

Generalized Extended Mapping Method and its Application to Zakharov-Kuznetsov Equation

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Abstract: Making use of a new generalized ansatzes, the generalized extended mapping method is used to derive new general exact solutions for the Zakharov-Kuznetsov (ZK) equation. As a result, many new and general periodic wave solutions are obtained which include new solitary and shock wave solutions. As an illustrative sample, the properties of some periodic and solitary wave solutions are shown by some figures.

Key words: Zakharov-Kuznetsov equation . the generalized extended mapping method . traveling periodic wave solutions . solitary wave solutions . jacobi elliptic functions

INTRODUCTION

It is well known that the nonlinear complex physical phenomena are related to Nonlinear Partial Differential Equations (NLPDEs), which are involved in many fields from physics to biology, chemistry mechanics, mechanics, etc. As mathematical models of the phenomena, the investigation of exact solutions of NLPDEs will help one to understand these phenomena better. In the past several decades, many methods for obtaining traveling periodic wave solutions of NLPDEs have been proposed, such as Hirota's method [1], the Backlund and Darboux transformation [2-5], Painleve expansions [6], homogenous balance method [7], Jacobi elliptic function [8, 9], extended tanh-function methods [10-12], extended F-expansion methods [13-17], A domain methods [18-20], Exp-function methods [21-22] and mapping method [23-32] which was proposed recently as an overall generalization of Jacobi elliptic expansion function method. In this manuscript, the improved extended mapping method has been proposed and applied to obtain the general traveling periodic wave solutions and the corresponding solitary solutions to ZK equation. It is obvious that the more formal solutions of NEDEs may provide a useful help for physicist in studying more complicated physical phenomena. On one hand, we recover the previously known solitary and shock wave solutions. On the second hand, more importantly, we also obtain other new and more general solutions of ZK equation.

METHOD AND ITS APPLICATION

Here, we give a brief description of the generalized extended mapping method. For the given nonlinear

evolution equation, say, with three independent variables x , y and t

$$F(u, u_t, u_x, u_y, \dots) = 0 \quad (1)$$

where F is in general a polynomial in u and its various partial derivatives. Seeking its traveling wave solution of Eq.(1) by taking

$$u(x, y, t) = u(\xi), \quad \xi = kx + ly - \omega t \quad (2)$$

where k , l and λ are constants to be determined later. Substituting (2) into (1) yields an ordinary differential equation of $u(\xi)$

$$F_0(u, u_\xi, u_{\xi\xi}, \dots) = 0 \quad (3)$$

Then $u(\xi)$ is expanded into a polynomial in $f(\xi)$ and $g(\xi)$:

$$u(\xi) = a_0 + \sum_{i=1}^n f^{i-1}(a_i f(\xi) + b_i g(\xi)) \quad (4)$$

in which a_i and b_i are constants to be determined and n is fixed by balancing the linear term of the highest order derivative with highest power nonlinear terms in Eq.(1), while $f(\xi)$ satisfies the equation:

$$\begin{aligned} f'(\xi) &= \sqrt{pf^2(\xi) + \frac{1}{2}qf^4(\xi) + \frac{1}{3}sf^6(\xi) + r} \\ f''(\xi) &= pf(\xi) + qf^3(\xi) + sf^5(\xi) \end{aligned} \quad (5)$$

where p , q , s , r are real parameters and the prime means the derivative with respect to ξ g satisfying

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the same equation with different coefficients possibly. In order to compute conveniently, however, in this manuscript, g is restricted to fulfilling the following relation:

$$\begin{aligned} g''(\xi) &= g(c_1 + c_2 f \xi^2) \\ g(\xi) &= \sqrt{c_3 + c_4 f(\xi)^2} \\ f'(\xi) g'(\xi) &= f(\xi) g(\xi) (c_5 + c_6 f(\xi)^2) \end{aligned} \quad (6)$$

where c_i are constants to be determined. Substituting Eq.(4) with Eqs.(5) and (6) into (3) and setting the coefficients of $f^j g^i (j=0,1)$ to zero will lead to algebraic system, from which $a_i, b_i, c_i, p, q, r, s, k, l, \omega$, can be determined.

THE PERIODIC SOLUTIONS OF ZK EQUATION

The nonlinear development of ion-acoustic waves in magnetized plasma under the restrictions of small

