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Analytic Solution Diffusivity Equation in Radial Form

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Abstract: In this paper, He's homotopy perturbation method is applied to diffusivity equation in radial form:

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = \frac{\partial p}{\partial t},$$
$$p(r_i, 0) = r.$$

The solutions are introduced in this paper are in recursive sequence forms which can be used to obtain the closed form of the solutions if they are required. The method is tested on example which is revealing the effectiveness and the simplicity of the method.

Key words: Diffusivity equation · Homotopy perturbation method

INTRODUCTION

Most scientific problems and physical phenomena occur nonlinearly. Except in a limited number of these problems, we have difficulty finding their exact analytical solutions. Therefore, there have been attempts to develop the new techniques for obtaining analytical solutions which reasonably approximate the exact solutions. In recent years, several such methods have drawn particular attention, such as Hirota's bilinear method [1], the homogeneous balance method [2, 3], the inverse scattering method [4], the Adomian decomposition method [5], the variational iteration method [6], the homotopy analysis method [7] and the homotopy perturbation method [8].

This paper presents an investigation of the use of the homotopy perturbation scheme for the solution of diffusivity equation:

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = \frac{\partial p}{\partial t},$$

with initial condition

$$p(r_i, 0) = r,$$

Homotopy Perturbation Method: The homotopy perturbation method was first proposed by the Chinese mathematician He [10-16]. The essential idea of this method is to introduce a homotopy parameter, say p, which takes values from 0 to 1. When p = 0, the system of equations usually reduces to a sufficiently simplified form, which normally admits a rather simple solution. As p is gradually increased to 1, the system goes through a sequence of "deformations," the solution for each of which is "close" to that at the previous stage of "deformation." Eventually at p = 1, the system takes the original form of the equation and the final stage of "deformation" gives the desired solution. One of the most remarkable features of the HPM is that usually just few perturbation terms are sufficient for obtaining a reasonably accurate solution. Homotopy perturbation method (HPM) is one of the most effective techniques among these methods, which has been employed to solve a large variety of linear and nonlinear problems such as fractional partial differential equations [17], the nonlinear Hirota-Satsuma coupled KdV partial differential equation [18], fractional KdV Burgers equation [19], nonlinear convective-radiative cooling equation, nonlinear heat equation (porous media equation) and nonlinear heat equation with cubic nonlinearity [20], evaluating the efficiency of straight fins with temperature-dependent thermal conductivity and determining the temperature distribution within the fin [21], the inverse parabolic equations and computing an unknown time-dependent parameter [22], finding improved approximate solutions to conservative truly nonlinear oscillators [23], complicated integrals which cannot be expressed in terms of

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elementary functions or analytical formulae [24], etc. In [25], Dehghan and Shakeri presented solution of a partial differential equation subject to temperature overspecification by HPM. The same authors worked an integro differential equation which describes the charged particle motion for certain configurations of oscillating magnetic fields [26] and solution of delay differential equations [27] by the HPM. The homotopy perturbation method is extended in [28] and is used to solve a kind of nonlinear evolution with the help of symbolic computation system Maple. This technique is applied in [29] to obtain approximate solutions of Klein-Gordon and Sine-Gordon equations. Also an efficient way of choosing the initial approximation is given by these authors.

The Blasius equation is solved in [30] using the homotopy perturbation method. This method is used in [31] to solve functional integral equations. Also comparison is made with an expansion method based on the Lagrange interpolation formula. The homotopy perturbation method is applied in [32] to solve pure strong nonlinear second-order differential equation. Using this approach the approximate analytic solution is obtained. In [33] solved the nonlinear matrix differential equations by homotopy perturbation method. A dynamic system and Burgers equation are taken as examples to illustrate its effectiveness and convenience. A reliable algorithm based on an adoption of the standard homotopy method is given in [34]. The homotopy perturbation method is treated as an algorithm in a sequence of intervals for finding accurate approximate solutions of linear and nonlinear system of ordinary differential equations. He's homotopy perturbation method employed in [35] to solve the singular initial boundary value problems of Lan7e-Emden type equations. The results show that construction of the homotopy for the perturbation problem plays a significant role for the accuracy of the solution. He's homotopy perturbation method combined with averaging applied to van der Pol oscillator with very strong nonlinearity in [36]. Chowdhury and Hashim [37] presented a new reliable algorithm based on an adaptation of the standard homotopy-perturbation method (HPM) applied to the Chen system which was a threedimensional system of ODEs with quadratic nonlinearities. Chowdhury and Hashim obtained approximate and exact analytical solutions of the generalized Emden-Fowler type equations in the second order ordinary differential equations (ODEs) by homotopy-perturbationmethod (HPM) in [38]. The two-dimensionalKdV-Burgers equation is solved byMolabahrami *et al.* [39] by using HPM. Zhang *et al.* [40] solved the modified Camassa-Holm and Degasperis-Procesi equations by the HPM. Moreover, many authors have developed and implemented methods of the new modifications [41-44].

Consider the following nonlinear differential equation:

For the purpose of applications illustration of the methodology of the proposed method, using homotopy perturbation method, we consider the following nonlinear differential equation,

$$A(u) - f(r) = 0,\ r \in \Omega, \tag{1}$$

$$B(u, \partial u/\partial n) = 0,\ r \in \Gamma, \tag{2}$$

Where A is a general differential operator, $f(r)$ is a known analytic function, B is a boundary condition and Γ is the boundary of the domain Ω.

