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Multiple Periodic Solutions of Delayed Predator-prey Systems with Harvesting Terms and Holing III Type Functional Responses on Time Scales

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Abstract: In this paper, we consider a periodic delayed predator-prey systems with harvesting terms and Holing III type functional responses on time scales. Using coincidence degree theory, some sufficient conditions are obtained for the global existence of multiple positive periodic solutions of the model.

Key words: Delayed predator-prey systems . harvesting terms . periodic solution . coincidence degree . time scales

INTRODUCTION

In recent years, the existence of periodic solutions for the predator-prey model has been widely studied. Since the exploitation of biological resources and the harvest of population species are commonly practiced in fishery, forestry and wildlife management, the study of population dynamics with harvesting is an important subject in mathematical bioeconomics, which is related to the optimal management of renewable resources [6, 16, 22]. Recently, many scholars investigated some predator-prey models with harvesting [9, 13, 27, 28].

It is well known that two species Lotka-Volterra predatory-prey model with harvesting terms can be formulated by [19, 23]:

$$\begin{cases} \dot{x}(t)=x(t)(a_1-b_1x(t)-c_1y(t))-h_1 \\ \dot{y}(t)=x(t)(a_2-b_2x(t)-c_2y(t))-h_2 \end{cases} \quad (1)$$

where x (t) and y (t) denote the densities of the prey and the predator, respectively; a_i and b_i(i = 1,2) are all positive constants and denote the intrinsic growth rates and the intra-specific competition rates, respectively; $c_1>0$ is the predation rate of the predator and $c_2>0$ represents the conversion rate at which the ingested prey in excess of what is needed for maintenance is translated into the predator population increase; h_i(i = 1,2) is the ith species harvesting terms standing for the harvests. Recently, Zhao and Ye [29] considered the following non-autonomous model:

$$\begin{cases} \dot{x}(t)=x(t)(a_1(t)-b_1(t)x(t)-c_1(t)y(t))-h_1(t) \\ \dot{y}(t)=x(t)(a_2(t)-b_2(t)x(t)-c_2(t)y(t))-h_2(t) \end{cases} \quad (2)$$

and some sufficient conditions are obtained for the existence of four positive periodic solutions of (2). Very recently, Wei [25] studied the following three-species periodic predator-prey system with Holling III type functional response and harvesting term:

$$\begin{cases} \dot{y}_1(t)=y_1(t)[r_1(t)-a_{11}(t)y_1(t)-\dfrac{a_{12}(t)y_1(t)N_2(t)}{1+b_{12}(t)y_1^2(t)}] \\ \qquad -h_1(t) \\ \dot{y}_2(t)=y_2(t)[r_2(t)+\dfrac{\theta_1(t)a_{12}(t)y_1(t)y_1(t)}{1+b_{12}(t)y_1^2(t)}-a_{22}(t)y_2(t) \\ \qquad -\dfrac{a_{23}(t)y_2(t)y_3(t)}{1+b_{23}(t)y_2^2(t)}]-h_2(t) \\ \dot{y}_3(t)=y_3(t)[r_3(t)+\dfrac{\theta_2(t)a_{23}(t)y_2(t)y_2(t)}{1+b_{23}(t)y_2^2(t)} \\ \qquad -a_{33}(t)y_3(t)]-h_3(t) \end{cases} \quad (3)$$

where the parameters in system (3) are continuous positive ω-periodic functions and proved that the system (3) exists at least eight periodic positive solutions. One discrete analogue of system (3) can be rewritten as follows:

$$\begin{cases} y_1(t+1)=y_1(t)e^{r_1(t)-a_{11}(t)y_1(t)-\frac{a_{12}(t)y_1(t)N_2(t)}{1+b_{12}(t)y_1^2(t)}-\frac{h_1(t)}{y_1(t)}} \\ y_2(t+1)=y_2(t)e^{r_2(t)+\frac{\theta_1(t)a_{12}(t)y_1(t)y_1(t)}{1+b_{12}(t)y_1^2(t)}-a_{22}(t)y_2(t)-\frac{a_{23}(t)y_2(t)y_3(t)}{1+b_{23}(t)y_2^2(t)}-\frac{h_2(t)}{y_2(t)}} \\ y_3(t)=y_3(t)e^{r_3(t)+\frac{\theta_2(t)a_{23}(t)y_2(t)y_2(t)}{1+b_{23}(t)y_2^2(t)}-a_{33}(t)y_3(t)-\frac{h_3(t)}{y_3(t)}} \end{cases} \quad (4)$$

where t is integers.

As was pointed out by Kuang [15] that any model of species dynamics without delays is an approximation

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at best. More detailed arguments on the importance and usefulness of time-delays in realistic models may also be found in the classical books of Gopalsamy [11] and Macdonald [18]. Many scholars studied the delay predator-prey systems and obtained some interesting results Egami [7], Fan [8], He *et al*. [12], Huo [14] and Xu [26].

On the other hand, recently, in order to unify differential and difference equations, people have done a lot of research about dynamic equations on time scales. Moreover, many results on this issue have been well documented in the monographs [1-5, 12, 17, 20, 21, 24]. And, in fact, continuous and discrete systems are very important in implementing and applications. But it is troublesome to study the existence and stability of periodic solutions for continuous and discrete systems, respectively. Therefore, it is meaningful to study that on time scale which can unify the continuous and discrete situations.