$$\begin{aligned} \frac{8}{3}ka_2s(l^2 + k^2) &= 0, b_2k(k^2 + l^2) = 0, a_2k(k^2 + l^2) = 0 \\ \frac{1}{2}kb_2^2c_4 + 3kl^2a_2q + \frac{1}{2}ka_2^2 + 3k^3a_2q &= 0 \\ 2kl^2b_2c_6 + 2k^3b_2c_6 + k^3b_2q + ka_2b_2 + kl^2b_2c_2 + k^3b_2c_2 + kl^2b_2q &= 0 \\ k^3a_1q + ka_1a_2 + k^3a_1q + kb_2c_4 &= 0 \\ ka_1b_1 + kl^2b_1c_2 + k^3b_1c_2 + kb_1a_2 &= 0 \\ \frac{1}{2}ka_1^2 + 4kl^2a_2p + ka_0a_2 + 4k^3a_2p - \omega a_2 + \frac{1}{2}kb_2^2c_4 + \frac{1}{2}kb_2^2c_3 &= 0 \\ kb_2c_1 + 2k^3b_2c_5 - \omega b_2 + k^3b_2p + k^3b_2p + 2kl^2b_2c_5 + k^3b_2c_1 + ka_0b_2 + ka_1b_1 &= 0 \\ ka_0a_1 - \omega a_1 + kb_1b_2c_3 + k^3a_1p + kb_1p &= 0 \\ k^3b_1c_1 + ka_0b_1 + kl^2b_1c_1 - \omega b_1 &= 0 \\ -c - \omega a_0 + 2k^3a_2r + \frac{1}{2}ka_0^2 + 2k^3a_2r + \frac{1}{2}kb_1^2c_3 &= 0 \end{aligned} \quad (10)$$

From which four sets of solutions are obtained

$$a_0 = a_1, a_1 = -\frac{2b_1c_2(k^2 + l^2)}{3b_2}, a_2 = -\frac{1}{3}c_2(l^2 + k^2), b_1 = b_1, b_2 = b_2, c_2 = c_2, s = 0 \quad (11a)$$

where constraints among the coefficients are given by

$$\begin{aligned} c_1k(k^2 + l^2) + ka_0 - \omega &= 0, 24c_6 + 7c_2 = 0, 24c_5b_2^2 - 7b_1^2c_2 = 0 \\ 12pb_2^2 - b_1^2c_2 &= 0, 12q + c_2 = 0 \\ 18c_3kb_2^4 - c_2(12ka_0l^2b_2^2 + 12k^3a_0b_2^2 - 12\omega l^2b_2^2 - 12\omega k^2b_2^2 + 2k^3b_1l^2c_2 + k_5b_1^2c_2) &= 0, 18c_4b_2^2 + 5c_2^2(l^4 + 2k^2l^2 + k^4) = 0 \\ 24rb_2^4k(k^2 + l^2)^2 + 36cb_2^4 + 36\omega a_0b_2^4 - 18ka_0^2b_2^4 - 12b_1^2k^3c_2b_2^2 - 2b_1^4k^3l^2c_2^2 &= 0 \\ -b_1^4k^5c_2^2 - b_1^4kl^4c_2^2 - 12b_1^2k^3a_0c_2b_2^2 + 12b_1^2\omega l^2c_2b_2^2 + 12b_1^2\omega k^2c_2b_2^2 &= 0 \end{aligned} \quad (11b)$$

wave amplitude, weak dispersion and strong magnetic fields is described by the ZK equation [33-37]:

$$\begin{aligned} u_t(x, y, t) + u(x, y, t)u_x(x, y, t) \\ + u_{xxx}(x, y, t) + u_{xyy}(x, y, t) = 0 \end{aligned} \quad (7)$$

Substituting Eq.(2) into Eq.(7) and integrating once, we have

$$k^3u_{\xi\xi} + k^2u_{\xi\xi\xi} + \frac{1}{2}ku^2 - \omega u = c \quad (8)$$

where c is the integration constant. According to the method described above, we assume that Eq.(8) has solution in the form

$$u = a_0 + a_1f(\xi) + b_1g(\xi) + a_2f(\xi)^2 + b_2f(\xi)g(\xi) \quad (9)$$

The substitution of Eq.(9) with Eqs.(5) and (6) into Eq.(8), equating the coefficients of like powers $f^j g^i (j=0,1)$, yields

$$a_0 = -\frac{c_1 k (k^2 + l^2) - \omega}{k}, b_1 = b_1, a_2 = -c_2 (l^2 + k^2), a_1 = b_2 = s = 0 \\ c_1 = c_1, c_2 = c_2, c_5 = c_5, c_6 = c_6, p = p, r = r, s = 0 \quad (12a)$$

where constraints among the coefficients are given by

$$6q - c_2 = 0 \\ c_3 k^2 b_1^2 - 2ck - \omega^2 - 4k^2 l^4 rc_2 - 8k^4 l^2 rc_2 + k^6 c_1^2 + 2k^4 l^2 c_1^2 + k^2 l^4 c_1^2 - 4k^6 rc_2 = 0 \\ c_4 b_1^2 - 2c_2 (4k^4 p - k^4 c_1 + 8k^2 l^2 p - 2k^2 l^2 c_1 - l^4 c_1 + 4l^4 p) = 0 \quad (12b)$$

$$a_0 = a_0, a_2 = \frac{2c + a_0(2\omega - ka_0)}{4kr(k^2 + l^2)}, b_2 = b_2, a_1 = b_1 = s = 0 \\ c_2 = c_2, c_4 = c_4, c_5 = c_5, c_6 = c_6, p = p, r = r \quad (13a)$$

