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New Explicit Solutions of Petviashvili Equation Arising in Physics

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Abstract: By means of Exp-function method with symbolic computation system, we obtain some new exact solutions of the Riccati equation. By means of the Riccati equation and its new exact solutions, we find some new solutions of the Petviashvili equation arising in mathematical physics. It is shown that the proposed method provides a very effective and powerful mathematical tool for solving nonlinear evolution equations arising in physics.

Key words: Exp-function method . nonlinear evolution equations . auxiliary equation method . new exact solutions

INTRODUCTION

A large variety of physical, chemical and biological phenomena is governed by nonlinear evolution equations. The analytical study of nonlinear partial differential equations was of great interest during the last decade years. The investigations of the travelling wave solution of nonlinear equations play an important role in the study of nonlinear physical phenomena. The importance of obtaining the exact solutions, if available, of those nonlinear equations facilitates the verification of numerical solvers and aids in the stability analysis of solutions. In the past decade, both mathematicians and physicists have made significant progression in this direction.

Searching and constructing exact solutions for nonlinear evolution equations is an ongoing research. These exact solutions when they exist can help one to well understand the mechanism of the complicated physical phenomena and dynamical processes modeled by these nonlinear evolution equations. Various powerful methods for obtaining explicit travelling solitary wave solutions to nonlinear evolution equations have proposed such as [1-20].

More recently, He, Abdou [2] and Abdou [14, 15, 20] proposed a straight-forward and concise method, called Exp-function method, to obtain generalized solitary solutions and periodic solutions, applications of the method can be found in [2, 12, 14, 15, 20] for solving nonlinear evolution equations arising in mathematical physics. The solution procedure of this method, with the aid of Maple, is of utter simplicity and this method can easily extended to other kinds of nonlinear evolution equations.

The goal of the present work, we used the Exp-function method, to seek new exact solutions of the Riccati equation, then employ the Riccati equation and its solutions to find more general exact solutions of Petviashvili equation [21] as:

$$\frac{\partial}{\partial t}[\nabla^2\phi - \phi] + C_R[1+\phi]\frac{\partial}{\partial x} + J(\phi, \nabla^2\phi) = 0$$

$$\nabla^2 = \frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2}$$

$$C_R = \frac{\beta L_R^2}{\sqrt{gH}}, J(A,B) = \frac{\partial A \partial B}{\partial x \partial y} - \frac{\partial A}{\partial y}\frac{\partial B}{\partial x} \tag{1}$$

is the linear zero-dimensional phase velocity of Rossby wave.

EXACT SOLUTIONS OF RICCATI EQUATION VIA EXP-FUNCTION METHOD

The aim of this paper is motivated by the desire to Exp-function method to generalized Riccati equation. For illustration, we consider [17].

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$$\phi'(\zeta) = r + p\phi(\zeta) + q\phi^2(\zeta) \qquad (2)$$

where p, q and r are constants to be determined later and the prime denotes differentiation with respect to ζ. If setting some new variables and parameters

$$\psi = \frac{p}{2q} + \phi,\ l_0 = \frac{4qr - p^2}{4q^2},\ q\zeta = \xi$$

Then Eq. (2) reduces to

$$\psi'(\xi) = l_0 + \psi^2(\xi) \qquad (3)$$

where l_0 is a constant.
Introducing a complex variable η defined as [17]

$$\eta = k\xi + \xi_0 \qquad (4)$$

where k and ξ_0 are constants. Then eq. (3) convert into ordinary different equations, which read

$$k\psi' - l_0 - \psi^2 = 0 \qquad (5)$$

where the prime denotes the derivative with respect to η.

In view of Exp-function method, we assume that the solution of Eq. (5) can be expressed as

$$\psi(\eta) = \frac{\sum_{n=-p}^{q} a_n \exp(n\eta)}{\sum_{m=-e}^{f} b_m \exp(m\eta)} \qquad (6)$$

where p, q, f and e are positive integers which are unknown to be determined later, a_n and b_m are unknown constants.