Motivated by the above reasons, in this paper, we are concerned with the global existence of multiple positive periodic solutions of the following predator-prey system with delays on time scales:

$$\begin{cases} x_1^{\Delta}(t)=r_1(t)-a_{11}(t)e^{x_1(t)}-\dfrac{a_{12}(t)e^{x_1(t-\tau(t))}e^{x_2(t)}}{1+b_{12}(t)e^{x_1(t)}e^{x_1(t-\tau(t))}} \\ \qquad\qquad -h_1(t)e^{-x_1(t)} \\ x_2^{\Delta}(t)=r_2(t)-a_{22}(t)e^{x_2(t)}+\zeta(t)-h_2(t)e^{-x_2(t)} \\ x_3^{\Delta}(t)=r_3(t)+\dfrac{\theta_2(t)a_{23}(t)e^{x_2(t-\tau(t))}e^{x_2(t-\tau(t))}}{1+b_{23}(t)e^{2x_2(t-\tau(t))}} \\ \qquad\qquad -a_{33}(t)e^{x_3(t)}-h_3(t)e^{-x_3(t)} \end{cases} \tag{5}$$

where

$$\zeta(t)=\frac{\theta_1(t)a_{12}(t)e^{x_1(t-\tau(t))}e^{x_1(t-\tau(t))}}{1+b_{12}(t)e^{2x_1(t-\tau(t))}}-\frac{a_{23}(t)e^{x_2(t-\tau(t))}e^{x_3(t)}}{1+b_{23}(t)e^{x_2(t-\tau(t))}e^{x_2(t)}}$$

x_j (t) stands for the density of jth species at time t, respectively; T is a time scale, i.e., T is an arbitrary nonempty closed subset of the real numbers R; r_j (t) represents the jth species intrinsic growth rates; a_{jj} (t) denotes the intra-specific competition rates of the jth species; θ_1 (t), θ_2 (t) are the nutrition conversion rates for the first species to the second species, the second species to the third species, respectively; h_j (t) is the harvesting term for the jth species. Moreover, $\tau(t)$, $r_j(t)$, $a_{jj}(t)$, $b_{jj}(t)$, $h_j(t)$ and $a_{12}(t)$, $a_{23}(t)$, $b_{12}(t)$, $b_{23}(t)$, $\theta_1(t)$, $\theta_2(t)$ are rd-continuous, bounded and strictly positive ω-periodic functions defined on $[0,+\infty(j=1,2,3)$. The symbol Δ stands for the delta-derivative and ω is periodic time scale which has the subspace topology inherited form the standard topology on R.

It is easy to see that the predator-prey system (5) with delays on time scales includes the systems (1-4) as special cases. Thus, the predator-prey system (5) provides a general setting for the study of the continuous and discrete predator-prey system with delays. To our best knowledge, (5) has not been investigated on time scales so far.

In this paper, by using the coincidence degree theory, some sufficient conditions are obtained which guarantee the global existence of multiple positive periodic solutions of system (5). Our results incorporate and extend a known result in the literature essentially when the time scale is chosen as the real numbers and the delay is zero.

PRELIMINARIES

In this section, we first recall some basic definitions, lemmas on time scales which are used in the following. For more details, we refer readers to [1, 3, 5].

Let T be a nonempty closed subset (time scale) of R. Throughout this paper, we assume that the time scale T is unbounded above and below, such as R,Z and $\bigcup_{k\in Z}[2k,2k+1]$.

The forward and backward jump operators σ, ρ:T→T and the graininess μ: T→R^+ are defined, respectively, by

$$\sigma(t)=\inf\{s\in T: s>t\}$$
$$\rho(t)=\sup\{s\in T: s<t\},\quad \mu(t)=\sigma(t)-t$$

A point t∈T is called left-dense if t> inf T and ρ(t) = t, left-scattered if ρ(t) < t, right-dense if t<sup T and σ (t) = t and right-scattered if σ (t)>t. If T has a left-scattered maximum m, then T^k = T\{m}; otherwise T^k = T. If T has a right-scattered minimum m, then T^k = T\{m}; otherwise T^k = T.

Let ω∈R and ω>0. We say that T is ω-periodic time scale if T is a nonempty closed subset of R such that t+ω∈T and μ(t) = μ(t+ω) whenever t∈T.

A function *f*:T→R is right-dense continuous provided it is continuous at right-dense point in T and its left-side limits exist at left-dense points in T. If *f* is continuous at each right-dense point and each left-dense point, then *f* is said to be a continuous function on T. We define C(J,R) = {u(t) is continuouson J}.

For y:T→R and t∈T^k, we define the delta derivative of t(t), $t^{\Delta}(t)$, to be the number (if it exists) with the property that for a given ε>0, there exists a neighborhood U of t such that

$$|[y(\sigma(t))-y(s)]-y^{\Delta}(t)[\sigma(t)-s]|<\varepsilon\,|\,\sigma(t)-s$$

for all $s\in U$.

If y is continuous, then y is right-dense continuous and if y is delta differentiable at t, then y is continuous at t.

Let y be right-dense continuous. If $Y^{\Delta}(t) = y(t)$, then we define the delta integral by

$$\int_a^t y(s)\Delta s = Y(t) - Y(a)$$

Definition 2.1: [3] We say that a time scale T is periodic if there exists p>0 such that if $t\in T$, then $t\pm p\in T$. For $T\neq R$, the smallest positive p is called the period of the time scale.

Definition 2.2: [3] Let $T\neq R$ be a periodic time scale with period p. We say that the function f: $T\to R$ is periodic with period ω if there exists a natural number n such that $\omega = np$, $f(t+\omega) = f(t)$ for all $t\in T$ and ω is the smallest number such that $f(t+\omega)$: $f(t)$.

If T = R, we say that f is periodic with period $\omega>0$ if ω is the smallest positive number such that $f(t+\omega)=f(t)$ for all $t\in T$.