where constraints among the coefficients are given by

$$ck(k^2 + l^2) + 2k^3 c_5 - \omega + k l^2 p + k^3 p + 2kl^2 c_5 + ka_0 = 0 \\ c_3 2 b_2^2 k^2 (k^2 + l^2) - 4 l^2 k^2 p a_0^2 + 8l^2 k p c + 8 l^2 k p \omega a_0 + 3 \omega k a_0^2 - 2 \omega^2 a_0 - 2 \omega c - k 2 a_0^3 - 4k^4 p a_0^2 + 8k^3 p c + 2ka_0 c + 8k^3 p \omega a_0 = 0 \\ 4qkr(l^2 + k^2)^2 - 2c - 2\omega a_0 - 8k^4 l^2 c_6 - 16k^3 l^2 r c_6 - 4k^4 l^2 r c_2 - 8k^3 l^2 r c_2 - ka_0^2 - 8k^5 r c_6 - 4k^5 r c_2 = 0 \\ 16 \zeta b_2^2 r^2 k^2 (k^2 + l^2) - 48ck l^4 r c_2 - 48\omega a_0 k l^4 r c_2 + 24k^2 l^4 r c_2 a_0^2 - 96\omega a_0 k l^4 r c_6 + 48k^2 l^4 r c_6 a_0^2 - 96ck l^4 r c_6 - 96\omega a_0 k l^4 r c_2 - 192ck^3 l^2 r c_6 + 96k^4 l^2 r \zeta a_0^2 - 96ck^3 l^2 r c_2 - 192\omega a_0 k l^3 r c_6 + 48k^4 l^2 r c_2 a_0^2 - 20\omega^2 a_0^2 + 20\omega^3 k - 40c \omega a_0 - 48\omega a_0 k^5 r c_2 - 96ck^5 r c_6 + 20cka_0^2 - 48ck^5 r c_2 + 48k^6 a_0^2 r c_6 - 96\omega a_0 k^5 r c_6 - 5k^2 a_0^4 + 24k^6 a_0^2 r c_2 - 20c^2 = 0 \quad (13b)$$

$$a_0 = -\frac{4pk(l^2 + k^2) - \omega}{k}, a_2 = -6q(l^2 + k^2), a_1 = b_1 = b_2 = s = 0 \\ c_1 = c_1, c_2 = c_2, c_3 = c_3, c_4 = c_4, c_5 = c_5, c_6 = c_6, p = p, q = q \quad (14a)$$

where the constraint among the coefficients is given by

$$24rk^2q(l^4 + 2k^2l^2 + k^4) + 2ck + \omega^2 - 16k^2l^4 p^2 - 32k^4l^2 p^2 - 16k^6 p^2 = 0 \quad (14b)$$

Therefore, we obtain four kind of exact solutions for Eq.(7) as follows:

$$u_1 = a_0 - \frac{2b_1 c_2 (k^2 + l^2)}{3b_2} f(\xi) + b_1 g(\xi) - \frac{1}{3} c_2 (l^2 + k^2) f^2(\xi) + b_2 f(\xi) g(\xi) \quad (15)$$

$$u_2 = -\frac{c_1 k (k^2 + l^2) - \omega}{k} + b_1 g(\xi) - c_2 (l^2 + k^2) f^2(\xi) \quad (16)$$

$$u_3 = a_0 + \frac{2c + a_0(2\omega - ka_0)}{4kr(k^2 + l^2)} f^2(\xi) + b_2 f(\xi) g(\xi) \quad (17)$$