In order to determine values of f and q, we balance the linear term of the highest order in Eq. (5) with the highest order nonlinear term ϕ' and ϕ^2, we have

$$\psi'(\eta) = \frac{c_1 \exp[(f+q)\eta] + \ldots\ldots}{c_2 \exp[2f\eta] + \ldots.} \qquad (7)$$

$$\psi^2(\eta) = \frac{c_3 \exp[2q\eta] + \ldots\ldots}{c_4 \exp[2f\eta] + \ldots.} \qquad (8)$$

where c_i are coefficients for simplicity. By balancing highest order of Exp-function in Eqs. (7) and (8), we have

$$q + f = 2q \qquad (9)$$

which leads to the results f = q. Proceeding the same manner as illustrated above, we can determine values of e and p. Balancing the linear term of lowest order in Eq. (5)

$$\psi'(\eta) = \frac{d_1 \exp[-(p+e)\eta] + \ldots\ldots}{d_2 \exp[-2e\eta] + \ldots.} \qquad (10)$$

$$\psi^2(\eta) = \frac{d_3 \exp[-2p\eta] + \ldots\ldots}{d_4 \exp[-2e\eta] + \ldots.} \qquad (11)$$

where d_i are coefficients for simplicity. By balancing highest order of Exp-function in Eqs. (11) and (10), we have

$$-(p+e) = -2e \qquad (12)$$

which leads to the result e = p.

Case (1): f = q = 1 and e = p = 1

For simplicity, we set f = q = 1 and e = p = 1, the trial function, Eq. (6) becomes

$$\psi(\eta) = \frac{a_1 \exp(\eta) + a_0 + a_{-1}\exp(-\eta)}{b_1 \exp(\eta) + b_0 + b_{-1}\exp(-\eta)} \qquad (13)$$

Substituting Eq. (13) into Eq. (5), equating to zero the coefficients of all powers of exp($n\eta$) yields a set of algebraic equations for a_0, b_0, a_1, a_{-1}, b_{-1} and k. Solving the system of algebraic equations with the aid of Maple, we obtain three sets of solutions

Case (i)

$$a_0 = b_0 = 0, a_1 = -\sqrt{-l_0}\,b_1,\ b_{-1} = \sqrt{\frac{-1}{l_0}}\,a_{-1}, k = -\sqrt{-l_0} \qquad (14)$$

with two arbitrary constants a_{-1} and b_1

Case (ii)

$$a_0 = b_0 = 0, a_1 = -i\sqrt{l_0}\,b_1, b_{-1} = -i\sqrt{\frac{1}{l_0}}\,a_{-1}, k = i\sqrt{l_0} \qquad (15)$$

with two arbitrary constants a_{-1} and b_1.

Case (iii)

$$a_1 = -\sqrt{-l_0}\,b_1,\ a_{-1} = -\sqrt{-l_0}\,b_{-1}, k = 2\sqrt{-l_0}, b_1 = \frac{a_0^2 + l_0 b_0^2}{4l_0 b - 1} \qquad (16)$$

with two arbitrary constants a_0, b_0, b_{-1} and

$$\eta = k\xi + \xi_0$$

According to case (1), we have the following generalized solitary wave solution

$$\psi(\eta)=\frac{-\sqrt{-l_0}b_1\exp(\eta)+a_{-1}\exp(-\eta)}{b_1\exp(\eta)+\sqrt{\frac{-1}{l_0}}a_{-1}\exp(-\eta)} \tag{17}$$

As some special examples, when $\xi_0=0$, $b_1=0$, $a_{-1}=\pm\sqrt{-l_0}$, then Eq. (17) reduces to

$$\psi(\eta)=-\sqrt{-l_0}\tanh(\sqrt{-l_0}\xi)$$

$$\psi(\eta)=-\sqrt{-l_0}\coth(\sqrt{-l_0}\xi) \tag{18}$$

which are these solitary wave solutions obtained by Yan *et al*. [17]

In view of Eq. (15), leads to

$$\psi(\eta)=\frac{-i\sqrt{l_0}b_1\exp(i\eta)+a_{-1}\exp(-i\eta)}{b_1\exp(i\eta)-i\sqrt{\frac{1}{l_0}}a_{-1}\exp(-i\eta)} \tag{19}$$

As some special examples, when $\xi_0=0, b_1=i$, $a_{-1}=\pm\sqrt{l_0}$, the solution (19) reads

$$\psi(\eta)=\sqrt{l_0}\tan(\sqrt{l_0}\xi)$$

$$\psi(\eta)=-\sqrt{l_0}\cot(\sqrt{l_0}\xi) \tag{20}$$

which are these triangular function solutions given by Yan *et al* [17].

