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Exact Analysis of Radiation Convective Flow Heat and Mass Transfer over an Inclined Plate in a Porous Medium

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Abstract: Analytical solutions are presented for the problem of convection heat and mass transfer of a viscous electrically conducting incompressible fluid over a semi-infinite inclined plate in a porous medium with radiation and heat generation. The exact solutions for concentration, temperature and velocity are obtained in terms of exponential functions. It is found that the velocity is increased on increasing the permeability of the porous medium. Nusselt number increases with increasing the conduction-radiation parameter and decreases with increasing the heat generation parameter.

Key words: Convection · Porous medium · Radiation · Heat generating parameter

INTRODUCTION

The study of convective flows with heat and mass transfer in a porous medium has attracted considerable attention in recent times due to numerous applications in geothermal energy, oil reservoir modelling, building insulation, food processing and grain storage. Raptis [1] studied free convection flow through a porous medium bounded by a vertical infinite porous plate in the presence of radiation. He found that the velocity decreases when the radiation parameter increases. Unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction is studied by Kim [2]. He found that the velocity decreases when the magnetic parameter increases and increases when the permeability parameter increases. Muthucumaraswamy [3] presented heat and mass transfer effects on a continuously moving isothermal vertical surface with uniform suction. He found that the concentration increases in the presence of the generative reaction.

Chamkha [4] reports analytical solutions for heat and mass transfer problem of an electrically conducting and heat generating/absorbing fluid on a uniformly moving surface. He concluded that the fluid velocity is decreased as the strength of the magnetic field is increased. Kumari and Nath [5] analyzed the effect of thermal radiation on the non-Darcy mixed convection flow over a non-isothermal horizontal surface immersed in a saturated porous medium. They found that the Nusselt number increases with the radiation parameter. The aim of the present study is to find the analytical solution of the problem of the heat and mass transfer over a semi-infinite inclined plate in the porous medium with radiation and heat generation.

MATHEMATICAL ANALYSIS

Consider the steady laminar flow of an incompressible viscous electrically conducting fluid past a semi-infinite inclined surface with an acute angle ϕ from the vertical embedded in a porous medium. The wall is maintained at a constant temperature T_w and concentration C_w which is higher than the ambient temperature T_∞ and concentration C_∞ of the surrounding fluid, respectively. All the fluid properties are assumed to be constant. The porous medium is assumed to be isotropic, homogeneous and in thermodynamic equilibrium with the fluid. The governing equations for continuity, momentum, energy and concentration can be written in the following form

$$\frac{\partial v}{\partial y} = 0 \tag{1}$$

$$v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta_T(T - T_\infty)\cos\phi + g\beta_C(c - c_\infty)\cos\phi - \frac{\nu}{k'}u \tag{2}$$

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$$v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} + \frac{Q}{\rho c_p}(T - T_\infty) \tag{3}$$

$$v\frac{\partial c}{\partial y} = D\frac{\partial^2 c}{\partial y^2} \tag{4}$$

with the boundary conditions

$$u = u_w,\ v = -v_w,\ T = T_w,\ c = c_w \quad \text{at } y = 0$$

$$u \to 0, \quad T \to T_\infty,\ c \to c_\infty \ \text{as } y\to\infty \tag{5}$$

Where y, u, v, T and c are the horizontal or transverse coordinate, the axial velocity, the transverse velocity, the temperature of the fluid and the species concentration, respectively. The parameters g, β_T, β_c, ϕ, ν, k', α, q_r, Q, ρ, c_p, D are the acceleration due to gravity, the coefficient of thermal expansion, the coefficient of concentration expansion, the angle of inclination, the kinamatic viscosity, the permeability of porous medium, the thermal diffusivity, the local radiative heat flux, the heat generation/absorption coefficient, the density, the specific heat and the mass diffusion coefficient, respectively. u_w is the surface velocity and $v_w > 0$ is the suction velocity.

The radiative heat flux term is simplified by using the Rosseland approximation in Sparrow and Cess [6],

$$q_r = -\frac{4\sigma_o}{3k^*}\frac{\partial T^4}{\partial y} \tag{6}$$

Where σ_o and k^* are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively.