$$u_4 = -\frac{4pk(l^2 + k^2) - \omega}{k} - 6q(l^2 + k^2) f^2(\xi) \quad (18)$$

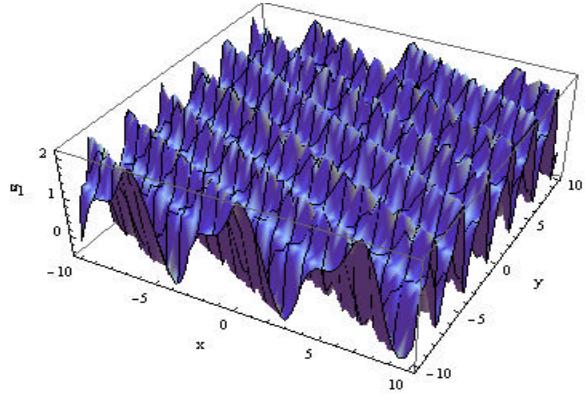


Fig. 1: The structure graph of Eq. 19

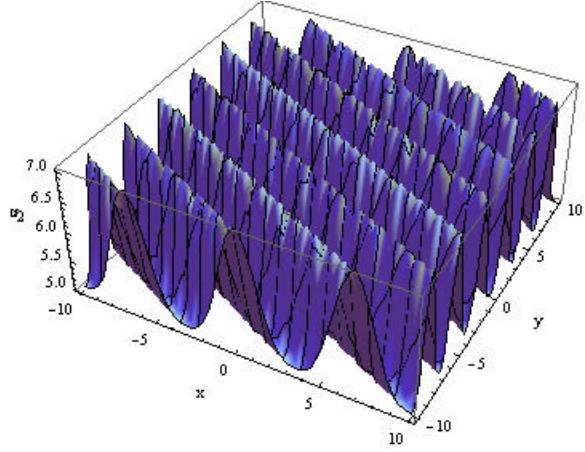


Fig. 2: The structure graph of Eq. 20

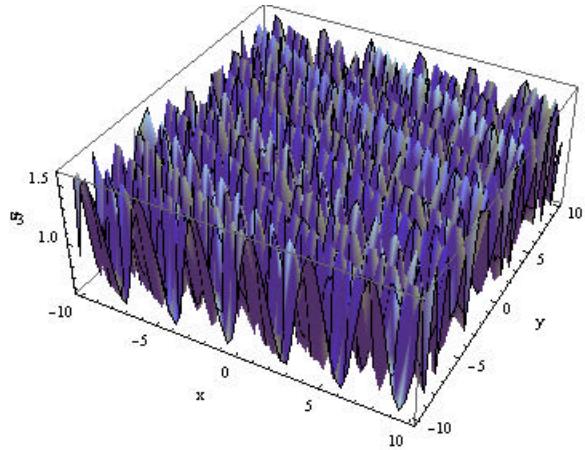


Fig. 3: The structure graph of Eq. 21

I would like to point out that the solution (18) has been obtained previously by Peng [40]. A series of new periodic wave solutions in terms of Jacobi elliptic functions can be obtained according to the different choice of the function $f(\xi)$ and $g(\xi)$ as follows:

Case 1:

$$\begin{aligned} p &= -(1+m^2), q = 2m^2, s = 0, r = 1, c_1 = -1, \\ c_2 &= 2m^2, c_3 = 1, c_4 = -1, c_5 = -1, c_6 = m^2 \end{aligned}$$

Eqs. (5) and (6) have the solution $f = \text{sn}\xi, g = \text{cn}\xi$. So, we get new general combined periodic wave solutions of Eq. (7)

$$\begin{aligned} u_1 &= a_0 - \frac{4b_1m^2(k^2+l^2)}{3b_2}\text{sn}(kx+ly-\omega t) \\ &\quad + b_1\text{cn}(kx+ly-\omega t) \\ &\quad - \frac{2m^2}{3}(l^2+k^2)\text{sn}^2(kx+ly-\omega t) \\ &\quad + b_2\text{sn}(kx+ly-\omega t)\text{cn}(kx+ly-\omega t) \end{aligned} \quad (19)$$

$$\begin{aligned} u_2 &= \frac{k(k^2+l^2)+\omega}{k} + b_1\text{cn}(kx+ly-\omega t) \\ &\quad - 2m^2(l^2+k^2)\text{sn}^2(kx+ly-\omega t) \end{aligned} \quad (20)$$

$$\begin{aligned} u_3 &= a_0 + \frac{2c+a_0(2\omega-ka_0)}{4k(k^2+l^2)}\text{sn}^2(kx+ly-\omega t) \\ &\quad + b_2\text{sn}(kx+ly-\omega t)\text{cn}(kx+ly-\omega t) \end{aligned} \quad (21)$$

$$\begin{aligned} u_4 &= \frac{4k(1+m^2)(l^2+k^2)+\omega}{k} \\ &\quad - 12m^2(l^2+k^2)\text{sn}^2(kx+ly-\omega t) \end{aligned} \quad (22)$$

where $\text{sn} = \text{sn}(\xi|m), \text{cn} = \text{cn}(\xi|m)$ are Jacobi elliptic functions with $m(0 < m < 1)$ is the modulus of the elliptic function. Detailed explanation about the Jacobi elliptic functions can be found in Refs. [38, 39]. The structure graph of Eqs. (19-21) are shown in Fig. 1-3, respectively, where the parameters are $m=0.2, k=1, l=2, \omega=1, t=0$.

As $m \rightarrow 1$, Eqs.(19-22) respectively degenerates to new combined solitary and shock wave solutions

$$\begin{aligned} u_1 &= a_0 - \frac{4b_1(k^2+l^2)}{3b_2}\tanh(kx+ly-\omega t) \\ &\quad + b_1\text{sech}(kx+ly-\omega t) \\ &\quad - \frac{2}{3}(l^2+k^2)\tanh^2(kx+ly-\omega t) \\ &\quad + b_2\tanh(kx+ly-\omega t)\text{sech}(kx+ly-\omega t) \end{aligned} \quad (23)$$

$$\begin{aligned} u_2 &= \frac{k(k^2+l^2)+\omega}{k} + b_1\text{sech}(kx+ly-\omega t) \\ &\quad - 2(l^2+k^2)\tanh^2(kx+ly-\omega t) \end{aligned} \quad (24)$$

$$\begin{aligned} u_3 &= a_0 + \frac{2c+a_0(2\omega-ka_0)}{4k(k^2+l^2)}\tanh^2(kx+ly-\omega t) \\ &\quad + b_2\tanh(kx+ly-\omega t)\text{sech}(kx+ly-\omega t) \end{aligned} \quad (25)$$