Definition 2.3: [3] A function f: $T\to R$ is said to be rd-continuous if it is continuous at right-dense points in T and its left-sides limits exist (finite) at left-dense points in T. The set of rd-continuous functions is denoted by

$$C_{rd} = C_{rd}(T) = C_{rd}(T,R)$$

Lemma 2.1: [3] Every rd-continuous function has an antiderivative.

Lemma 2.2: [3] If $a,b\in T$, α, $\beta\in R$ and f, $g\in C(T,R)$, then the following conclusions hold:

- $\int_a^b [\alpha f(t)+\beta g(t)]\Delta t = \alpha\int_a^b f(t)\Delta t+ \beta\int_a^b g(t)\Delta t$
- if $f(t)\geq 0$ for all $a\leq t\leq b$, then $\int_a^b f(t)\Delta t \geq 0$
- if $|f(t)|\leq g(t)$ on $[a,b) := \{t\in T: a\leq t<b\}$, then $|\int_a^b f(t)\Delta t| \leq \int_a^b g(t)\Delta t$

The following concepts and facts can be found in the book due to Gains and Mawhin [10].

Let X,Z be normed vector spaces, $L:\text{Dom } L\subset X\to Z$ be a linear mapping and $N:X\times[0,1]\to Z$ be a continuous mapping. The mapping L will be called a Fredholm mapping of index zero if $\dim\text{Ker } L = \text{codim} \text{Im} <+\infty$ and Im L is closed in Z. If L is a Fredholm mapping of index zero, then there exist continuous projectors $P: X\to X$ and $Q:Z\to Z$ such that $\text{Im } P = \text{Ker}L$ and $\text{Im}L=\text{Ker}Q=\text{Im}I-Q$ and $X=\text{Ker}L\otimes\text{Im}Q$, $Z=\text{Im } L\otimes\text{Im}Q$. It follows that mapping $L_{\text{Dom}L\bigcap\text{Ker}P}:(I-P)X\to\text{Im}L$ is invertible and its inverse is denoted by K_P. If Ω is a bounded open subset of X, the mapping N is called L-compact on $\bar{\Omega}\times[0,1]$, if $QN\ (\bar{\Omega}\times[0,1])$ is bounded and $K_P(I-Q)N: \bar{\Omega}\times[0,1]\to X$ is compact. Since ImQ is isomorphic to KerL, we know that there exists an isomorphism $J: \text{Im}Q\to\text{Ker}L$.

The following Lemma is the Mawhin continuous theorem.

Lemma 2.3: [10] Let L be a Fredholm mapping of index zero and let N be L-compact on $\bar{\Omega}$. Assume that

- for each $\lambda\in(0,1)$, every solution x of $Lx = \lambda N(x,\lambda)$ is such that $x\notin\partial\Omega\bigcap\text{Dom}L$;
- $QN(x,0)x \neq 0$ for each $x\in\partial\Omega\bigcap\text{Dom}L$;
- $\deg(JQN(x,0),\Omega\bigcap\text{Ker}L,0)\neq 0$.

Then the equation Lx = Nx has at least one solution in $\bar{\Omega}\bigcap\text{Dom}L$.

In the following, we denote the notation

$$R^{+} = [0,+\infty), k=\min\{R^{+}\bigcap T\}, I_{\omega} = [k,k+\omega]\bigcap T$$

To facilitate the discussion below, throughout this paper we adopt the following notation and all the other notations appearing in this paper are defined analogously.

$$f^{L} = \min_{t\in I_{\omega}} f(t), \quad f^{M} = \max_{t\in I_{\omega}} f(t), \quad \bar{f} = \frac{1}{\omega}\int_{I_{\omega}} f(s)\Delta s$$

where $f\in C_{rd}(T)$ is an ω-periodic real function.

For convenience, we introduce some notations as follows:

$$l_1^{\pm} = \frac{1}{2a^{L}11}(r_1^{M} \pm \sqrt{(r_1^{M})^2 - 4a_{11}^{L}h_1^{L}})$$

$$l_2^{\pm} = \frac{1}{2a^{L}22}(r_2^{M} + \frac{a_{12}^{M}}{b_{12}^{L}} \pm \sqrt{(r_2^{M} + \frac{a_{12}^{M}}{b_{12}^{L}})^2 - 4a_{22}^{L}h_2^{L}})$$

$$l_3^{\pm} = \frac{1}{2a^{L}33}(r_3^{M} + \frac{a_{23}^{M}}{b_{23}^{L}} \pm \sqrt{(r_3^{M} + \frac{a_{23}^{M}}{b_{23}^{L}})^2 - 4a_{33}^{L}h_3^{L}})$$

$$H_1^{\pm} = \frac{1}{2a_{11}^{M}}(r_1^{L} \pm \sqrt{(r_1^{L})^2 - 4a_{11}^{M}(\frac{a_{12}^{M}}{b_{12}^{L}} + h_1^{M})})$$

$$H_2^{\pm} = \frac{1}{2a_{22}^M}\left(r_2^L \pm \sqrt{(r_2^L)^2 - 4a_{22}^M\left(\frac{a_{23}^M l_3^+}{b_{23}^L} + h_2^M\right)}\right)$$

$$H_3^{\pm} = \frac{1}{2a_{33}^M}\left(r_3^L \pm \sqrt{(r_3^L)^2 - 4a_{33}^M h_3^M}\right)$$