We assume that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \tag{7}$$

The equation of continuity (1) with boundary condition (5) gives $v = -v_w$ (8)

The non-dimensional variables are

$$Y = \frac{y v_w}{\nu},\ U = \frac{u}{u_w},\ \theta = \frac{T - T_\infty}{T_w - T_\infty},\ C = \frac{c - c_\infty}{c_w - c_\infty} \tag{9}$$

Using (6), (7), (8) and (9), equations (2)-(4) becomes,

$$\frac{d^2U}{dY^2} + \frac{dU}{dY} + Gr_T\theta\cos\phi + Gr_C C\cos\phi - \frac{1}{K}U = 0 \tag{10}$$

$$\left(1 + \frac{4}{3N}\right)\frac{d^2\theta}{dY^2} + \Pr\frac{d\theta}{dY} + \Pr S\theta = 0 \tag{11}$$

$$\frac{d^2C}{dY^2} + Sc\frac{dC}{dY} = 0 \tag{12}$$

With the boundary conditions

$$\begin{array}{ll} U=1,\ \theta=1,\ C=1 & at\ Y=0 \\ U\to 0,\ \theta\to 0,\ C\to 0 & as\ Y\to\infty \end{array} \tag{13}$$

Where $Gr_T = \frac{g\beta_T(T_w - T_\infty)\nu}{u_w v_w^2}$ is he thermal Grashof number, $Gr_C = \frac{g\beta_c(c_w - c_\infty)\nu}{u_w v_w^2}$ is the solutal Grashof number, $K = \frac{k' v_w^2}{\nu^2}$ is the dimensionless permeability parameter, $\Pr = \frac{\rho\nu c_p}{k} = \frac{\alpha}{\nu}$ is the Prandtl number, $N = \frac{kk^*}{4\sigma T_\infty^3}$ is the conduction-radiation parameter, $S = \frac{Q\nu}{\rho c_p v_w^2}$ is the heat generation parameter, $Sc = \frac{\nu}{D}$ is the Schmidt number.

Solving the equation (12) with (13) we get

$$C = \exp(-ScY) \tag{14}$$

Solving the equation (11) with (13) we get

$$\theta = \exp(-m_2 Y) \tag{15}$$

Where ; $m_2 = \frac{1}{2\left(1 + \frac{4}{3N}\right)}\left(\Pr + \sqrt{\Pr^2 - 4\left(1 + \frac{4}{3N}\right)S\Pr}\right)$

Substituting the values of C and θ from equations (14) and (15) into (10) and solving equation (10) with boundary condition (13), we get the solution for U as

$$U = n_1\exp(-m_1 Y) + n_2\exp(-m_2 Y) + n_3\exp(-ScY) \tag{16}$$

Where $m_1 = \frac{1}{2}\left(1 + \sqrt{1 + (4/K)}\right)$

and n_1, n_2, n_3 are given by

$$n_2 = \frac{-Gr_T \cos\phi}{m_2^2 - m_2 - (1/K)},\ n_3 = \frac{-Gr_C \cos\phi}{Sc^2 - Sc - (1/K)},\ n_1 = 1 - n_2 - n_3$$

It is now important to calculate the physical quantities of primary interest, which are the local wall shear stress, the local surface heat and mass fluxes.

Given the velocity field in the boundary layer, we can now calculate the wall shear stress

$$\tau_f = \mu \left.\frac{du}{dy}\right|_{y=0}$$

and the skin-friction coefficient is defined as

$$C_f = \frac{\tau_f}{\rho u_w v_w} = \frac{dU}{dY}(0) = -n_1 m_1 - n_2 m_2 - n_3 Sc$$

It is interesting to study the effect of the convection on the wall heat transfer rate

$$q_w = -k \left.\frac{dT}{dy}\right|_{y=0}$$

and the Nusselt number is defined as

$$Nu = \frac{q_w v}{(T_w - T_\infty) k v_w} = -\frac{d\theta}{dY}(0) = m_2$$

The definition of the wall mass transfer rate is given by

$$J_w = -D \left.\frac{dc}{dy}\right|_{y=0}$$

and the Sherwood number is defined as

$$Sh = \frac{J_w v}{(c_w - c_\infty) D v_w} = -\frac{dC}{dY}(0) = Sc$$

RESULTS AND DISCUSSION

In order to study the behavior of velocity field, temperature and concentration fields, a numerical computation is carried out for various values of the parameters involving in the study. The heat generating/absorption parameter $S>0$ corresponds to heat generation, $S=0$ corresponds to no heat generation/absorption effects and $S>0$ corresponds to heat absorption.

Table 1 shows the local skin friction coefficient, Nusselt numbers and Sherwood numbers for various combinations of parameters. It is interesting to observe that the present results in the absence of radiation and permeability of the porous medium are well agreement with the solutions of Chamkha [5]. It is clearly observed from the table that the values of skin friction coefficient increase with increasing the permeability of the porous medium and internal heat generation/absorption parameter and decreases with increasing conduction-radiation parameter. Further, it is analyzed from these tables that the values of Nusselt number increase with increasing the conduction-radiation parameter and decrease with increasing the heat generation/absorption parameter. Also no effect is seen in Nusselt number with increasing the permeability of the porous medium.