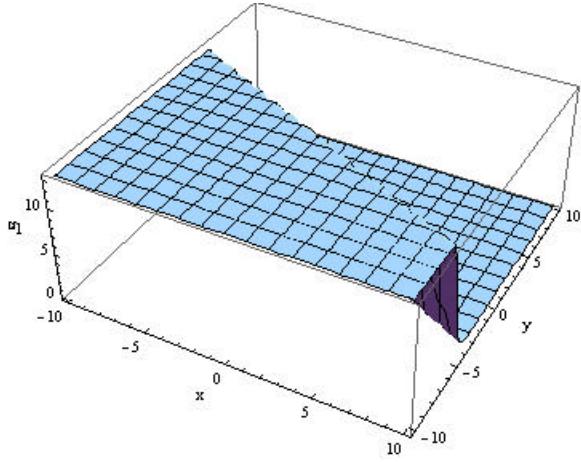


Fig. 4: The structure graph of Eq. 23

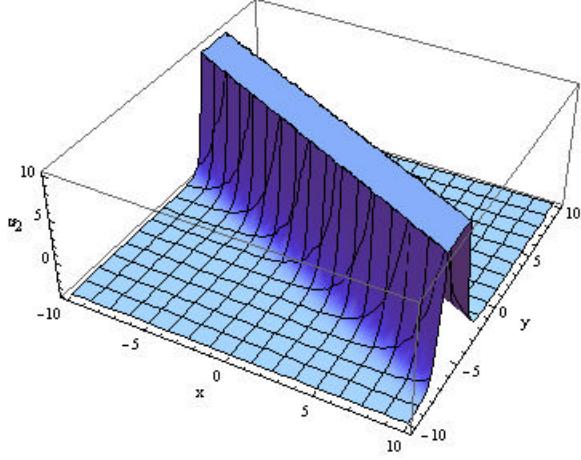


Fig. 5: The structure graph of Eq. 24

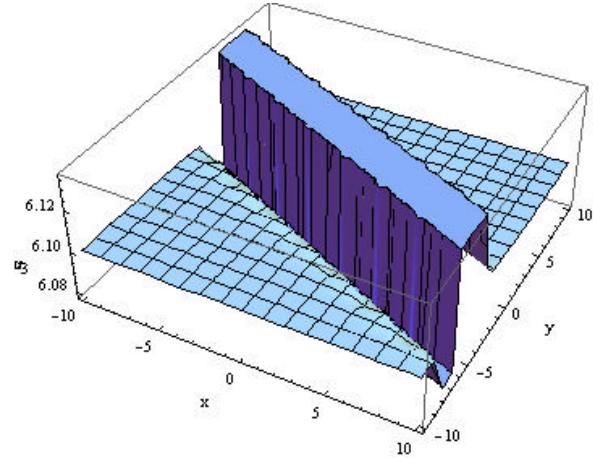


Fig. 6: The structure graph of Eq. 25

$$u_4 = \frac{8k(l^2 + k^2) + \omega}{k} - 12(l^2 + k^2)\tanh^2(kx + ly - \omega t) \quad (26)$$

The structure graph of Eqs.(23-25) are shown in Fig. 4-6, respectively.

Case 2:

$$\begin{aligned} p &= -(1 + m^2), q = 2m^2, s = 0, r = 1, c_1 = -m^2, \\ c_2 &= 2m^2, c_3 = 1, c_4 = -m^2, c_5 = -m^2, c_6 = m^2 \end{aligned}$$

The solution of Eqs.(5) and (6) is $f = \text{sn}\xi, g = \text{dn}\xi$. Hence another periodic wave solutions to Eq.(7) reads

$$\begin{aligned} u_1 &= a_0 - \frac{4b_1m^2(k^2 + l^2)}{3b_2} \text{sn}(kx + ly - \omega t) + b_1 \text{dn}(kx + ly - \omega t) \\ &\quad - \frac{2m^2}{3}(l^2 + k^2) \text{sn}^2(kx + ly - \omega t) + b_1 \text{sn}(kx + ly - \omega t) \text{dn}(kx + ly - \omega t) \end{aligned} \quad (27)$$

$$u_2 = \frac{m^2 k (k^2 + l^2) + \omega}{k} + b_1 \text{dn}(kx + ly - \omega t) - 2m^2(l^2 + k^2) \text{sn}^2(kx + ly - \omega t) \quad (28)$$

$$u_3 = a_0 + \frac{2c + a_0(2\omega - ka_0)}{4k(k^2 + l^2)} \text{sn}^2(kx + ly - \omega t) + b_1 \text{sn}(kx + ly - \omega t) \text{dn}(kx + ly - \omega t) \quad (29)$$

$$u_4 = \frac{4k(1 + m^2)(l^2 + k^2) + \omega}{k} - 12m^2(l^2 + k^2) \text{sn}^2(kx + ly - \omega t) \quad (30)$$

Case 3:

$$p = -(1 + m^2), q = 2m^2, s = 0, r = 1, c_1 = -m^2, c_2 = 2m^2, c_3 = \frac{1}{1 - m^2}, c_4 = \frac{-m^2}{1 - m^2}, c_5 = -m^2, c_6 = m^2$$