The operator A can be generally divided into two operators, L and N where L is a linear, while N is a nonlinear operator. Equation (1) can be, therefore, written as follows:

$$L(u) + N(u) - f(r) = 0. \tag{3}$$

Using the homotopy technique, we construct a homotopy $U(r,p)$: $\Omega \times [0,1] \rightarrow \mathbb{R}$ which satisfies:

$$H(U, p) = (1 - p)[L(U) - L(u_0))] + p[A(U) - f(r)] = 0,\ p \in [0,1]\ r \in \Omega, \tag{4}$$

or

$$H(U, p) = L(U) - L(u_0) + pL(u_0) + p[N(U) - f(r)] = 0. \tag{5}$$

Where $p \in [0, 1]$, is called homotopy parameter and u_0 is an initial approximation for the solution of Eq.(1), which satisfies the boundary conditions. Obviously from Esq. (4) and (5) we will have.

$$H(U, 0) = L(U) - L(u_0) = 0. \tag{6}$$

$$H(U, 1) = A(U) - f(u) = 0. \tag{7}$$

We can assume that the solution of (4) or (5) can be expressed as a series in p, as follows:

$$U = U_0 + pU_1 + p^2U_2 + ... \tag{8}$$

Setting p =1, results in the approximate solution of Eq. (1)

$$u = \lim_{p \to 1} U = U_0 + U_1 + U_2 + \dots \tag{9}$$

HPM for Diffusivity Equation in Radial Form: The diffusivity equation in radial form [9].

$$\frac{\partial^2 p}{\partial r^2}+\frac{1}{r}\frac{\partial p}{\partial r}=\frac{\partial p}{\partial t}, \qquad (10)$$
$$p(r_i,0)=r.$$

With initial condition,

$$p(r_i,0)=0. \qquad (11)$$

To solve Eq. (10) by homotopy perturbation method, we construct the following homotopy

$$(1-q)\left(\frac{\partial^2 P}{\partial r^2}-\frac{\partial^2 p_0}{\partial r^2}\right)+q\left(\frac{\partial^2 P}{\partial r^2}+\frac{1}{r}\frac{\partial P}{\partial r}-\frac{\partial P}{\partial t}\right)=0,$$

or

$$\frac{\partial^2 P}{\partial r^2}-\frac{\partial^2 p_0}{\partial r^2}+q\left(\frac{\partial^2 p_0}{\partial r^2}+\frac{1}{r}\frac{\partial P}{\partial r}-\frac{\partial P}{\partial t}\right)=0, \qquad (12)$$

Suppose the solution of Eq. (12) has the following form

$$P=P_0+qP_1+q^2P_2+\ldots \qquad (13)$$

Substituting (13) into (12) and equating the coefficients of the terms with the identical powers of p leads to

$$p^0:\frac{\partial^2 P_0}{\partial r^2}-\frac{\partial^2 p_0}{\partial r^2}=0,$$
$$p^1:\frac{\partial^2 P_1}{\partial r^2}+\frac{1}{r}\frac{\partial P_0}{\partial r}-\frac{\partial P_0}{\partial t}=0,$$
$$p^2:\frac{\partial^2 P_1}{\partial r^2}+\frac{1}{r}\frac{\partial P_0}{\partial r}-\frac{\partial P_0}{\partial t}=0,$$
$$\vdots$$
$$p^j:\frac{\partial^2 P_j}{\partial r^2}+\frac{1}{r}\frac{\partial P_{j-1}}{\partial r}-\frac{\partial P_{j-1}}{\partial t}=0,$$
$$\vdots$$

We take

$$P_0=p_0=r. \qquad (14)$$

We have the following recurrent equations for j = 1,2,3,....

$$P_j=-\int_0^t \frac{1}{r}\frac{\partial P_{j-1}}{\partial r}-\frac{\partial P_{j-1}}{\partial t}dt. \qquad (15)$$

With the aid of the initial approximation given by Eq. (14) and the iteration formula (15) we get the other of component as follows

$$P_1=\frac{t}{r},$$
$$P_2=\frac{t^2}{2r^3},$$
$$\vdots$$
$$P_n=\frac{1^2\times 3^2\times\ldots\times(2n-3)^2t^n}{n!\times r^{2n-1}}.$$

Approximate solution of (10) can be obtained by setting q =1

$$p=\lim_{q\to 1}P=P_0+P_1+P_2+\ldots. \qquad (16)$$

In the same manner, the rest of components of the iteration were obtained using Maple Package. Using Taylor series into (16), we find the closed form solution.

$$p(r,t)=r+\sum_{n=1}^{\infty}\frac{1^2\times 3^2\times\ldots\times(2n-3)^2t^n}{n!\times r^{2n-1}}.$$

Which is the exact solution of the problem.

CONCLUSION

In this paper, we construct the exact solutions of various kinds of diffusivity equation in radial form by He's homotopy perturbation method. The method is effective, easy to use and reliable and the main benefit of the method is to offer an analytical approximation, in many cases an exact solution, in a rapid convergent series form. The solution process of homotopy perturbation method is compatible with those methods in the literature providing analytical approximation such as Adomian decomposition method [9]. In our work, we use the maple package to carry the computations.

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