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A Solution for Statistical Control of Correlated Processes

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Abstract: Production systems as a principal process, normally include numerous associated sub-processes (stages) in which output quality of earlier stages affects quality of next stages. This study recommends a solution which allows us measure effect of each stage on output quality of the next stage. A distinct aspect of this solution is the use of information which is produced by different degrees of correlation between product quality and that of within and between the production stages of the system under study. In this study, we show how the estimates obtained by this method can be used to measure the effect of operational variables at each stage on variation of production quality, identification of stages in production system on which managers and engineers should concentrate their quality improvement efforts and to assess potential effect of process improvement on output and final product quality.

Key words: Quality control . correlated processes . process improvement

INTRODUCTION

Today, with ever increasing growth of human knowledge and technology and strong dependence of societies on its advantages, the issue regarding quality of technology design, manufacturing, supports and its economics is of high importance in business decisions. On the other hand, increasing competition between firms and societies in acquiring high quality and low cost technologies has sharpen fighting against economic and social losses arising from pure quality and systems (processes, goods and services). In this regard, since most of these systems are affected by random factors and deviation is considered part of their functional nature, statistical quality control in the sense of application of statistical principles and techniques throughout design, production, supports and service stages of these technologies aiming to satisfy economic desires have a special position.

The conventional methods of process control such as Shewharts' charts, exponential weighted moving average chart, cumulative sum chart and other similar techniques are very easy to use and when properly applied, they can be effective in giving necessary signals about existence of instability of the process. The conventional control methods, generally, assume that product quality variables (characteristics) different stages of a production system are independent, while in many instances, these variables to a large extent are dependent on each other. It has been found that the control methods which ignore dependence between variables, compare to methods that take into account these interdependence, are less able to explain causes of deviation from control Hawkin [1]. Hence, many researchers have emphasized on necessity of using multi-variable methods in statistical control of process. So far, several control charts have been suggested for multi-variable data, but most of these methods concern systems of only one production stage, while many production systems have more than one stage in which output quality of one stage not only indicates operational effect of that stage, but also comprises operational effects of previous stages. Therefore, despite high ability of multi-variable control methods in detecting causes of deviations from control, when these methods are used in multi-stage systems, since they have not been designed specifically in these systems' environment, they are not able to systematically use the information related to correlation between characteristics of product quality through the stages and hence they are unable to identify the stages out-of-control.

In recent years, many researchers have focused on study and monitoring of multi-stage systems. A prominent feature of multi-stage production systems is that the output quality of stage i in addition to indication of operational affect of the stage i, represents quality of input to this stage as well. Hence, if one stage goes out-of-control, the conventional process control methods may mistakenly imply that earlier stages are

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out-of-control. Therefore, if operational effect of each stage can be separated from earlier stages, not only condition of being out of process control can be determined, but also the stages out of the system control can be identified.

This article recommends a new multi-variable method for control of process and product quality in systems with interdependent stages. This method, unlike the conventional methods of process control, clearly uses the information related to dependence between product's quality specifications within and between stages. In addition, this method is not only used to determine condition of being out-of-control, but also by taking into account the input quality to each stage is very useful in identification of the system's out-of-control stages. Identification of out-of-control stages, especially in those operations is very crucial that examining sources of deviation from control is very expensive. Besides, identification of out-of-control stages enables producer to resolve failures in operation as quickly as possible and hence to reduce number of products produced in out-of-control conditions in these production stages.

Another problem that producers are confronted is control and explanation of a large volume of data obtained from process and products. Unlike the existing control methods, the method that we develop in this research is very suitable for control of a large number of variables. An important aspect of this method that reduces the observed variables to fewer numbers of factors not only makes interpretation simpler but also protects major part of information regarding main variables. Reduction in data volume can make the process control simpler for production employees.