From Eq. (16), we obtain the new exact solutions

$$\psi(\eta)=\frac{-\sqrt{-l_0}b_1\exp(2\eta)+a_0-\sqrt{-l_0}b_{-1}\exp(-2\eta)}{\frac{a_0^2+l_0b_0^2}{4l_0b_{-1}}\exp(2\eta)+b_0+b_{-1}\exp(-2\eta)} \tag{21}$$

Case [2]: $f=q=2$ and $e=p=1$

We consider the case $f=q=2$ and $e=p=1$, Eq. (6) can be expressed as

$$\psi(\eta)=\frac{a_2\exp(2\eta)+a_1\exp(\eta)+a_0+a_{-1}\exp(-\eta)}{b_2\exp(2\eta)+b_0+b_1\exp(\eta)+b_{-2}\exp(-2\eta)} \tag{22}$$

Inserting Eq. (22) into Eq. (5), equating to zero the coefficients of all powers of $\exp(n\eta)$ yields a set of algebraic equations for a_2, b_1, a_0, b_0, a_1, a_{-1}, b_{-2}, b_2 and k. Solving the system of algebraic equations with the aid of Maple, we obtain two sets of solutions

Case (i)

$$a_2=\sqrt{-l_0}b_2,\quad a_{-1}=-\frac{\sqrt{-l_0}}{l_0}b_{-1}, b_0=\frac{a_1^2+l_0b_1^2+2\sqrt{-l_0}a_0b_2}{2l_0b_2}$$
$$b_{-1}=\frac{(a_1+\sqrt{-l_0}b_1)(4a_0b_2-\sqrt{-l_0}a_1^2-l_0\sqrt{-l_0}b_1^2)}{8l_0^2b_2^2}, k=2\sqrt{-l_0} \tag{23}$$

with arbitrary constants a_0, a_1, b_1 and b_2

Case (ii)

$$b_{-1}=-\frac{\sqrt{-l_0}}{l_0}a_{-1}, b_0=\frac{\sqrt{-l_0}\,a_{-1}b_2}{l_0b_1}, a_0=-\frac{a_{-1}b_2}{b_1}$$
$$a_1=-\sqrt{-l_0}b_1, k=-2\sqrt{-l_0}, a_2=-\sqrt{-l_0}b_2 \tag{24}$$

with arbitrary constants a_{-1}, b_1 and b_2

In view of Eq. (23), we obtain exact solution of eq. (1) as follows

$$\psi(\eta)=\frac{\sqrt{-l_0}b_2\exp(2\eta)+a_1\exp(\eta)+a_0-\frac{\sqrt{-l_0}}{l_0}b_{-1}\exp(-\eta)}{b_2\exp(2\eta)+b_1\exp(\eta)+\frac{a_1^2+l_0b_1^2+2\sqrt{-l_0}a_0b_2}{2l_0b_2}+\frac{(a_1+\sqrt{-l_0}b_1)(4a_0b_2-\sqrt{-l_0}a_1^2-l_0\sqrt{-l_0}b_1^2)}{8l_0^2b_2^2}\exp(-\eta)}$$

$$\eta=2\sqrt{-l_0}\xi+\xi_0 \tag{25}$$

According to Eq. (24), we have the following solutions

$$\psi(\eta)=\frac{\sqrt{-l_0}b_2\exp(2\eta)-\sqrt{-l_0}b_1\exp(\eta)-\frac{a_{-1}b_2}{b_1}+a_{-1}\exp(-\eta)}{b_2\exp(2\eta)+\frac{\sqrt{-l_0}a_{-1}b_2}{l_0b_1}+b_1\exp(\eta)+\frac{\sqrt{-l_0}}{l_0}a_{-1}\exp(-\eta)} \tag{26}$$

$$\eta=-2\sqrt{-l_0}\xi+\xi_0$$

AUXILIARLY EQUATION METHOD

For a given nonlinear evolution equation, say, in two independent variables as follows

$$N(u,u_t,u_x,\ldots)=0 \tag{27}$$

and its travelling wave solution

$$\phi(x,t)=\phi(\xi),\quad \xi=kx-ct \tag{28}$$

where k and c are constants to be determined later.

