\bar{r}_3 - \bar{r}_3)h \right] \times \left[\bar{\bar{r}}_4 - (\bar{\bar{r}}_4 - \bar{\bar{r}}_4)h \right] \\ & + \left[\bar{r}_1 + (\bar{r}_1 - \bar{r}_1)h \right] \times \left[\bar{\bar{r}}_2 - (\bar{\bar{r}}_2 - \bar{\bar{r}}_2)h \right] \times \left[\bar{\bar{r}}_3 - (\bar{\bar{r}}_3 - \bar{\bar{r}}_3)h \right] \times \left[\bar{\bar{r}}_4 - (\bar{\bar{r}}_4 - \bar{\bar{r}}_4)h \right] \\ & + \left[\bar{\bar{r}}_1 - (\bar{\bar{r}}_1 - \bar{\bar{r}}_1)h \right] \times \left[\bar{r}_2 + (\bar{r}_2 - \bar{r}_2)h \right] \times \left[\bar{\bar{r}}_3 - (\bar{\bar{r}}_3 - \bar{\bar{r}}_3)h \right] \times \left[\bar{\bar{r}}_4 - (\bar{\bar{r}}_4 - \bar{\bar{r}}_4)h \right] \\ & + \left[\bar{\bar{r}}_1 - (\bar{\bar{r}}_1 - \bar{\bar{r}}_1)h \right] \times \left[\bar{\bar{r}}_2 - (\bar{\bar{r}}_2 - \bar{\bar{r}}_2)h \right] \times \left[\bar{r}_3 + (\bar{r}_3 - \bar{r}_3)h \right] \times \left[\bar{\bar{r}}_4 - (\bar{\bar{r}}_4 - \bar{\bar{r}}_4)h \right] \\ & + \left[\bar{\bar{r}}_1 - (\bar{\bar{r}}_1 - \bar{\bar{r}}_1)h \right] \times \left[\bar{\bar{r}}_2 - (\bar{\bar{r}}_2 - \bar{\bar{r}}_2)h \right] \times \left[\bar{\bar{r}}_3 - (\bar{\bar{r}}_3 - \bar{\bar{r}}_3)h \right] \times \left[\bar{r}_4 + (\bar{r}_4 - \bar{r}_4)h \right] \\ & \left. + \left[\bar{\bar{r}}_1 - (\bar{\bar{r}}_1 - \bar{\bar{r}}_1)h \right] \times \left[\bar{\bar{r}}_2 - (\bar{\bar{r}}_2 - \bar{\bar{r}}_2)h \right] \times \left[\bar{\bar{r}}_3 - (\bar{\bar{r}}_3 - \bar{\bar{r}}_3)h \right] \times \left[\bar{\bar{r}}_4 - (\bar{\bar{r}}_4 - \bar{\bar{r}}_4)h \right] \right\} dh / \int_0^1 h \, dh \end{aligned}$$

$$\begin{aligned}
& + \left(\bar{r}_1 \bar{r}_2 + \bar{r}_1 \bar{r}_2 + \bar{r}_1 \bar{r}_2 + \bar{r}_1 \bar{r}_2 \right) \left(\bar{r}_3 \bar{r}_4 + \bar{r}_3 \bar{r}_4 - \bar{r}_3 \bar{r}_4 - \bar{r}_3 \bar{r}_4 - \bar{r}_3 \bar{r}_4 - \bar{r}_3 \bar{r}_4 + \bar{r}_3 \bar{r}_4 + \bar{r}_3 \bar{r}_4 \right) \\
& + \left(\bar{r}_1 \bar{r}_2 + \bar{r}_1 \bar{r}_2 + \bar{r}_1 \bar{r}_2 + \bar{r}_1 \bar{r}_2 + \bar{r}_1 \bar{r}_2 + \bar{r}_1 \bar{r}_2 \right) \left(\bar{r}_3 \bar{r}_4 - \bar{r}_3 \bar{r}_4 - \bar{r}_3 \bar{r}_4 + \bar{r}_3 \bar{r}_4 \right) \\
& + \left(\bar{r}_1 \bar{r}_2 + \bar{r}_1 \bar{r}_2 + \bar{r}_1 \bar{r}_2 + \bar{r}_1 \bar{r}_2 + \bar{r}_1 \bar{r}_2 + \bar{r}_1 \bar{r}_2 \right) \left(\bar{r}_3 \bar{r}_4 + \bar{r}_3 \bar{r}_4 - \bar{r}_3 \bar{r}_4 - \bar{r}_3 \bar{r}_4 \right)
\end{aligned}$$

For example, let

$$T_1 = (\bar{r}_1, \bar{r}_1, \bar{r}_1, \bar{r}_1) = (0, 2, 4, 6), T_2 = (\bar{r}_2, \bar{r}_2, \bar{r}_2, \bar{r}_2) = (2, 4, 6, 8), T_3 = (\bar{r}_3, \bar{r}_3, \bar{r}_3, \bar{r}_3) = (4, 6, 8, 10)$$

and

$$T_4 = (\bar{r}_4, \bar{r}_4, \bar{r}_4, \bar{r}_4) = (6, 8, 10, 12).$$

be four trapezoidal fuzzy numbers. The representation of T_1 is $P(T_1) = 3$, The representation of T_2 is $P(T_2) = 5$, The representation of T_3 is $P(T_3) = 7$ and The representation of T_4 is $P(T_4) = 9$.

$$P(T_1) \otimes P(T_2) \otimes P(T_3) \otimes P(T_4) = 3 \times 5 \times 7 \times 9 = 945.$$

By formula (*) proposed in this paper, the representation of $T_1 \otimes T_2 \otimes T_3 \otimes T_4$ is

$$P(T_1 \otimes T_2 \otimes T_3 \otimes T_4) = 3 \times 5 \times 7 \times 9 = 945.$$

We have that

$$P(T_1) \times P(T_2) \times P(T_3) \times P(T_4) = 945 = P(T_1 \otimes T_2 \otimes T_3 \otimes T_4)$$

EXAMPLE

The basic data is shown as follows.