This paper suggests a solution by means of which operational effect of each stage on output quality of final product can be measured. One distinct aspect of this solution is use of information related to correlation of product's quality specifications inside and between the system's stages. In general, this solution suggests that relationship between quality specification of one stage and the measured quality specifications at earlier stages is expressed by means of a linear regression relation. This method intends to control a stage by taking effect of earlier stages' quality specifications from quality specification of that stage. The recommended control method is carried out in two steps. In the first step, using the obtained data from production system, regression model parameters are estimated then by looking backwards, it is tested whether at the time of data gathering the system has been under control or not? Purpose of the first stage is obtaining a set of under control data which are used for control of production system's future observations. In the second stage, the obtained model in the first stage is used to test under control being of the system when next units are producing.

The aim of this paper is to answer the following principal question:

Does use of the control method which takes dependence between stages in to account in identification of sources and conditions of being out-of-control has more efficient relative to the conventional methods of process control?

In the following, after analysis of research background, the recommended model will be explained.

The remainder of the article proceeds as follows. The next section presents, describes and analyses the literature review. Section 3 presents the mathematical development to study and present the research's model for statistical control of correlated processes. Sources of variation in production quality, data reduction and estimation of model's parameters are other essential points that we consider in this section. In section 4 we applied the developed model and described the results. At the end of this section we discourse about the effect of process improvement on production quality. Section 5 presents the procedure of execution of control method. The conclusions are in the last section.

LITERATURE REVIEW

In some continuous product-manufacturing operations, a basic statistical assumption of independence is often violated, i.e. data collected at regular time interval from these processes are serially correlated Cook and Chiu [2], Montgomery and Friedman [3]. Due to this autocorrelation, traditional control charts such as $\overline{X}$ and R charts result in a large number of false out-of-control alarms (Cook & Chiu). Thus, reactions to these false alarms often result in costly over-control of a process.

To use a control chart such as $\overline{X}$ chart for process mean monitoring or R chart for deviation monitoring, some samples in the course of time are taken from process and the statistics and values related to them are drawn on a chart. In the introduced chart by Shewhart, upon the moment when the calculated statistic by a sample falls out the control limit, an out-of-control signal is given by this chart Shewhart [4]. This limit normally is set at ± 3 times of the drawn statistic's standard deviation from a central line which is called process mean. These limits are called 3 sigma control limits. For more information regarding the concepts and instances used from control charts refer to Ryan [5] and Woodall & Adams [6].

Several attempts have been made in some literature to extend traditional SPC techniques to deal with correlated parameters. Alwan and Roberts [7], Montgomery and Friedman, Wardell *et al*. [8], and Moskowitz and Plante [9] recommended the use of time series modeling techniques for monitoring correlated processes. Wardell *et al*. conducted an extensive study of the performance of four different control charts, i.e. Shewhart, EWMA, the common-cause control (CCC) and the special-causes control (SCC which is also called residual chart). Their results showed that the EWMA chart was quite robust to data correlation while CCC and SCC worked very well when the process mean shifts exceeded two standard deviations. But they found that in many cases SCC did not perform well when the processes are positively correlated.

Runger, Willemain and Prabhu proposed the use of cumulative sum (CUSUM) control chart for monitoring the residuals produced by an ARMA model [10]. However, this model does not perform effectively when the processes are positively correlated. Wright, Booth and Hu suggested a joint estimation outlier detection approach to control short-run correlated processes [11]. In their research, ARIMA model was used as characterizing time series data to detect and identify four different abnormal types of correlated process. Other authors such as Schmid [12], Adams and Tseng [13] and Timmer [14] have also proposed different views for monitoring correlated observations. These control chart methods have been shown to improve monitoring performance in the presence of autocorrelation.