Inserting Eq. (28) into Eq. (27) yields an ordinary differential equation of $\phi(\xi)$. Then $\phi(\xi)$ is expanded into a polynomial in $f(\xi)$ and $g(\xi)$

$$\phi(\xi) = A_0 + \sum_{i=0}^{M} f^{i-1}(\xi)[A_i f(\xi) + B_i g(\xi)] \qquad (29)$$

where A_0, A_i and B_i are constants to be determined later, M is fixed by balancing the linear term of the highest order derivative with nonlinear term, while $f(\xi)$ and $g(\xi)$ satisfy the system of equations

$$f' = \sqrt{pf^2 + \frac{1}{2}qf^4 + r}, f'' = pf + qf^3$$

$$g'' = g(c_1 + c_2 f^2), g^2 = c_3 + c_4 f^2$$

$$g' f' = fg(c_5 + c_6 f^2) \qquad (30)$$

where the prime denotes derivative with respect to ξ and p, q, r and c_i (i = 1, …6) are constants to be determined.

To look for the travelling wave solution of Eq. (1), we use the gauge transformation

$$\phi = \phi(\xi), \xi = kx + ly - wt \qquad (31)$$

Inserting Eq. (31) into (1), we have

$$-w(k^2 + l^2)\phi'''(\xi) + kC_R\phi(\xi)\phi'(\xi) + (w + kC_R)\phi'(\xi) = 0 \quad (32)$$

Integrating obtained equation once and setting the integration constant as 0, we have

$$-w(k^2 + l^2)\phi''(\xi) + \frac{k}{2}C_R\phi^2(\xi) + (w + kC_R)\phi(\xi) = 0 \qquad (33)$$

Eq. (33) can be rewritten as

$$A\phi''(\xi) + B\phi(\xi) + C\phi^2(\xi) = 0 \qquad (34)$$

$$A = 1,\ B = -\frac{(w + kC_R)}{w(k^2 + l^2)}, C = -\frac{kC_R}{2w(k^2 + l^2)}$$

Considering the homogeneous balance between $\phi''(\xi)$ and $\phi^2(\xi)$ in Eq. (34), we have M = 2. Therefore, we assume that $\phi(\xi)$ can be expressed as

$$\phi(\xi) = A_0 + A_1 f(\xi) + Bg(\xi) + A_2 f^2(\xi) + B_2 f(\xi)g(\xi) \quad (35)$$

where A_0, A_i and B_i are constants to be determined and $f(\xi)$ and $g(\xi)$ satisfy the system of equations (30). We substitute anzatz (35) into (34), make use of Eq. (30) with computerized symbolic computation, equating to zero the coefficients of all powers of $f^i(\xi)g^j(\xi)$ yields a set of algebraic equations for A_0, A_i and B_i. Solving the system of algebraic equations with the aid of Maple, we have

Case [1]

$$A_0 = -\frac{4p - \alpha}{2\beta}, A_1 = 0, A_2 = -\frac{3q}{\beta}, B_1 = B_2 = 0 \qquad (36)$$

Case [2]

$$A_0 = \frac{(p + c_1 + 2c_5) -}{2\beta}, A_1 = 0$$

$$B_1 = 0, A_2 = -\frac{(q + c_2 + 2c_6)A}{2\beta}$$

$$B_2^2 = \frac{(3p - c_1 - 2c_5)(q + c_2 + 2c_6)}{2c_3\beta^2} \qquad (37)$$

$$c_3 = (-5q + c_2 + 2c_6) + 2c_4(3p - c_1 - 2c_5) = 0$$

By means of Eqs. (36) and (35), admits to the new exact travelling wave solutions of Eq. (1) as follows

$$\phi(\xi) = -\frac{4p - \alpha}{2\beta} - \frac{3q}{\beta}f^2(\xi) \qquad (38)$$