We assume the following hypothesis throughout this paper:

$$(A_1)\ r_1^L > 2\sqrt{a_{11}^M\left(\frac{a_{12}^M l_2^+}{b_{12}^L} + h_1^M\right)}$$

$$(A_2)\ r_2^L > 2\sqrt{a_{22}^M\left(\frac{a_{23}^M l_3^+}{b_{23}^L} + h_2^M\right)}$$

$$(A_3)\ r_3^L > 2\sqrt{a_{33}^M h_3^M}$$

Lemma 2.4: [25] Let x>0, y>0, z>0 and $x > 2\sqrt{yz}$. For functions

$$f(x,y,z) = \frac{x + \sqrt{x^2 - 4yz}}{2z}$$

and

$$g(x,y,z) = \frac{x - \sqrt{x^2 - 4yz}}{2z}$$

the following assertions hold:

- $f(x,y,z)$ and $g(x,y,z)$ are monotonically increasing and monotonically decreasing on the variable $x \in (0,+\infty)$, respectively;
- $f(x,y,z)$ and $g(x,y,z)$ are monotonically decreasing and monotonically increasing on the variable $y \in (0,+\infty)$, respectively;
- $f(x,y,z)$ and $g(x,y,z)$ are monotonically decreasing and monotonically increasing on the variable $z \in (0,+\infty)$, respectively.

Lemma 2.5: The following statements are true.

$$l_1^+ > H_1^+,\quad l_1^- < H_1^-,\quad l_2^+ > H_2^+$$
$$l_2^- < H_2^-,\quad l_3^+ > H_3^+,\quad l_3^- < H_3^-$$

Proof: According to Lemma 2.4, we have

$$l_1^+ = f(r_1^M, h_1^L, a_{11}^L) > f\left(r_1^L, \frac{a_{12}^M l_2^+}{b_{12}^L} + h_1^M, a_{11}^M\right) = H_1^+$$

$$l_1^- = g(r_1^M, h_1^L, a_{11}^L) < g\left(r_1^L, \frac{a_{12}^M l_2^+}{b_{12}^L} + h_1^M, a_{11}^M\right) = H_1^-$$

$$l_2^+ = f\left(r_2^M + \frac{a_{12}^M}{b_{12}^L}, h_2^L, a_{22}^L\right) > f\left(r_2^L, \frac{a_{23}^M l_3^+}{b_{23}^L} + h_2^M, a_{22}^M\right) = H_2^+$$

$$l_2^- = g\left(r_2^M + \frac{a_{12}^M}{b_{12}^L}, h_2^L, a_{22}^L\right) < g\left(r_2^L, \frac{a_{23}^M l_3^+}{b_{23}^L} + h_2^M, a_{22}^M\right) = H_2^-$$

$$l_3^+ = f\left(r_3^M + \frac{a_{23}^M}{b_{23}^L}, h_3^L, a_{33}^L\right) > f(r_3^L, h_3^M, a_{33}^M) = H_3^+$$

and

$$l_3^- = g\left(r_3^M + \frac{a_{23}^M}{b_{23}^L}, h_3^L, a_{33}^L\right) < g(r_3^L, h_3^M, a_{33}^M) = H_3^-$$

This completes the proof.

MAIN RESULTS

We are now in a position to state and prove our main result of this paper.

Theorem 3.1: Suppose that (A_1), (A_2), (A_3) hold. Then dynamic system (5) has at least eight positive ω-periodic solutions.

Proof: In order to apply Lemma 2.3 to system (5), let

$$X = Z = \{x = (x_1(t), x_2(t), x_3(t))^T : x_i(t) \in C_{rd},$$
$$x_j(t+\omega) = x_j(t),\ t \in T, j = 1,2,3\}$$

be equipped with the norm

$$||x|| = \max_{t\in I_\omega}|x_1(t)| + \max_{t\in I_\omega}|x_2(t)| + \max_{t\in I_\omega}|x_3(t)|$$

where $|\cdot|$ is the Euclidean norm. Then X and Z are Banach spaces. Let

$$N(x,\lambda) = (\Phi_1(t,\lambda), \Phi_2(t,\lambda), \Phi_3(t,\lambda))^T$$

$$Lx = (x_1^\Delta(t), x_2^\Delta(t), x_3^\Delta(t))^T$$

$$Px = \left(\frac{1}{\omega}\int_{I_\omega} x_1(t)\Delta t, \frac{1}{\omega}\int_{I_\omega} x_2(t)\Delta t, \frac{1}{\omega}\int_{I_\omega} x_3(t)\Delta t\right)^T$$

$$Qz = \left(\frac{1}{\omega}\int_{I_\omega} z_1(t)\Delta t, \frac{1}{\omega}\int_{I_\omega} z_2(t)\Delta t, \frac{1}{\omega}\int_{I_\omega} z_3(t)\Delta t\right)^T$$

where $x \in X$, $z \in Z$, $\lambda \in (0,1)$ and

$$\Phi_1(t,\lambda) = r_1(t) - a_{11}(t)e^{(x_1(t))} - \frac{\lambda a_{12}(t)e^{x_1(t-\tau(t))}e^{x_2(t)}}{1 + b_{12}(t)e^{x_1(t)}e^{x_1(t-\tau(t))}} - h_1(t)e^{-x_1(t)}$$

$$\Phi_2(t,\lambda)=r_2(t)+\frac{\lambda\theta_1(t)a_{12}(t)e^{x_1(t-\tau(t))}e^{x_1(t-\tau(t))}}{1+b_{12}(t)e^{2x_1(t-\tau(t))}}-a_{22}(t)e^{x_2(t)}$$
$$-\frac{\lambda a_{23}(t)e^{x_2(t-\tau(t))}e^{x_3(t)}}{1+b_{23}(t)e^{x_2(t-\tau(t))}e^{x_2(t)}}-h_2(t)e^{-x_2(t)}$$

$$\Phi_3(t,\lambda)=r_3(t)+\frac{\lambda\theta_2(t)a_{23}(t)e^{x_2(t-\tau(t))}e^{x_2(t-\tau(t))}}{1+b_{23}(t)e^{2x_2(t-\tau(t))}}$$
$$-a_{33}(t)e^{x_3(t)}-h_3(t)e^{-x_3(t)}$$