In issue of process statistical control, control of multi-variable data has got considerable attention. So far, several control charts for multi-variate data have been suggested including T-Hotelling Charts and Multi-Variable Cumulative Sum Charts. However, the obvious limitation of these charts is that they cannot be used for identification of out-of-control quality specifications. To solve this problem, Hawkin suggested a multi-variable control method which has been based on the adjusted regression variables. This researcher with little changes in his proposed method, presented a new method for control of systems in which despite presence of dependence between quality specifications, a change in one variable will not necessarily lead to change in another variable [15]. Although Hawkin's studies are powerful with regard to control of multi-variable data, these methods have not been designed specifically for use in multi-stage systems and hence they are not able to systematically identify out-of-control stages. Wade and Woodall by formulating relationships between input and output variables presented a new method called Cause Selection Charts for two-stage systems which was based on the assumption that relationships between the model's variables are predetermined and correctly estimated [16]. Shu and Tsung while referring to problems of Wade and Woodall's model, by applying some changes to charts of cause selection, tried to identify out-of-control stages in two-stage systems [17]. In addition, these researchers by generalizing their suggested method and introducing Multi-Variable Cause Selection Charts have offered a new method for control of two-stage systems [18].

Following the work of Yang and Hancock [19], where each rational subgroup is assumed to be a realization of a multivariate normally distributed vector with an arbitrary correlation matrix, Liu *et al*. [20] studied the effect of the correlation on the economic design of the X chart. This study was extended to X charts with double sampling or with variable parameters: Chen and Chiou [21] considered variable sampling intervals, and Torng *et al*. [22] considered double sampling. Bin Wu and Yu, J.B., present a neural network-based identification model is proposed for both mean and variance shifts in correlated processes [23]. Costa and Machado presented a pure Markov chain approach to investigate the properties of the X chart with variable parameters (VP) and the X chart with Double Sampling (DS) [24].

As was mentioned, most of these methods have been designed for control of two-stage systems. One interesting exception to this is the solution presented by Zantek *et al*. [25] which is a monitoring method for multi-stage systems and not only is capable in identifying the out-of-control stages, but also at each stage, it can identify the factors contributing to being out-of-control. Also, these researchers by taking input quality of each stage offered another solution for control of multi-stage systems [26].

Methods such as T-Hotelling and CUSUM charts for control of multi-variable data have been recommended. Since these methods simultaneously examine all quality specifications of product, in case the process is out-of-control, they are not able to specify the out-of-control specifications. To solve this problem, Hawkin introduced a new multi-variable control method which is founded based on regression relationships of quality specifications. In this method, to examine whether a quality specification is under control, the residue control obtained from the intended specification regression over other quality specifications will be used. Since in many systems due to high correlation of quality specifications with change in one specification some other specifications may undergo changes, use of these specifications as auxiliary variables in regression relation is not suitable. Therefore, by little changes in his recommended method, Hawkin offered a new method

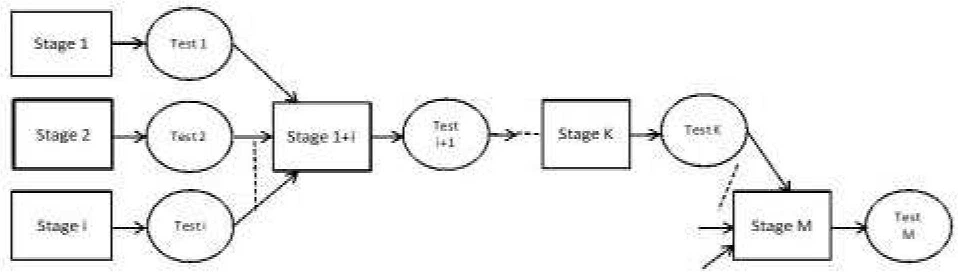

Fig. 1: Production stages of a product

for control of these systems. In this method, for control of a quality specification, in the respective regression relation, only the specifications are taken into consideration as auxiliary variable that by change of the intended specification do not undergo change. The method recommended by Hawkin [8], despite its high capability in control of multi-variable data, since it is designed specifically for control of multi-stage systems, in case of being applied to these systems, it won't be able to identify out-of-control stages.