Using Eqs. (37) and (35), admits to the new exact travelling wave solutions of Eq. (1) as follows

$$\phi(\xi) = -\frac{(p + c_1 + 2c_5) - \alpha}{2\beta} - \frac{(q + c_2 + 2c_6)}{2\beta}f^2(\xi) + \sqrt{\frac{(3p - c_1 - 2c_5)(q + c_2 + 2c_6)}{2c_3\beta^2}}f(\xi)g(\xi) \qquad (39)$$

where $f(\xi)$ and $g(\xi)$ satisfy Eq. (30) with the constraint among the parameters

$$c_3 = (-5q + c_2 + 2c_6) + 2c_4(3p - c_1 - 2c_5) = 0$$

where $\xi = kx + ly - wt$. Different classes of new periodic wave solutions can be obtained according to the different choice of the two functions $f(\xi)$ and $g(\xi)$. So we study only the solution of Eq. (39) in what follows.

New periodic wave solutions

Case (1): When $p = (2m^2 - 1)$, $q = (-2m^2)$, $r = (1 - m^2)$, $c_1 = m^2$, $c_2 = -2m^2$, $c_3 = (1 - m^2)$, $c_4 = m^2$, $c_5 = m^2$, $c_6 = -m^2$. We have $f(\xi)$ = cn(ξ) and $g(\xi)$ = dn(ξ). Thus the new periodic wave solution of Eq. (1) is

$$\phi_1(\xi) = \frac{(5m^2-1)-\alpha}{2\beta} - \frac{2m^2}{\beta}cn^2(\xi)$$

$$\pm\sqrt{\frac{9m^2(m^2-1)}{(1-m^2)^2}}\,cn(\xi)dn(\xi) \tag{40}$$

For $m\to 1$, Eq. (40) admits to solitary wave solution

$$\phi_{11}(\xi) = \frac{4-\alpha}{2\beta} - \frac{2}{\beta}\text{sech}^2(\xi)$$

Case (2): When $p = (2-m^2)$, $q = -2(1-m^2)$, $r = -1$, $c_1 = 1$, $c_2 = -2(1-m^2)$, $c_3 = -\frac{1}{m^2}$, $c_4 = \frac{1}{m^2}$, $c_5 = 1$, $c_6 = -(1-m^2)$, Here, we have $f(\xi) = nd(\xi)$ and $g(\xi) = sd(\xi)$ and the corresponding new periodic wave solution is

$$\phi_2(\xi) = \frac{(5-m^2)-\alpha}{2\beta} + \frac{(-6+6m^2)}{2\beta}nd^2(\xi) + \frac{1}{2}\sqrt{-\frac{36(1-m^2)(-1+m^2)}{\beta^2}}nd(\xi)sd(\xi) \tag{41}$$

For $m\to 0$, Eq. (41) admits to triangular function solution as

$$\phi_{20}(\xi) = \frac{5-\alpha}{2\beta} - \frac{3}{\beta} - \frac{3}{\beta}\sin(\xi)$$

Case (3): If we select $p = -(1+m^2)$, $q = 2$, $r = m^2$, $c_1 = -m^2$, $c_2 = 2$, $c_3 = -1$, $c_4 = 1$, $c_5 = -m^2$, $c_6 = 1$. We have $f(\xi) = ns(\xi)$ and $g(\xi) = cs(\xi)$ and we obtain the new periodic wave solution as follows

$$\phi_3(\xi) = \frac{(-1-4m^2)-\alpha}{2\beta} + \frac{3}{\beta}ns^2(\xi) + 3\sqrt{\frac{1}{\beta^2}}ns(\xi)cs(\xi) \tag{42}$$

When $m\to 1$, Eq. (42) admits to new solitary wave solution as follows

$$\phi_{31}(\xi) = \frac{-5-\alpha}{2\beta} + \frac{3}{\beta}\coth^2(\xi) + 3\sqrt{\frac{1}{\beta^2}}\coth(\xi)\text{csch}(\xi)$$

For $m\to 0$, Eq. (42) admits to triangular wave solution as

$$\phi_{30}(\xi) = \frac{-1-\alpha}{2\beta} + \frac{1}{\beta}\csc^2(\xi) + 3\sqrt{\frac{1}{\beta^2}}\csc(\xi)\cot(\xi)$$