One has $f = cd\xi, g = nd\xi$. So, the periodic wave solution to Eq.(7) reads:

$$\begin{aligned} u_1 &= a_0 - \frac{4b_1m^2(k^2 + l^2)}{3b_2} cd(kx + ly - \omega t) + b_1nd(kx + ly - \omega t) \\ &\quad - \frac{2m^2}{3}(l^2 + k^2)cd^2(kx + ly - \omega t) + b_2cd(kx + ly - \omega t)nd(kx + ly - \omega t) \end{aligned} \quad (31)$$

$$u_2 = \frac{m\bar{k}(k^2 + l^2) + \omega}{k} + b_1nd(kx + ly - \omega t) - 2m^2(l^2 + k^2)cd^2(kx + ly - \omega t) \quad (32)$$

$$u_3 = a_0 + \frac{2c + a_0(2\omega - ka_0)}{4k(k^2 + l^2)} cd^2(kx + ly - \omega t) + b_2cd(kx + ly - \omega t)nd(kx + ly - \omega t) \quad (33)$$

$$u_4 = \frac{4k(1 + m^2)(l^2 + k^2) + \omega}{k} - 12m^2(l^2 + k^2)cd^2(kx + ly - \omega t) \quad (34)$$

Case 4:

$$p = -(1 + m^2), q = 2m^2, s = 0, r = 1, c_1 = -1, c_2 = 2m^2, c_3 = \frac{1}{1 - m^2}, c_4 = \frac{-m^2}{1 - m^2}, c_5 = -1, c_6 = m^2$$

The solution of Eqs.(5) and (6) is $f = cd\xi, g = sd\xi$. Thus, we get:

$$\begin{aligned} u_1 &= a_0 - \frac{4b_1m^2(k^2 + l^2)}{3b_2} cd(kx + ly - \omega t) + b_1sd(kx + ly - \omega t) \\ &\quad - \frac{2m^2}{3}(l^2 + k^2)cd^2(kx + ly - \omega t) + b_2cd(kx + ly - \omega t)sd(kx + ly - \omega t) \end{aligned} \quad (35)$$

$$u_2 = \frac{k(k^2 + l^2) + \omega}{k} + b_1sd(kx + ly - \omega t) - 2m^2(l^2 + k^2)cd^2(kx + ly - \omega t) \quad (36)$$

$$u_3 = a_0 + \frac{2c + a_0(2\omega - ka_0)}{4k(k^2 + l^2)} cd^2(kx + ly - \omega t) + b_2cd(kx + ly - \omega t)sd(kx + ly - \omega t) \quad (37)$$

$$u_4 = \frac{4k(1 + m^2)(l^2 + k^2) + \omega}{k} - 12m^2(l^2 + k^2)cd^2(kx + ly - \omega t) \quad (38)$$

Case 5:

$$p = -(1 + m^2), q = 2, s = 0, r = m^2, c_1 = -m^2, c_2 = 2, c_3 = -1, c_4 = 1, c_5 = -m^2, c_6 = 1$$

The solution of Eqs.(5) and (6) is $f = ns\xi, g = cs\xi$. Thus, Eq.(7) has the periodic wave solutions:

$$\begin{aligned} u_1 &= a_0 - \frac{4b_1(k^2 + l^2)}{3b_2} ns(kx + ly - \omega t) + b_1cs(kx + ly - \omega t) \\ &\quad - \frac{2}{3}(l^2 + k^2)ns^2(kx + ly - \omega t) + b_2ns(kx + ly - \omega t)cs(kx + ly - \omega t) \end{aligned} \quad (39)$$

$$u_2 = \frac{m\bar{k}(k^2 + l^2) + \omega}{k} + b_1cs(kx + ly - \omega t) - 2(l^2 + k^2)ns^2(kx + ly - \omega t) \quad (40)$$

$$u_3 = a_0 + \frac{2c + a_0(2\omega - ka_0)}{4km^2(k^2 + l^2)} ns^2(kx + ly - \omega t) + b_2ns(kx + ly - \omega t)cs(kx + ly - \omega t) \quad (41)$$

$$u_4 = \frac{4k(1 + m^2)(l^2 + k^2) + \omega}{k} - 12(l^2 + k^2)ns^2(kx + ly - \omega t) \quad (42)$$

Case 6:

$$p = -(1 + m^2), q = 2, s = 0, r = m^2, c_1 = -1, c_2 = 2, c_3 = -m^2, c_4 = 1, c_5 = -1, c_6 = 1$$

In this case, one has $f = ns\xi, g = ds\xi$. Thus,

$$\begin{aligned} u_1 &= a_0 - \frac{4b_1(k^2 + l^2)}{3b_2} ns(kx + ly - \omega t) + b_1 ds(kx + ly - \omega t) \\ &\quad - \frac{2}{3}(l^2 + k^2) ns^2(kx + ly - \omega t) + b_2 ns(kx + ly - \omega t) ds(kx + ly - \omega t) \end{aligned} \quad (43)$$

$$u_2 = \frac{k(k^2 + l^2) + \omega}{k} + b_1 ds(kx + ly - \omega t) - 2(l^2 + k^2) ns^2(kx + ly - \omega t) \quad (44)$$

$$u_3 = a_0 + \frac{2c + a_0(2\omega - ka_0)}{4km^2(k^2 + l^2)} ns^2(kx + ly - \omega t) + b_2 ns(kx + ly - \omega t) ds(kx + ly - \omega t) \quad (45)$$

$$u_4 = \frac{4k(1 + m^2)(l^2 + k^2) + \omega}{k} - 12(l^2 + k^2) ns^2(kx + ly - \omega t) \quad (46)$$