The issue of multi-stage systems is a new subject and began with studies of Wade and Woodall. These researchers by formulating relationships between input and output quality specifications in a two-stage system embarked on identification of out-of-control stages. In this regard, to control the first stage, the quality specification control measured at this stage is used, whereas for control of the second stage, the obtained residue from regression of the measured quality specification at this stage over the first stage quality specification will be used. In this method, it is supposed that regression relation of input and output quality specifications have been correctly estimated or they are predetermined. Shu and Tsung by generalization of Wade and Woodall's recommended method presented a new technique for control of two-stage systems which includes two different phases. In the first phase, parameters of regression model using the under-control observations will be estimated and in the second phase, by using the observations during manufacturing the process and the estimated model in the first phase are monitored. In this method, unlike the recommended method by Wade and Woodall, the regression model stability in the course of time is tested. These researchers in 2004 generalized their recommended method and presented a new method for control of two-stage systems in which quality specification of the second stage is a function of several quality specifications measured in the first stage. Besides, Zantek *et al.* by modeling quality link between stages offered a technique for control of the systems which are made out of more than two stages. These researchers using a simulation study and analytical study examined their recommended method in identification of out-of-control conditions.

RESEARCH'S MODEL INTRODUCTION

To describe the problem, suppose a production system includes M stages. As is shown in Fig. 1, the stages are numbered in ascending order so as if the stage i is ahead of the stage k, we will have: i<k.

At each stage of the system, one or more operations are executed and after each stage, the product is tested. This test includes measurement of one or more products specifications called quality specifications.

Suppose the continuous variable y_{ij} indicate quality specification j of the product which has been measured at stage i so that $i = 1,\ldots M$, $j = 1,\ldots.q_i$, in this case:

$$q = \sum_{i=1}^{M} q_i \qquad (1)$$

represents total number of product's quality specifications which are measured in the whole system. In addition to product's quality specifications, operational variables (controllable factors) like environmental condition and equipments' efficiency degree may be measured as well. Also, suppose x_{ij} represents the operational variable j which has been measured at stage i so as $i = 1,\ldots M$, $j = 1,\ldots.q_i$, in which case:

$$p = \sum_{i=1}^{M} p_i \qquad (2)$$

indicates total number of operational variables which are measured in the whole system.

Now, supposing that product's quality specifications which are measured at stage i, are directly under influence of operational variables at stage i and quality specification of input product is to stage i, product's quality specification, y_{ij}, can be modeled as a function of the following instances:

A) Operational variables at stage i and
B) Product's Quality specifications which have been measured in output of previous stages.
$(i = 1,\ldots M, j = 1,\ldots q_i,)$

In this relation, in order to provide a general formula for the model, suppose:

α_{ijl}: Represents direct effect of the operational variable j of the stage i on the measured quality specification of product in output of the stage i.
D_i: Represents sum of the stages before stage i.
β_{ijkl}:Direct effect of the measured quality specification j of product in output of stage i the measured quality specification l of product in output of stage k, in which:

$$k = 1,\ldots M \quad jj = 1,\ldots q_i \quad l = 1,\ldots q_k \quad i \in D_k$$

Thus, by supposing the relation linearity, the general model with required details is as follows:

$$y_{kl} = \sum_{j=1}^{p_k} \alpha_{kjl} x_{kj} + \sum_{i \in D_k} \sum_{j=1}^{q_i} \beta_{ijkl} y_{ij} + \varepsilon_{kl} \quad (3)$$

In which, ε_{kl} is random error with mean zero and variance of σ_{kl}^2.

The model of simultaneous equations (3) indicates the point that the product's quality specification, y_{kl}, is as a linear combination of the following instances:
A) Operational variables at stage k:

$$\sum_{j=1}^{p_k} \alpha_{kjl} x_{kj}$$

B) Measured quality specifications of product in output of previous stages:

$$\sum_{i \in D_k} \sum_{j=1}^{q_i} \beta_{ijkl} y_{ij}$$

C) Random error: ε_{kl}

In this relation, it is assumed that ε_{kl} represents effects of operational variables not-considered at stage k on y_{kl} and for all i, j, k and ls, ε_{kl} and x_{ij} are uncorrelated. Now, to facilitate next discussions with regard to the model, the below matrix (vectorial) symbols are introduced:

$y_i = (y_{ij})$: Represents a $1{*}q_i$ vector of the measured quality specifications of product at stage i $(i = 1,\ldots M, j = 1,\ldots q_i)$

$x_i = (x_{ij})$: Represents a $1{*}p_i$ of the measured operational variables at stage i $(j = 1,\ldots p_i)$.