With the above definitions, we obtain

$$\mathrm{KerL}=\{x\in X: x=h=(h_1,h_2,h_3)^T\in R^3, t\in T\}$$

$$\mathrm{ImL}=\{z\in Z:\int_{I_\omega}z_j(t)\Delta t=0, j=1,2,3, t\in T\}$$

P,Q are continuous projectors such that

$$\mathrm{ImP=KerL, ImL=KerQ=Im}(I-Q)$$

ImL is closed in Z and

$$\mathrm{dimKerL=3=codimImL}$$

Therefore, L is is a Fredholm mapping of index zero. Furthermore, the generalized inverse (to L) $K_p:\mathrm{ImL}\to\mathrm{DomL}\bigcap\mathrm{KerP}$ exists and is given by

$$K_p x=(X_1,X_2,X_3)^T$$

where

$$X_j=\int_k^t x_j(s)\Delta s-\frac{1}{\omega}\int_{I_\omega}\int_k^t x_j(s)\Delta s\Delta t$$

for j = 1,2,3. Thus

$$QN(x,\lambda)=\begin{pmatrix}\frac{1}{\omega}\int_{I_\omega}\Phi_1(s,\lambda)\Delta s,\frac{1}{\omega}\int_{I_\omega}\Phi_2(s,\lambda)\Delta s,\\ \frac{1}{\omega}\int_{I_\omega}\Phi_3(s,\lambda)\Delta s\end{pmatrix}^T$$

and

$$K_p(I-Q)N(x,\lambda)=(\Theta_1(t,\lambda),\Theta_2(t,\lambda),\Theta_3(t,\lambda))^T$$

where

$$\Theta_j(t,\lambda)=\int_k^t\Phi_j(s)\Delta s-\frac{1}{\omega}\int_{I_\omega}\int_k^t\Phi_j(s)\Delta s\Delta t$$
$$-[t-k-\frac{1}{\omega}\int_{I_\omega}(t-k))\Delta t]\overline{\Phi_j}$$

for j = 1,2,3.

Obviously, QN and K_P(I-Q)N are continuous. By using the Arzela-Ascoli theorem, one can show that $\overline{(K_P(I-Q)N)(\Omega)}$ is compact for any open bounded set $\Omega\subset X$. Moreover, $N(\overline{\Omega})$ is clearly bounded. Thus, N is L-compact on $\overline{\Omega}$ for any open bounded set $\Omega\subset X$.

Now we reach the position to search for at least eight appropriate open bounded subsets Ω for the application of Lemma 2.3.

Consider the operator equation

$$Lx=\lambda N(x,\lambda),\quad \lambda\in(0,1) \tag{1}$$

we have

$$\begin{cases}x_1^\Delta(t)=\lambda\Phi_1(t,\lambda)\\ x_2^\Delta(t)=\lambda\Phi_2(t,\lambda)\\ x_3^\Delta(t)=\lambda\Phi_3(t,\lambda)\end{cases} \tag{2}$$

Assume that $(x_1(t), x_2(t), x_3(t))^T\in X$ is a ω-periodic solution of (2) for a certain $\lambda\in(0,1)$. Then there exist $\xi_j,\eta_j\in I_\omega (j = 1,2,3)$ such that

$$x_j(\xi_j)=\max_{t\in I_\omega}x_j(t), x_j(\eta_i)=\min_{t\in I_\omega}x_j(t),\quad j=1,2,3 \tag{3}$$

From (2) and (3), we have

$$\begin{cases}\Phi_1(\xi_1,\lambda)=0\\ \Phi_2(\xi_2,\lambda)=0\\ \Phi_3(\xi_3,\lambda)=0\end{cases} \tag{4}$$

and

$$\begin{cases}\Phi_1(\eta_1,\lambda)=0\\ \Phi_2(\eta_2,\lambda)=0\\ \Phi_3(\eta_3,\lambda)=0\end{cases} \tag{5}$$

According to the first equation of (4), we have

$$a_{11}^L e^{x_1(\xi_1)}+h_1^L e^{-x_1(\xi_1)}\le a_{11}(\xi_1)e^{x_1(\xi_1)}+h_1(\xi_1)e^{-x_1(\xi_1)}$$
$$=r_1(\xi_1)-\frac{\lambda a_{12}(\xi_1)e^{x_1(\xi_1-\tau(\xi_1))}e^{x_2(\xi_1)}}{1+b_{12}(\xi_1)e^{x_1(\xi_1)}e^{x_1(\xi_1-\tau(\xi_1))}}<r_1^M$$

or

$$a_{11}^L e^{2x_1(\xi_1)}-r_1^M e^{x_1(\xi_1)}+h_1^L<0$$

which implies that

$$\ln l_1^-<x_1(\xi_1)<\ln l_1^+ \tag{6}$$

By analogue arguments to the first equation of (5) yields

$$\ln l_1^-<x_1(\eta_1)<\ln l_1^+ \tag{7}$$

By the second equation of (4), we obtain

$$a_{22}^{L}e^{x_2(\xi_2)}+h_2^{L}e^{-x_2(\xi_2)}\le a_{22}(\xi_2)e^{x_2(\xi_2)}+h_2(\xi_2)e^{-x_2(\xi_2)}$$
$$<r_2(\xi_2)+\frac{a_{12}(\xi_2)}{b_{12}(\xi_2)}<r_2^{M}+\frac{a_{12}^{M}}{b_{12}^{L}}$$