Case 7:

$$p = -(1 + m^2), q = 2, s = 0, r = m^2, c_1 = -1, c_2 = 2, c_3 = -\frac{m^2}{1-m^2}, c_4 = \frac{1}{1-m^2}, c_5 = -1, c_6 = 1$$

Now, we have $f = dc\xi, g = nc\xi$. So,

$$\begin{aligned} u_1 &= a_0 - \frac{4b_1(k^2 + l^2)}{3b_2} dc(kx + ly - \omega t) + b_1 c(kx + ly - \omega t) \\ &\quad - \frac{2}{3}(l^2 + k^2) dc^2(kx + ly - \omega t) + b_2 dc(kx + ly - \omega t) nc(kx + ly - \omega t) \end{aligned} \quad (47)$$

$$u_2 = \frac{k(k^2 + l^2) + \omega}{k} + b_1 c(kx + ly - \omega t) - 2(l^2 + k^2) dc^2(kx + ly - \omega t) \quad (48)$$

$$u_3 = a_0 + \frac{2c + a_0(2\omega - ka_0)}{4km^2(k^2 + l^2)} dc^2(kx + ly - \omega t) + b_2 dc(kx + ly - \omega t) nc(kx + ly - \omega t) \quad (49)$$

$$u_4 = \frac{4k(1 + m^2)(l^2 + k^2) + \omega}{k} - 12(l^2 + k^2) dc^2(kx + ly - \omega t) \quad (50)$$

Case 8:

$$p = -(1 + m^2), q = 2, s = 0, r = m^2, c_1 = -m^2, c_2 = 2, c_3 = -\frac{1}{1-m^2}, c_4 = \frac{1}{1-m^2}, c_5 = -m^2, c_6 = 1$$

The solution of Eqs.(5) and (6) reads $f = dc\xi, g = sc\xi$. Thus, one has

$$\begin{aligned} u_1 &= a_0 - \frac{4b_1(k^2 + l^2)}{3b_2} dc(kx + ly - \omega t) + b_1 s(kx + ly - \omega t) \\ &\quad - \frac{2}{3}(l^2 + k^2) dc^2(kx + ly - \omega t) + b_2 dc(kx + ly - \omega t) sc(kx + ly - \omega t) \end{aligned} \quad (51)$$

$$u_2 = \frac{m^2 k (k^2 + l^2) + \omega}{k} + b_1 s(kx + ly - \omega t) - 2(l^2 + k^2) dc^2(kx + ly - \omega t) \quad (52)$$

$$u_3 = a_0 + \frac{2c + a_0(2\omega - ka_0)}{4km^2(k^2 + l^2)} dc^2(kx + ly - \omega t) + b_2 dc(kx + ly - \omega t) sc(kx + ly - \omega t) \quad (53)$$

$$u_4 = \frac{4k(1+m^2)(l^2 + k^2) + \omega}{k} - 12(l^2 + k^2) dc^2(kx + ly - \omega t) \quad (54)$$

Case 9:

$$p = 2m^2 - 1, q = -2m^2, s = 0, r = 1 - m^2, c_1 = m^2, c_2 = -2m^2, c_3 = 1 - m^2, c_4 = m^2, c_5 = m^2, c_6 = -m^2$$

Eqs.(5) and (6) has the solution $f = cn\xi, g = dn\xi$. The corresponding solutions of Eq.(7) are

$$\begin{aligned} u_1 &= a_0 + \frac{4m^2 b_1 (k^2 + l^2)}{3b_2} cn(kx + ly - \omega t) + b_1 dn(kx + ly - \omega t) \\ &\quad + \frac{2m^2}{3} (l^2 + k^2) cn^2(kx + ly - \omega t) + b_2 cn(kx + ly - \omega t) dn(kx + ly - \omega t) \end{aligned} \quad (55)$$

$$u_2 = -\frac{m k (k^2 + l^2) - \omega}{k} + b_1 dn(kx + ly - \omega t) + 2m^2 (l^2 + k^2) cn^2(kx + ly - \omega t) \quad (56)$$

$$u_3 = a_0 + \frac{2c + a_0(2\omega - ka_0)}{4k(1-m^2)(k^2 + l^2)} cn^2(kx + ly - \omega t) + b_2 cn(kx + ly - \omega t) dn(kx + ly - \omega t) \quad (57)$$

$$u_4 = -\frac{4k(2m^2 - 1)(l^2 + k^2) - \omega}{k} + 12m^2 (l^2 + k^2) cn^2(kx + ly - \omega t) \quad (58)$$