A_i: Denotes a $p_i{*}q_i$ parameter matrix the element j of which is equal to α_{ijl} so as A_i includes direct effects of operational variables at stage i on product's quality specifications which are measured in output of stage i.

B_{ik}: Denotes a $q_i{*}q_k$ parametric matrix the element i of which is equal to β_{ijkl} and includes direct effects of product's quality specifications at stage i on product's quality specifications in stage k where $i \in D_k$.

These definitions enable us to rewrite the model of simultaneous equations into a matrix form:

$$y_k = x_k A_k + \sum_{i \in D_k} y_i B_{ik} + \varepsilon_k \quad (4)$$

In which, ε_k is vector of $1{*}q_k$ random errors the element l of which is ε_{kl} and its mean is zero.

For simplification of the next discussions, suppose A is diagonal-block matrix of parameters so that its block i is equal to matrix A_i and A includes direct effects of operational variables on product's quality specifications. In addition, B is a matrix of parameters in which if the stage i is ahead of k, the ik sub-matrix of which is equal to B_{ik}, else it is equal to zero $q_i{*}q_k$ matrix and B includes direct effects between product's quality specifications.

These definitions enable us to rewrite the model (4) as follows:

$$y = xA + yB + \varepsilon \quad (5)$$

where,

$$x = (x_1,\ldots x_M), y = (y_1,\ldots y_M)$$

$$\varepsilon = (\varepsilon_1,\ldots \varepsilon_M)$$

have a zero mean and variance-covariance matrix of

$$E(\varepsilon'\varepsilon) = \sum$$

Sources of variation in production quality: Now, we obtain a formula which will enable us to specify operational effect of each stage on variation in product's quality specifications. Since variation, in general, leads to improvement of quality, reduced losses, reduced reworking and guarantee costs, identification of variation sources is considered an important part in quality improvement activities. We begin this topic with changeability analysis in product's quality specifications. With deduction yB from both sides of the relation (5), we have:

$$y(I - B) = xA + \varepsilon \quad (6)$$

Since matrix B is an upper-triangular matrix, there is $C = (I-B)^{-1}$ and by multiplying it with the relation (6) we will have:

$$y = xAC + \varepsilon C \qquad (7)$$

By taking variance from both sides of the relation (7) (under assumption of non-correlation of errors and operational variables) we will have:

$$\mathrm{var}(y) = C'A'\mathrm{var}(x)AC + C'\Sigma C \qquad (8)$$

In which, var(.) denotes variance-covariance matrix (.).

The relation (8) enables us to determine share of each stage in changeability of product's quality specifications. In addition, the relation (8) also enables us to assess potential effects of process changeability on production system's efficiency and changeability in product's quality specifications.

Data reduction: In many production systems, some variables are highly correlated because they are produced by similar basic factors or variables. This situation is regarded undesirable, because some of these variables are redundant and estimation of the model's parameters may be affected by them. Therefore, if data are multi-dimensional, we use factorial analysis to explain observational variables with fewer variables which are called factors. After determining the factors and their observational values, for estimation of n observations of each factor, we will use an algorithm called Partial Least Squares (PLS). This algorithm, estimates observations of each factor by writing that factor as weighted sum of its observed values.

Estimation of model's parameters: Now, for estimation of the parametric matrices A and B, we write model (3) for all observational combinations as follows:

$$Y_{kl} = Z_{kl}\delta_{kl} + \varepsilon_{kl}$$

$$(k = 1,....M \qquad l = 1,....q_k) \qquad (9)$$

In which, Y_{kl} represents a n*1 vector of observations related to the product's quality variable y_{kl}.