or

$$a_{22}^{L}e^{2x_2(\xi_2)}-(r_2^{M}+\frac{a_{12}^{M}}{b_{12}^{L}})e^{x_2(\xi_2)}+h_2^{L}<0$$

which implies that

$$\ln l_2^{-}<x_2(\xi_2)<\ln l_2^{+} \qquad (8)$$

By analogue arguments to the second equation of (5) yields

$$\ln l_2^{-}<x_2(\eta_2)<\ln l_2^{+} \qquad (9)$$

From the third equation of (4), we obtain

$$a_{33}^{L}e^{x_3(\xi_3)}+h_3^{L}e^{-x_3(\xi_3)}\le a_{33}(\xi_3)e^{x_3(\xi_3)}+h_3(\xi_3)e^{-x_3(\xi_3)}$$
$$<r_3(\xi_3)+\frac{a_{23}(\xi_3)}{b_{23}(\xi_3)}<r_3^{M}+\frac{a_{23}^{M}}{b_{23}^{L}}$$

or

$$a_{33}^{L}e^{2x_3(\xi_3)}-(r_3^{M}+\frac{a_{23}^{M}}{b_{23}^{L}})e^{x_3(\xi_3)}+h_3^{L}<0$$

which implies that

$$\ln l_3^{-}<x_3(\xi_3)<\ln l_3^{+} \qquad (10)$$

By analogue arguments to the third equation of (5) yields

$$\ln l_3^{-}<x_3(\eta_3)<\ln l_3^{+} \qquad (11)$$

On the other hand, from the first equation (4), we obtain

$$a_{11}^{M}e^{2x_1(\xi_1)}-r_1^{L}e^{x_1(\xi_1)}+\frac{a_{12}^{M}l_2^{+}}{b_{12}^{L}}+h_1^{M}>0$$

which implies that

$$x_1(\xi_1)>\ln H_1^{+}\text{ or }x_1(\xi_1)<\ln H_1^{-} \qquad (12)$$

Similarly, by the first equation (5), we have

$$x_1(\eta_1)>\ln H_1^{+}\text{ or }x_1(\eta_1)<\ln H_1^{-} \qquad (13)$$

According to Lemma 2.5, it follows from (6), (7), (12) and (13) that

$$\ln l_1^{-}<x_1(\eta_1)<x_1(\xi_1)<\ln H_1^{-}$$

or

$$\ln H_1^{+}<x_1(\eta_1)<x_1(\xi_1)<\ln l_1^{+}$$

and so

$$\ln l_1^{-}<x_1(t)<\ln H_1^{-}\text{ or }\ln H_1^{+}<x_1(t)<\ln l_1^{+}\text{ for }\forall t\in T \qquad (14)$$

According to the second equation of (4), we have

$$a_{22}^{M}e^{2x_2(\xi_2)}-r_2^{L}e^{x_2(\xi_2)}+\frac{a_{23}^{M}l_3^{+}}{b_{23}^{L}}+h_2^{M}>0$$

which implies

$$x_2(\xi_2)>\ln H_2^{+}\text{ or }x_2(\xi_2)<\ln H_2^{-} \qquad (15)$$

Similarly, by the second equation of (5), we obtain

$$x_2(\eta_2)>\ln H_2^{+}\text{ or }x_2(\eta_2)<\ln H_2^{-} \qquad (16)$$

According to Lemma 2.5, from (8), (9), (15) and (16), we have

$$\ln l_2^{-}<x_2(\eta_2)<x_2(\xi_2)<\ln H_2^{-}$$

or

$$\ln H_2^{+}<x_2(\eta_2)<x_2(\xi_2)<\ln l_2^{+}$$

and so

$$\ln l_2^{-}<x_2(t)<\ln H_2^{-}\text{ or }\ln H_2^{+}<x_2(t)<\ln l_2^{+}\text{ for }\forall t\in T \qquad (17)$$

By the third equation of (4), we obtain

$$a_{33}^{M}e^{2x_3(\xi_3)}-r_3^{L}e^{x_3(\xi_3)}+h_3^{M}>0$$

which yields

$$x_3(\xi_3)>\ln H_3^{+}\text{ or }x_3(\xi_3)<\ln H_3^{-} \qquad (18)$$

Similarly, by the third equation of (5), we obtain

$$x_3(\eta_3)>\ln H_3^{+}\text{ or }x_3(\eta_3)<\ln H_3^{-} \qquad (19)$$

By using Lemma 2.5, from (10), (11), (18) and (19), we get

$$\ln l_3^{-}<x_3(\eta_3)<x_3(\xi_3)<\ln H_3^{-}$$

or

$$\ln H_3^{+}<x_3(\eta_3)<x_3(\xi_3)<\ln l_3^{+}$$

and so
$$\ln l_3^- < x_3(t) < \ln H_3^- \ \text{ or } \ \ln H_3^+ < x_3(t) < \ln l_3^+, \forall t \in T \tag{20}$$