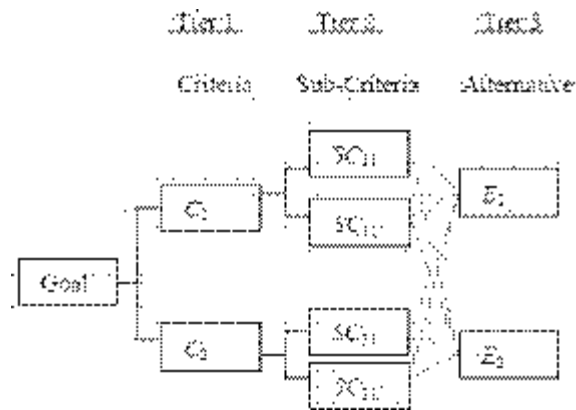


Fig. 1: The hierarchy structure for three-tier FMCDM problem

Alternatives: E_1 and E_2 ,

Criteria: C_1 and C_2 ,

Sub-Criteria: SC_{11} , SC_{12} , SC_{21} and SC_{22} ,

Importance weight of C_1 : $W_{C1} = (0.1, 0.3, 0.5, 0.7)$,

Importance weight of C_2 : $W_{C2} = (1.1, 1.3, 1.5, 1.7)$,

Importance weight of C_{11} : $W_{SC11} = (0.9, 1.1, 1.3, 1.5)$,

Importance weight of C_{12} : $W_{SC12} = (0.3, 0.5, 0.7, 0.9)$,

Importance weight of C_{21} : $W_{SC21} = (0.9, 1.1, 1.3, 1.5)$,

Importance weight of C_{22} : $W_{SC22} = (0.3, 0.5, 0.7, 0.9)$,

Preference of E_1 under SC_{11} : $P_{E1SC11} = (1.3, 1.5, 1.7, 1.9)$,

Preference of E_1 under SC_{12} : $P_{E1SC12} = (1.5, 1.7, 1.9, 2.1)$,

Preference of E_1 under SC_{21} : $P_{E1SC21} = (0.9, 1.1, 1.3, 1.5)$,

Preference of E_1 under SC_{22} : $P_{E1SC22} = (1.1, 1.3, 1.5, 1.7)$,

Preference of E_2 under SC_{11} : $P_{E2SC11} = (0.9, 1.1, 1.3, 1.5)$,

Preference of E_2 under SC_{12} : $P_{E2SC12} = (1.1, 1.3, 1.5, 1.7)$,

Preference of E_2 under SC_{21} : $P_{E2SC21} = (1.3, 1.5, 1.7, 1.9)$,

Preference of E_2 under SC_{22} : $P_{E2SC22} = (1.5, 1.7, 1.9, 2.1)$,

Let the total performance of alternative E_1 is P_{E1} ,

Let the total performance of alternative E_2 is P_{E2}

$$P_{E1} = P_{E1SC11} \otimes W_{SC11} \otimes W_{C1} \otimes W_{C2} +$$

$$P_{E1SC12} \otimes W_{SC12} \otimes W_{C1} +$$

$$P_{E1SC21} \otimes W_{SC21} \otimes W_{C2} \otimes W_{E1SC22} \otimes W_{SC22}$$

$$= (1.3, 1.5, 1.7, 1.9) \otimes (0.9, 1.1, 1.3, 1.5) \otimes (0.1, 0.3, 0.5, 0.7) +$$

$$+ (1.5, 1.7, 1.9, 2.1) \otimes (0.3, 0.5, 0.7, 0.9) \otimes (0.1, 0.3, 0.5, 0.7) +$$

$$+ (0.9, 1.1, 1.3, 1.5) \otimes (0.9, 1.1, 1.3, 1.5) \otimes (1.1, 1.3, 1.5, 1.7) +$$

$$+ (1.1, 1.3, 1.5, 1.7) \otimes (0.3, 0.5, 0.7, 0.9) \otimes (1.1, 1.3, 1.5, 1.7)$$

$$P_{E2} = P_{E2SC11} \otimes W_{SC11} \otimes W_{C1} \otimes W_{C2} +$$

$$P_{E2SC12} \otimes W_{SC12} \otimes W_{C1} +$$

$$P_{E2SC21} \otimes W_{SC21} \otimes W_{C2} \otimes W_{E2SC22} \otimes W_{SC22}$$

$$= (0.9, 1.1, 1.3, 1.5) \otimes (0.9, 1.1, 1.3, 1.5) \otimes (0.1, 0.3, 0.5, 0.7) +$$

$$+ (1.1, 1.3, 1.5, 1.7) \otimes (0.3, 0.5, 0.7, 0.9) \otimes (0.1, 0.3, 0.5, 0.7) +$$

$$+ (1.3, 1.5, 1.7, 1.9) \otimes (0.9, 1.1, 1.3, 1.5) \otimes (1.1, 1.3, 1.5, 1.7) +$$

$$+ (1.5, 1.7, 1.9, 2.1) \otimes (0.3, 0.5, 0.7, 0.9) \otimes (1.1, 1.3, 1.5, 1.7)$$

By formula (*), $P_{E1} = 4.392$, $P_{E2} = 5.112$. We have that $P_{E1} < P_{E2}$. The alternative P_{E2} is selected.

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