Case 10:

$$p = 2 - m^2, q = -2(1 - m^2), s = 0, r = -1, c_1 = 1, c_2 = -2(1 - m^2), c_3 = -\frac{1}{m^2}, c_4 = \frac{1}{m^2}, c_5 = 1, c_6 = -(1 - m^2)$$

One gets $f = nd\xi, g = sd\xi$. The solutions of Eq.(7) read

$$\begin{aligned} u_1 &= a_0 + \frac{4(1-m^2) b_1 (k^2 + l^2)}{3b_2} nd(kx + ly - \omega t) + b_1 sd(kx + ly - \omega t) \\ &\quad + \frac{2(1-m^2)}{3} (l^2 + k^2) nd^2(kx + ly - \omega t) + b_2 nd(kx + ly - \omega t) sd(kx + ly - \omega t) \end{aligned} \quad (59)$$

$$u_2 = -\frac{k(k^2 + l^2) - \omega}{k} + b_1 sd(kx + ly - \omega t) + 2(1 - m^2)(l^2 + k^2) nd^2(kx + ly - \omega t) \quad (60)$$

$$u_3 = a_0 - \frac{2c + a_0(2\omega - ka_0)}{4k(k^2 + l^2)} nd^2(kx + ly - \omega t) + b_2 nd(kx + ly - \omega t) sd(kx + ly - \omega t) \quad (61)$$

$$u_4 = -\frac{4k(2 - m^2)(l^2 + k^2) - \omega}{k} + 12(1 - m^2)(l^2 + k^2) nd^2(kx + ly - \omega t) \quad (62)$$

Case 11:

$$p = 2 - m^2, q = 2, s = 0, r = 1 - m^2, c_1 = 1, c_2 = 2, c_3 = 1 - m^2, c_4 = 1, c_5 = 1, c_6 = 1$$

Eqs.(5) and (6) has the solution $f = cs\xi, g = ds\xi$. The corresponding solutions of Eq.(7) are

$$\begin{aligned} u_1 &= a_0 - \frac{4 b_1 (k^2 + l^2)}{3b_2} cs(kx + ly - \omega t) + b_1 ds(kx + ly - \omega t) \\ &\quad - \frac{2}{3} (l^2 + k^2) cs^2(kx + ly - \omega t) + b_2 cs(kx + ly - \omega t) ds(kx + ly - \omega t) \end{aligned} \quad (63)$$

$$u_2 = -\frac{k(k^2 + l^2) - \omega}{k} + b_1 s(kx + ly - \omega t) - 2(l^2 + k^2) \cos^2(kx + ly - \omega t) \quad (64)$$

$$u_3 = a_0 + \frac{2c + a_0(2\omega - ka_0)}{4k(1-m^2)(k^2 + l^2)} \cos^2(kx + ly - \omega t) + b_2 \cos(kx + ly - \omega t) ds(kx + ly - \omega t) \quad (65)$$

$$u_4 = -\frac{4k(2-m^2)(l^2 + k^2) - \omega}{k} - 12(l^2 + k^2) \cos^2(kx + ly - \omega t) \quad (66)$$

Case 12:

$$p = 2m^2 - 1, q = 2(1-m^2), s = 0, r = -m^2, c_1 = m^2, c_2 = 2(1-m^2), c_3 = -1, c_4 = 1, c_5 = m^2, c_6 = 1-m^2$$

From Eqs.(5) and (6), one obtains $f = nc\xi$, $g = sc\xi$. And the solutions of Eq.(7) read

$$\begin{aligned} u_1 &= a_0 - \frac{4(1-m^2)b_1(k^2 + l^2)}{3b_2} nc(kx + ly - \omega t) + b_1 s(kx + ly - \omega t) \\ &\quad - \frac{2(1-m^2)}{3}(l^2 + k^2) nc^2(kx + ly - \omega t) + b_2 nc(kx + ly - \omega t) sc(kx + ly - \omega t) \end{aligned} \quad (67)$$

$$u_2 = -\frac{m k (k^2 + l^2) - \omega}{k} + b_1 s(kx + ly - \omega t) - 2(1-m^2)(l^2 + k^2) nc^2(kx + ly - \omega t) \quad (68)$$

$$u_3 = a_0 - \frac{2c + a_0(2\omega - ka_0)}{4km^2(k^2 + l^2)} nc^2(kx + ly - \omega t) + b_2 nc(kx + ly - \omega t) sc(kx + ly - \omega t) \quad (69)$$

$$u_4 = -\frac{4k(2m^2 - 1)(l^2 + k^2) - \omega}{k} - 12(1-m^2)(l^2 + k^2) nc^2(kx + ly - \omega t) \quad (70)$$

CONCLUSION

In this work, the generalized extended mapping method with a computerized symbolic computation has been proposed to obtain the new general exact solutions to ZK equation. As a result, many different new forms of traveling wave solutions such as periodic wave solutions, solitary wave solutions, shock wave solutions, are obtained. The limiting solutions are found as long as the modulus m of the elliptic approaches 0 or 1. Some of the properties of the periodic and solitary solutions are shown graphically. It can be easily seen that the method used in this work is straightforward and concise and it can also be applicable to other nonlinear evolution equations.

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