Figures 1 and 2 illustrate that the influence of *N, K* and *S*. It is observed from the Figure 1 that the velocity increases with increasing the permeability of the porous medium or the heat generating parameter *S* and decreases with increasing the conduction-radiation parameter *N*. It is clearly seen from Figure 2 that the temperature profiles increase with increasing the heat generation parameter *S* and decrease with increasing with conduction-radiation parameter *N*.

Concluding Remarks: Analytical solutions of convection heat and mass transfer of a viscous incompressible fluid over a semi-infinite inclined porous medium in the presence of heat generation and radiation are derived.

Table 1: The effects of all parameters on C_f, *Nu* and *Sh* for $Gr_T = Gr_C = 1$, $Sc = 0.6$.

ϕ	K	Pr	S	N	C_f	Nu	Sh
0	∞	0.71	0	∞	2.0751	0.7100	0.6000
0	∞	7.0	0	∞	0.8095	7.0000	0.6000
45	2	0.71	0.1	2	0.0094	0.2656	0.6000
45	4	0.71	0.1	2	0.5550	0.2656	0.6000
45	4	0.71	0.1	4	0.3104	0.3991	0.6000
45	4	0.71	0.1	6	0.2401	0.4525	0.6000
45	4	0.71	-0.1	4	0.0800	0.6186	0.6000

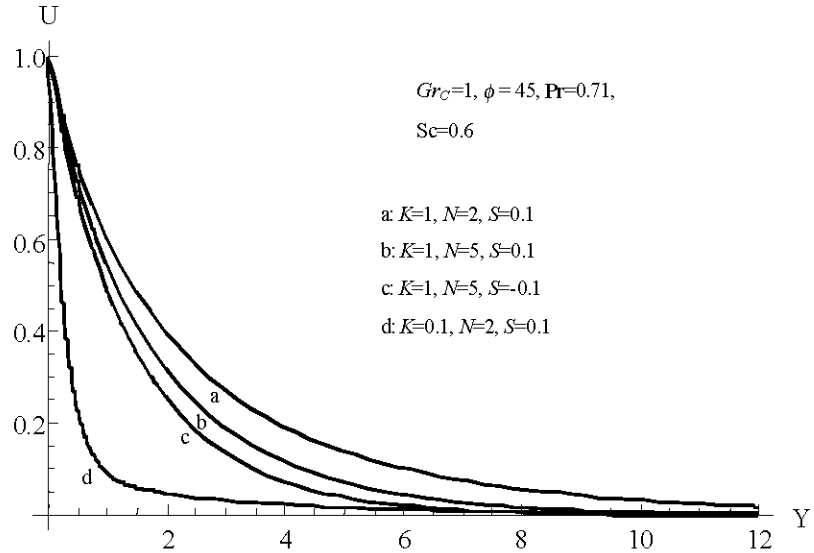

Fig. 1: Velocity profiles for different values of the permeability of the porous medium, conduction-radiation parameter and heat generating parameter

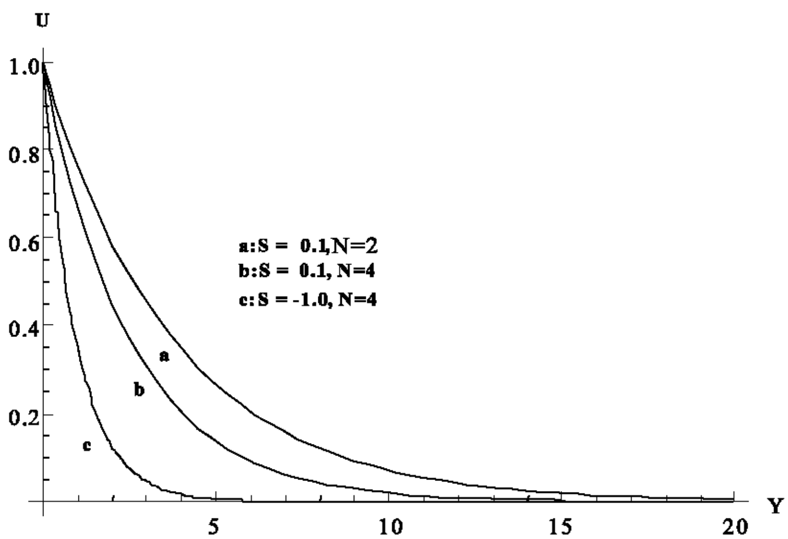

Fig. 2: Temperature profiles for different heat generating parameter and conduction-radiation parameter

The exact solutions for concentration, temperature and velocity are obtained in terms of exponential functions. The following results are concluded from the study.

- The velocity is increased on increasing the permeability of the porous medium.
- The temperature increases with increasing values of the heat generation parameter and decreases with increasing values of the conduction-radiation parameter.
- Skin friction coefficient increases with increasing the permeability of the porous medium.
- Nusselt number increases with increasing the conduction-radiation parameter and decreases with increasing the heat generation parameters.

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