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Isotropic Biharmonic Curves in the Complex Space C^3

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Abstract: In this paper, we study isotropic biharmonic curves in the complex space C^3. We prove that there exist no non-geodesic isotropic biharmonic curve in the complex space C^3.

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INTRODUCTION

In [1], the notion of an isotropic curve is studied in space C^3. In this space, author constructs E. Cartan frame and derivative equations by means of classical differential geometry methods.

Let $f:(M, g) \to (N, h)$ be a smooth map between two Lorentzian manifolds. The bienergy $E_2(f)$ of f over compact domain $\Omega \subset M$ is defined by

$$E_2(f) = \int_\Omega h(\tau(f), \tau(f))dv_g,$$

Where $\tau(f) = \mathrm{trace}_g \nabla df$ is the tension field of f and dv_g is the volume form of M, [2-5]. Using the first variational formula one sees that f is a biharmonic map if and only if its bitension field vanishes identically, i.e.,

$$\tau_2(f) := -\Delta^f(\tau(f)) - \mathrm{trace}_g R^N(df, \tau(f))df = 0,$$

Where

$$\Delta^f = -\mathrm{trace}^g(\nabla^f)^2 = -\mathrm{trace}_g(\nabla^f\nabla^f - \nabla^f_{\nabla M})$$

is the Laplacian on sections of the pull-back bundle $f^{-1}(TN)$ and R^N is the curvature operator of (N,h) defined by

$$R(X, Y)Z = [\nabla_X, \nabla_Y]Z - \nabla_{[X,Y]}Z.$$

In the last decade there have been a growing interest in the theory of biharmonic maps which can be divided into two main research directions. On the one side, the differential geometric aspect has driven attention to the construction of examples and classification results. The other side is the analytic aspect from the point of view of PDE: biharmonic maps are solutions of a fourth order strongly elliptic semilinear PDE [6-10].

In this paper, we study isotropic biharmonic curves in the complex space C^3. We prove that there exist no non-geodesic isotropic biharmonic curve in the complex space.

Preliminaries

Definition 2.1: ([1]) Let x_p *be a complex analytic function of a complex variable t. Then the vector function*

$$x = \sum_{p=0}^{3} x_p(t)\vec{k}_p \tag{2.1}$$

is called an imaginary curve, where $x: C \to C^3$ *and* $\vec{k}_p$ *are standard basis unit vectors of* E^3.

Definition 2.2: *([1]) The curves, of which the square of the distance between the two points equal to zero, are called minimal or isotropic curves.*

Definition 2.3: *([1]) Let s denote the arclength. A curve is a isotropic (minimal) curve if and only if* $ds^2 = 0$.

Let $x = x(t)$ be a isotropic (minimal) curve in space with t complex variable. Then above definitions follow that $ds^2 = dx^2 = 0$. For every regular point, we know that $x'(x) \neq 0$. Via this, it is safe to report that, isotropic curves in space $x = x(t)$ satisfy the vector differential equation.

$$\left[x'(t)\right]^2 = 0.$$

By differentiation, we have $x'(t)\ x''(t) = 0$. And by trivial calculus, it can be written that $[x'(t) \wedge x''(t)]^2$. This means that it is also an isotropic vector which is perpendicular to itself. Then,

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$$x'(t) \wedge x''(t) = \Psi . x'(t), \Psi \neq 0 \qquad (2.2)$$

can be written. By vector product with $x''(t)$, we know $\Psi = -[x'']$ and therefore

$$x' = \frac{x' \wedge x''}{\sqrt{-\left[x''\right]^2}}. \qquad (2.3)$$

For another complex variable t^*, $t = f(t^*)$ and $\frac{df}{dt^*} = \dot{f}$ we may write

$$dx = \frac{x' \wedge x''}{\sqrt{-\left[x''\right]^2}} dt = \frac{\dot{x} \wedge \ddot{x}}{\sqrt{-\left[\ddot{x}\right]^2}} dt^*, \qquad (2.4)$$

Where $\dot{x} = x'.\dot{f}$, $\ddot{x} = x''.\left(\dot{f}\right) + x'.\dot{f}$. The equality $\left(\ddot{x}\right)^2 = x''.\left(\dot{f}\right)^4$ can be written in the form

$$-\left[\left(\ddot{x}\right)^2\right]^{\frac{1}{4}} dt^* = -\left[\left(x''\right)^2\right]^{\frac{1}{4}} dt. \qquad (2.5)$$

If we choose t^* such that $\left(\ddot{x}\right)^2 = -1$, the by integration

$$t^* = s = -\int_{t_0}^{t} \left[\left(x''\right)^2\right]^{\frac{1}{4}} dt, s = s_1 + is_2$$

is obtained. It is called the pseudo arclength of the minimal curve which is invariant with respect to parameter t.

For each point x of the isotropic curve, E. Cartan frame is defined (for well-known complex number i^2) as follows:

$$e_1 = \dot{x},$$

$$e_2 = \ddot{x},$$

$$e_3 = -\frac{\beta}{2}\dot{x} + \dddot{x}, \qquad (2.6)$$

Where $\beta = \left(\ddot{x}\right)^2$ and $(e_1, e_2, e_3) = i$ The Frenet formulas are as follows

$$\dot{e}_1 = -ie_2$$

$$\dot{e}_2 = i(ke_1 + e_3)$$

$$\dot{e}_3 = -ike_2, \qquad (2.7)$$

Where $k = \beta/2$ is called pseudo curvature of the isotropic curve $x = x(s)$. These equations can be used if the isotropic curve is at least of class $C^4(C^3)$.

Isotropic Biharmonic Curves in the Complex Space C^3: Biharmonic equation for the curve x reduces to

$$\dddot{e}_1 = 0, \qquad (3.1)$$

that is, x is called a biharmonic curve if it is a solution of the equation (3.1), [11].

Theorem 3.1: *There exist no isotropic biharmonic curve in the complex space* C^3.

Proof: Assume that x is isotropic biharmonic curve in the complex space C^3. Then using (3.1), we have

$$\dot{k}e_1 - 2ike_2 = 0. \qquad (3.2)$$

From (2.6), we get

$$\begin{aligned} e_1 \cdot e_1 &= 1, \\ e_2 \cdot e_2 &= 1, \\ e_1 \cdot e_1 &= 0, \\ e_1 \cdot e_2 &= 0, \\ e_3 \cdot e_3 &= 0, \end{aligned} \qquad (3.3)$$

Substituting (3.3) into (3.2), we have

$$\dot{k} = 1, k = -\frac{1}{2i}.$$

This is a contradiction. Therefore, there exist no isotropic biharmonic curve in the complex space C^3.

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