It is obvious that $\ln l_j^\pm$ and $\ln H_j^\pm, (j=1,2,3)$ are independent of λ. Now, let

$$\Omega_1 = \{ x = (x_1, x_2, x_3)^T \in X \mid \ln l_1^- < x_1 < \ln H_1^-, \ln l_2^- < x_2 < \ln H_2^-, \ln l_3^- < x_3 < \ln H_3^- \}$$

$$\Omega_2 = \{ x = (x_1, x_2, x_3)^T \in X \mid \ln l_1^- < x_1 < \ln H_1^-, \ln l_2^- < x_2 < \ln H_2^-, \ln l_3^+ < x_3 < \ln H_3^+ \}$$

$$\Omega_3 = \{ x = (x_1, x_2, x_3)^T \in X \mid \ln l_1^- < x_1 < \ln H_1^-, \ln l_2^+ < x_2 < \ln H_2^+, \ln l_3^- < x_3 < \ln H_3^- \}$$

$$\Omega_4 = \{ x = (x_1, x_2, x_3)^T \in X \mid \ln l_1^- < x_1 < \ln H_1^-, \ln l_2^+ < x_2 < \ln H_2^+, \ln l_3^+ < x_3 < \ln H_3^+ \}$$

$$\Omega_5 = \{ x = (x_1, x_2, x_3)^T \in X \mid \ln l_1^+ < x_1 < \ln H_1^+, \ln l_2^+ < x_2 < \ln H_2^+, \ln l_3^- < x_3 < \ln H_3^- \}$$

$$\Omega_6 = \{ x = (x_1, x_2, x_3)^T \in X \mid \ln l_1^+ < x_1 < \ln H_1^+, \ln l_2^- < x_2 < \ln H_2^-, \ln l_3^- < x_3 < \ln H_3^- \}$$

$$\Omega_7 = \{ x = (x_1, x_2, x_3)^T \in X \mid \ln l_1^+ < x_1 < \ln H_1^+, \ln l_2^- < x_2 < \ln H_2^-, \ln l_3^+ < x_3 < \ln H_3^+ \}$$

$$\Omega_8 = \{ x = (x_1, x_2, x_3)^T \in X \mid \ln l_1^+ < x_1 < \ln H_1^+, \ln l_2^+ < x_2 < \ln H_2^+, \ln l_3^+ < x_3 < \ln H_3^+ \}$$

Then it is obvious that Ω_j (j = 1,2,...,8) are bounded open subsets of X, $\Omega_i \bigcap \Omega_j = \varnothing$ for $i \neq j$ with i, j = 1, 2, ..., 8. Thus, Ω_j (j = 1,2,...,8) satisfy the requitement (a) in Lemma 2.3.

In the next, we show that condition (b) of Lemma 2.3 holds, that is, $QN(x,0) \neq (0,0,0)^T$ for $x \in \partial\Omega_j \bigcap R^3 (j = 1, 2, \cdots, 8)$. If is not true, then there exists constant vector $x = (x_1, x_2, x_3)^T \in \partial\Omega_j$ and $t_i \in I_\omega (i = 1, 2, 3)$ satisfy

$$\begin{cases} r_1(t_1) - a_{11}(t_1) e^{x_1} - h_1(t_1) e^{-x_1} = 0 \\ r_2(t_2) - a_{22}(t_2) e^{x_2} - h_2(t_2) e^{-x_2} = 0 \\ r_3(t_3) - a_{33}(t_3) e^{x_3} - h_3(t_3) e^{-x_3} = 0 \end{cases} \tag{21}$$

Therefore, we obtain

$$\ln l_1^- < x_1 < \ln H_1^- \ \text{ or } \ \ln H_1^+ < x_1 < \ln l_1^+$$

$$\ln l_2^- < x_2 < \ln H_2^- \ \text{ or } \ \ln H_2^+ < x_2 < \ln l_2^+$$

$$\ln l_3^- < x_3 < \ln H_3^- \ \text{ or } \ \ln H_3^+ < x_3 < \ln l_3^+$$

This yields $x \in \Omega_j \bigcap R^3 (j = 1, 2, \cdots, 8)$, which contracts with the fact that $x \in \partial\Omega_j \bigcap R^3 (j = 1, 2, \cdots, 8)$. The condition (b) of Lemma 2.3 is valid.

Finally, we show that the condition (c) of Lemma 2.3 holds. Noting that the algebraic equations:

$$\begin{cases} r_1(t_1) - a_{11}(t_1) e^{u} - h_1(t_1) e^{-u} = 0 \\ r_2(t_2) - a_{22}(t_2) e^{v} - h_2(t_2) e^{-v} = 0 \\ r_3(t_3) - a_{33}(t_3) e^{w} - h_3(t_3) e^{-w} = 0 \end{cases} \tag{22}$$

has eight distinct solutions:

$$(u_1^*, v_1^*, w_1^*) = (\ln u_-, \ln v_-, \ln w_-)$$

$$\ln H_3^+ < x_3(\eta_3) < x_3(\xi_3) < \ln l_3^+$$

$$(u_3^*, v_3^*, w_3^*) = (\ln u_-, \ln v_+, \ln w_-)$$

$$(u_4^*, v_4^*, w_4^*) = (\ln u_-, \ln v_+, \ln w_+)$$

$$(u_5^*, v_5^*, w_5^*) = (\ln u_+, \ln v_+, \ln w_-)$$

$$(u_6^*, v_6^*, w_6^*) = (\ln u_+, \ln v_-, \ln w_-)$$

$$(u_7^*, v_7^*, w_7^*) = (\ln u_+, \ln v_-, \ln w_+)$$

$$(u_8^*, v_8^*, w_8^*) = (\ln u_+, \ln v_+, \ln w_+)$$

where

$$u_\pm = \frac{r_1(t_1) \pm \sqrt{r_1^2(t_1) - 4a_{11}(t_1) h_1(t_1)}}{2a_{11}(t_1)}$$

$$v_\pm = \frac{r_2(t_2) \pm \sqrt{r_2^2(t_2) - 4a_{22}(t_2) h_2(t_2)}}{2a_{22}(t_2)}$$

$$w_{\pm} = \frac{r_3(t_3) \pm \sqrt{r_3^2(t_3) - 4a_{33}(t_3)h_3(t_3)}}{2a_{33}(t_3)}$$