Case (4): Now with $p = -(1+m^2)$, $q = 2m^2$, $r = 1$, $c_1 = -1$, $c_2 = 2m^2$, $c_3 = 1$, $c_4 = -1$, $c_5 = -1$, $c_6 = m^2$. In this case, we have $f(\xi) = sn(\xi)$ and $g(\xi) = cn(\xi)$ and thus the corresponding new periodic wave solution is

$$\phi_4(\xi) = \frac{(-4-m^2)-\alpha}{2\beta} + \frac{(3m^2)}{\beta}sn^2(\xi) + 3\sqrt{-\frac{m^4}{\beta^2}}sn(\xi)cn(\xi) \tag{43}$$

For $m\to 0$, Eq. (43) admits to rational solution

$$\phi_{40}(\xi) = \frac{-4-\alpha}{2\beta}$$

In case of $m\to 1$, Eq. (43) admits to solitary wave solution as follows

$$\phi_{41}(\xi) = \frac{-5-\alpha}{2\beta} + \frac{3}{\beta}\tanh^2(\xi) + 3\sqrt{-\frac{1}{\beta^2}}\tanh(\xi)\text{sech}(\xi)$$

Case (5): If $p = -(1+m^2)$, $q = 2m^2$, $r = 1$, $c_1 = -m^2$, $c_2 = 2m^2$, $c_3 = -1$, $c_4 = -m^2$, $c_5 = -m^2$, $c_6 = m^2$. In this case, we have $f(\xi) = sn(\xi)$ and $g(\xi) = dn(\xi)$ and thus the corresponding new periodic wave solution is

$$\phi_5(\xi) = \frac{(-1-4m^2)-\alpha}{2\beta} + \frac{3m^2}{\beta}sn^2(\xi) + 3\sqrt{\frac{-m^2}{\beta^2}}sn(\xi)dn(\xi) \tag{44}$$

When $m\to 1$, Eq. (44) admits to solitary wave solution

$$\phi_{51}(\xi) = \frac{-5-\alpha}{2\beta} + \frac{3}{\beta}\tanh^2(\xi) + 3\sqrt{\frac{-1}{\beta^2}}\tanh(\xi)\text{sech}(\xi)$$

Case (6): If we select $p = -(1+m^2)$, $q = 2$, $r = m^2$, $c_1 = -1$, $c_2 = 2$, $c_3 = -m^2$, $c_4 = 1$, $c_5 = -1$, $c_6 = 1$. We have $f(\xi) = ns(\xi)$ and $f(\xi) = ds(\xi)$ and we obtain the new periodic wave solution as

$$\phi_6(\xi) = \frac{(-4-m^2)-\alpha}{2\beta} + \frac{2}{\beta}ns^2(\xi) + 3\sqrt{\frac{1}{\beta^2}}ns(\xi)ds(\xi) \tag{45}$$

As long as $m\to 0$, Eq. (45), admits to triangular periodic wave solution

$$\phi_{60}(\xi) = \frac{-4-\alpha}{2\beta} + \frac{2}{\beta}\csc^2(\xi) + 3\sqrt{\frac{1}{\beta^2}}\csc^2(\xi)$$

As long as $m\to 0$, Eq. (45), admits to triangular periodic wave solution

$$\phi_{60}(\xi) = \frac{-4-\alpha}{2\beta} + \frac{2}{\beta}\csc^2(\xi) + 3\sqrt{\frac{1}{\beta^2}}\csc^2(\xi)$$

For $m\to 1$, Eq. (45) admits to solitary wave solution as

$$\phi_{61}(\xi) = \frac{-5\alpha}{2\beta} + \frac{2}{\beta}\operatorname{csch}^2(\xi) + 3\sqrt{\frac{1}{\beta^2}}\operatorname{csch}(\xi)\coth(\xi)$$

SUMMARY AND DISCUSSION

In summary, we use the Exp-function method with a computerized symbolic computation to seek new exact solutions of the Riccati equation. Then we employ the Riccati equation (2) and its new solutions to find some new exact solutions of Petviashvili equation. It worth noting that the Exp-function method is more effective and simple than other methods and a lot of solutions can be obtained in the same time. In addition, this method is also comupterizable, which allows us to perform complicated and tedious algebraic calculation on a computer.