As is specified in model (3), Z_{kl} is the observational matrix of data n of y_{kl} descriptive variables, δ_{kl} is a perpendicular vector of parameters and ε_{kl} is a n*1 vector of uncorrelated random errors.

Since the matrix B in relation (5) is an upper-triangular matrix, in case equation errors are uncorrelated (matrix Σ being diagonal), model of the introduced simultaneous equations will be recessive. In this regard, parameters of each equation are estimated by normal PLS method. Therefore we will have:

$$\hat{\delta}_{kl} = (Z'_{kl}Z_{kl})^{-1}Z'_{kl}y_{kl} \text{ for all l and k} \qquad (10)$$

If $\hat{A}$ and $\hat{B}$ denote the obtained matrices by replacement of the estimated parameters using relation (10) in matrices A and B. the matrix which gives fitting to values is obtained by the following relation:

$$\hat{Y} = X\hat{A} + Y\hat{B} \qquad (11)$$

In which, X is n*p matrix from observation of operational variables and Y is matrix of data n*q from observation of product's quality specifications.
n*q residues matrix is calculated from the below relation:

$$E = Y - \hat{Y} \qquad (12)$$

And variance-covariance matrix of errors is estimated using the following relation:

$$\hat{\Sigma} = E'E / n \qquad (13)$$

MODEL APPLICATION AND RESULTS

In this section, the introduced solution in the previous section will be applied to mobile phone production line of Motorola Company. Figure 2 shows production line of this company which include M = 2 assembly phases board and Module. Board assembly phase which starts from soldering machine and end in manual soldering comprises 9 sections. In section soldering machine, a solder material is used for each board. In each one of the three next sections, an electrical part is placed on the board. Next, in section of manual assembly, a worker assembles additional parts on the board. In thermal section, the board is put in a furnace and soldering material is melted. Next, the board is cooled again and the soldering material becomes solid once again. Then, in manual section, a worker assembles other additional pieces on the board. To solder these parts on the board, we thrust them into the soldering material. After completion of the board assembly, in section of board resting, 11 product's quality specifications are measured. In module assembly phase, the final product by combination of the board with output of other production lines is assembled and in section of module testing, 30 product's quality specifications in the final product are measured.

Table 1: Observational factors and variables

Weight***	Correlation coefficient**	Mean variance*	Observed factors and sizes
0.50	0.99	0.99	x_{11} automatic assembly machine 1: observational variable 1
0.60	0.99		Automatic assembly machine 1: observational variable 2
0.47	0.97	0.96	x_{12} automatic assembly machine 2: observational variable 1
0.60	0.99		Automatic assembly machine 2: observational variable 2
0.50	1.00	100	x_{13} automatic assembly machine 3: observational variable 1
0.50	1.00		Automatic assembly machine 3: observational variable 2
0.58	0.92	0.58	y_{11} board test: observational variable 1
0.52	0.90		Board test: observational variable 2
0.64	0.89	0.70	y_{12} module test: observational variable 1
0.49	0.77		Module test: observational variable 2

*It is a ratio of variance which explains every factor

**It is equal to correlation coefficient between observational variables and factors

***Weights are used for calculation of factors' values. For example, y_{11} is calculated as follows

(Board test: observational variable 2)×(o.52) + (board test: observational variable 1)×(0.58)