By Lemma 2.4, it easy to verify that

$$\ln l_1^- < \ln u_- < \ln H_1^- < \ln H_1^+ < \ln u_+ < \ln l_1^+, \quad \ln l_2^- < \ln v_- < \ln H_2^- < \ln H_2^+ < \ln v_+ < \ln l_2^+, \quad \ln l_3^- < \ln w_- < \ln H_3^- < \ln H_3^+ < \ln w_+ < \ln l_3^+$$

and so $(u_i^*, v_i^*, w_i^*) \in \Omega_i$ for j = 1,2,...,8. Since Ker L = ImQ, we can take J = I. A direct computation yields that

$$\deg\{JQN(x,0), \Omega_j, (0,0,0)^T\} = \mathrm{sign}[(-a_{11}(t_1)u^* + \frac{h_1(t_1)}{u^*})(-a_{22}(t_2)v^* + \frac{h_2(t_2)}{v^*})(-a_{33}(t_3)w^* + \frac{h_3(t_3)}{w^*})]$$

From the fact

$$\begin{cases} r_1(t_1) - a_{11}(t_1)u^* - \dfrac{h_1(t_1)}{u^*} = 0 \\ r_2(t_2) - a_{22}(t_2)v^* - \dfrac{h_2(t_2)}{v^*} = 0 \\ r_3(t_3) - a_{33}(t_3)w^* - \dfrac{h_3(t_3)}{w^*} = 0 \end{cases} \tag{23}$$

we have

$$\deg\{JQN(x,0), \Omega_j, (0,0,0)^T\} = \mathrm{sign}[(r_1(t_1) - 2a_{11}(t_1)u^*)(r_2(t_2) - 2a_{22}(t_2)v^*)(r_3(t_3) - 2a_{33}(t_3)w^*)]$$

for j = 1,2,...,8. Therefore,

$$\deg\{JQN(x,0), \Omega_1, (0,0,0)^T\} = \mathrm{sign}[(r_1(t_1) - 2a_{11}(t_1)u_-)(r_2(t_2) - 2a_{22}(t_2)v_-)(r_3(t_3) - 2a_{33}(t_3)w_-)] = 1$$

$$\deg\{JQN(x,0), \Omega_2, (0,0,0)^T\} = \mathrm{sign}[(r_1(t_1) - 2a_{11}(t_1)u_-)(r_2(t_2) - 2a_{22}(t_2)v_-)(r_3(t_3) - 2a_{33}(t_3)w_+)] = -1$$

$$\deg\{JQN(x,0), \Omega_3, (0,0,0)^T\} = \mathrm{sign}[(r_1(t_1) - 2a_{11}(t_1)u_-)(r_2(t_2) - 2a_{22}(t_2)v_+)(r_3(t_3) - 2a_{33}(t_3)w_-)] = -1,$$

$$\deg\{JQN(x,0), \Omega_4, (0,0,0)^T\} = \mathrm{sign}[(r_1(t_1) - 2a_{11}(t_1)u_-)(r_2(t_2) - 2a_{22}(t_2)v_+)(r_3(t_3) - 2a_{33}(t_3)w_+)] = 1$$

$$\deg\{JQN(x,0), \Omega_5, (0,0,0)^T\} = \mathrm{sign}[(r_1(t_1) - 2a_{11}(t_1)u_+)(r_2(t_2) - 2a_{22}(t_2)v_+)(r_3(t_3) - 2a_{33}(t_3)w_-)] = 1$$

$$\deg\{JQN(x,0), \Omega_6, (0,0,0)^T\} = \mathrm{sign}[(r_1(t_1) - 2a_{11}(t_1)u_+)(r_2(t_2) - 2a_{22}(t_2)v_-)(r_3(t_3) - 2a_{33}(t_3)w_-)] = -1$$

$$\deg\{JQN(x,0), \Omega_7, (0,0,0)^T\} = \mathrm{sign}[(r_1(t_1) - 2a_{11}(t_1)u_+)(r_2(t_2) - 2a_{22}(t_2)v_-)(r_3(t_3) - 2a_{33}(t_3)w_+)] = 1$$

$$\deg\{JQN(x,0), \Omega_8, (0,0,0)^T\} = \mathrm{sign}[(r_1(t_1) - 2a_{11}(t_1)u_+)(r_2(t_2) - 2a_{22}(t_2)v_+)(r_3(t_3) - 2a_{33}(t_3)w_+)] = -1$$

It follows that the condition (c) of Lemma 2.3 holds. Hence, the system (5) has at least eight positive ω-periodic solutions. This completes the proof.

Since the continuous system (3) (when $T = R, \tau(t) = 0, y_j(t) = e^{x_j(t)}$ with j = 1,2,3 and the discrete system (4) (when $T = Z, \tau(t) = 0, y_j(t) = e^{x_j(t)}$ with j = 1, 2, 3 are two special cases of dynamic system (5), a direct consequence of Theorem 3.1 is the following corollary.

Corollary 3.1: Under the requirements of Theorem 3.1, (3) and (4) has at least eight positive ω-periodic solutions, respectively.

Remark 3.1: Our result indicates that the existence result of eight periodic solutions for continuous system (3) and discrete system (4) are verified. Therefore, the study of dynamic system (5) on time scales avoids proving the result twice, once for the continuous system and once for the discrete system.

Remark 3.2: Since there are many other time scales than just the set of real numbers R or the set of integers Z, we obtained a much more general result of dynamic system (5) on time scales. Thus, our results obtained (see Theorem 3.1) incorporate and extend a known result [25] in the literature essentially a special cases when the time scale is chosen as the real numbers and the delay is zero.

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