In view Exp-function method, we give a very simple and straightforward method for nonlinear evolution equations arising in mathematical physics. We make some important remarks on the method as follows

1. The method leads to both generalized solitary solutions and periodic solutions.
2. The expression of the Exp-function is more general than the sinh-function and the tanh-function, so we can found more general solutions in the Exp-function method.
3. The Exp-function method can be employed in both the straightforward way and the sub-equation way. But we suggest that it is better to use this method directly, not only for its convenience, but also because it is sometimes possible to lose some information and solutions if we apply it in the sub-equation way.

Finally, it can be easily seen that the method used in this paper must futher be improved to solve more nonlinear partial differential equations arising in mathematical physics. This is our task in the future.

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REFERENCES

1. El-Wakil, S. A., M. A. Madkour, M. T. Attia, A. Elhanbaly and M. A. Abdou, 2011. Construction of periodic and solitary wave solutions for nonlinear evolution equations. International Journal of Nonlinear Science, In Press.
2. Ji-Huan He and M. A. Abdou, 2007. New periodic solutions for nonlinear evolution equations using Exp function method. Chaos, Solitons and Fractals, 34: 1421-1429.
3. El-Wakil, S. A. and M. A. Abdou, 2006. The extended mapping method and its applications for nonlinear evolutions equations. Phys. Letter A, 358: 275-282.
4. Abdou, M. A. and S. Zhang, 2009. Comm. Non. Sci. Numer. Sim., 14: 2-11.
5. Abdou, M. A., 2008. Exact periodic wave solutions to some nonlinear evolution equations. International Journal of Nonlinear Science, 5: 1-9.
6. Peng, Y. Y., 2003. Exact periodic wave solutions to a Hamiltonian Amplitude Equations. J. Phys. Soc. Jpn., 72: 1356-1359.
7. He, J. H., 2006. Non perturbative method for strongly nonlinear problems. dissertation, de Verlag in Internt GmbH, Berlin.
8. Yu, S. and Lix in Tian, 2008. Nonsymmetrical Kink solution of generalized KdV equation with variable coefficients. International Journal of Nonlinear Science, 5: 71-78.
9. Abdou, M. A., 2008. An Extended Riccati equation rational equation method and its applications. International Journal of Nonlinear Science, (In Press).
10. Zhang, X. L. and H. Q. Zhang, 2007. A new generalized Riccati equation rational expansion method to a class of nonlinear evolution equations with nonlinear terms of any order. Appl. Math. and Comput., 186: 705-714.
11. Chen, Y., Q. Wang and B. Li, 2005. Elliptic equation rational expansion method and new exact travelling solutions for Whitham BroerKaup equations. Chaos, Solitons and Fractals, 26: 231-246.
12. He, J. H. and Xu-Hong Wu, 2006. Exp-function method for nonlinear wave equations, Chaos Solitons and Fractals, 30: 700-708.
13. Yomba, E., 2004. On exact solutions of the coupled Klein Gordon Schrodinger and the complex coupled KDV equations using mapping method. Chaos Solitons & Fractals, 21: 209-229.
14. El-Wakil, S. A., M. A. Abdou and A. Hendi, 2008. Physics Letter A, 372: 830-840.
15. Abdou, M. A., 2008. Generalized solitary and periodic solutions for nonlinear partial differential equations by the Exp-function method. Nonlinear Dynamics, 52 (1-2): 1-9.
16. Wang, M. and X. Li, 2005. Application of F-expansion to periodic wave solutions of a new Hamiltionian amplitude equation. Chaos, Solitons and Fractals, 24: 1257-1268.

17. Chao-Qing Dai and Yue-Yue Wang, 2009. New variable separation solutions of the (2+1)-dimensional asymmetric Nizhnik Novikov Veselov system, Nonlinear Analysis, 71: 14961503.
18. Abdou, M. A. and A. A. Soliman, 2005. New applications of Variational iteration method. Physica D, 211: 1-8.
19. Abdou, M. A., 2007. Further improved F-expansion and new exact solutions for nonlinear evolution equations. J. of Nonlinear Dynamics, 52 (3): 277-288.
20. El-Wakil, S.A., M. A. Madkour and M. A. Abdou, 2007. Application of Exp-function method for nonlinear evolution equations with variable coefficients. Physics Letter A, 369: 62-69.
21. Zhao, Q. and S. K. Liu, 2006. Chin. J. Geophys., 49: 965.