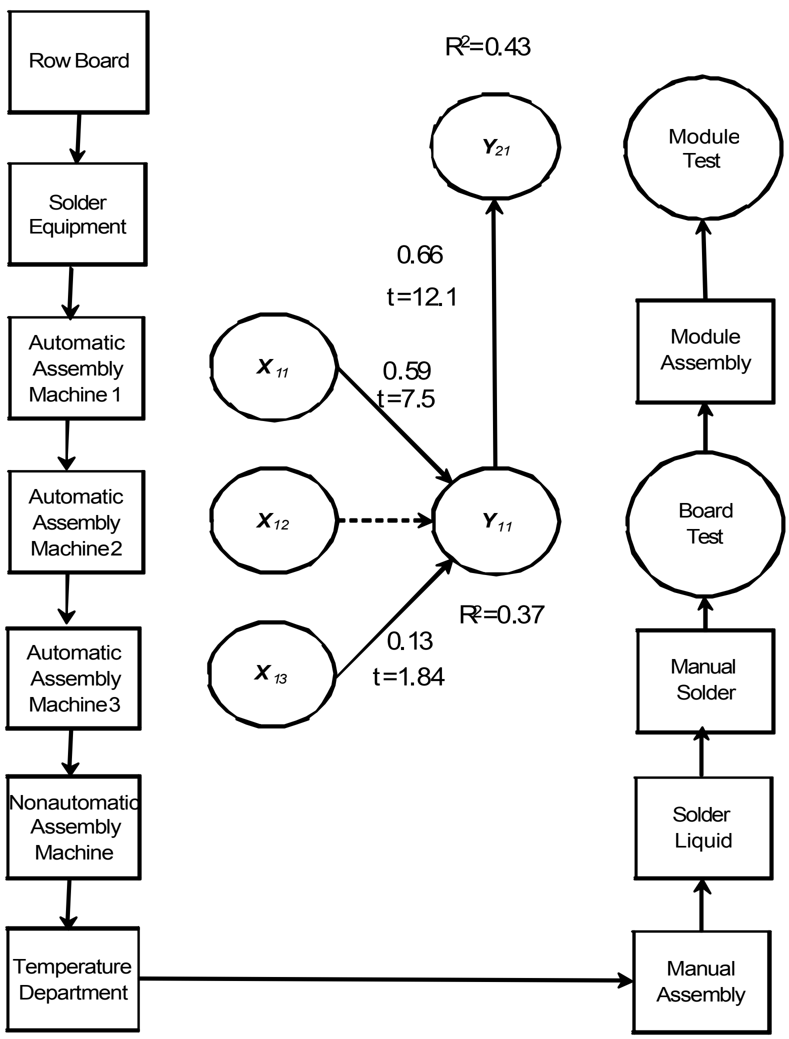

Fig. 2: Production line

Table 2: General effect of factors*

		Quality variables 'products	
Stage	Variable	y_{11}	y_{21}
Automatic assembly machine 1	x_{11}	0.59	0.39
Automatic assembly machine 2	x_{12}	0.00	0.00
Automatic assembly machine 3	x_{13}	0.13	0.09
Board test	y_{11}	0.00	0.66
Module test	y_{21}	0.00	0.00

*General effects have been obtained from estimation replacement of parameters of matrices A and B in equations 5 and 6

Table 3: Changeability sources in product's quality specifications

Source	*Percent of deviation in y_{11}	*Percent of deviation in y21
Automatic assembly machine 1: x_{11}	36	15
Automatic assembly machine 2: x_{12}	0	0
Automatic assembly machine 3: x_{13}	2	1
Ignored variables in board assembly stage	62	27
Ignored variables in module assembly stage	0	57

Data: Data of this study have been selected from several databases of Motorola Company. Each case in this database corresponds with a board and includes 11 sizes of the measured product's quality specifications in board testing section and 30 sizes of the measured product's quality specifications in section of module testing together with operational variables of equipments in each one of which the three sections of automatic equipments have been measured. All variables for purpose of confidentiality protection have been coded (In phase module assembly, no operational variable is measured).

Data reduction: Empirical (experimental) analysis begins with execution of factorial analysis in sections where multiple variables have been observed. Based on results of these factorial analyses, 47 observed variables have been reduced to 31 factors so as for each automatic assembly machine; the two observational variables have been reduced to one factor. Similarly, 11 product's quality specifications which had been observed in section of board testing were reduced to 8 factors and 30 product's quality specifications which had been observed in section of Module testing were reduced to 20 factors. Table 1 lists a number of the factors which have been specified through factorial analysis. To avoid presentation of a large volume of results, only one of the factors which had been specified in sections board testing and module testing are reported. Name of all variables for sake of data confidentiality is eliminated. Values of each factor using Partially Least Square (PLS) algorithm have been estimated. Table 1 also shows weights used for calculation of each factor's values.

Model and experimental results: Production line model and this model's parameters estimation using a solution explained earlier together with their t-statistic and R^{2-}statistic value of each equation are shown in Fig. 2. By study of results in Fig. 2 it is seen that direct effect of operational variable (factor) x_{11} on product's quality specification y_{11} is equal to 0.59. Since these coefficients are standardized, this value implies that as a result of one unit increase of standard deviation in x_{11}, the expected increase in y_{11} is equal to 0.59 unit. In addition, this figure shows that the operational factor x_{12} has no effect on y_{11}, but the factor x_{13} has a direct effect of 0.13 on y_{11}. Also, it is seen that output quality of module assembly phase is significantly affected by output quality of assembly phase. Specifically, it is seen that y_{11} have direct effect of 0.66 on y_{21}. Finally x_{11} and x_{13} each has indirect effect on the product's quality specification y_{21}.

Table 2 shows estimation of each factor's general effects on product's quality. Given this table, we find out that general effect of x_{11} on y_{21} is equal to 0.39 which suggest that as a result of one unit increase of standard deviation in x_{11}, the expected value increase in standard deviation of y_{21} is equal to 0.39 unit. In addition, given this table we find out that general effect x_{13} on y_{21} is equal to 0.09.

Determining the improvement opportunity: To break down variance of product's quality specifications into 5 constituents, relation (8) and the estimated parameters are used. Results are shown in Table 3.

This table shows that 36% of y_{11} variation has been the result of change in variable x_{11}. Remaining variation of y_{11} is the result of change in variable x_{13} (2%) and

Table 4: Prediction of effect of process improvement on changeability of production quality

Process improvement plan	Change in functional variables variation x_{11}	x_{13}	*Forecasting of variation in product's quality specification y_{11}	y_{21}
A	-25%	0.00%	-8.60%	3.70%
B	50%	0.00%	17.30%	-7.50%
C	0.00%	-25%	0.04%	-0.02%
D	0.00%	-50%	0.80%	-0.40%
E	25%	-25%	-9.10%	-3.90%
F	-50%	-50%	-18.10%	-7.90%

*They are obtained from parameters estimation and equation (8)

other operational variables which have not been measured in assembly phase (62%). In addition, by further study of Table 3 we find out that the main variation sources of y_{21}, is variable x_{11} (15%) and other operational variables which have not been measured in module assembly phase (57%). The remaining variation in y_{21} is the result of x_{13} (1%) and other operational variables which have not been measured in assembly phase (27%).

Results of Table 3 definitely suggest that managers and engineers should concentrate their improvement efforts on the automatic assembly machine (1) and module assembly phase. Table 3 also indicates that reduction in variation of y_{21} requires improvement of both module assembly and assembly phases.

Effect of process improvement on production quality: To predict a degree of variation decrease in product's quality which arises from changeability decrease in process relation (8) and the estimated parameters are used. Table 4 presents predicted effects of 6 different programs of process improvement on changeability of product's quality specifications.

Similar to results of Table 3, results of this table shows that variation decrease in the operational variable x_{11} has significant potential effect in variation decrease in product's quality. For example, given this table, it is predicted that by 50% v reduction in x_{11}, variation in y_{11} and y_{21} decreases to 17.3% and 7.5%, respectively. In addition, it is predicted that variation reduction in x_{13} has slight effect on changeability of product's quality specifications. This kind of analysis helps manager determine whether investment on a quality improvement program is justifiable or not.

The obtained results are also useful for interpretation of process control statistical charts and determination of stages which are responsible for deviation from statistical controls. As was earlier discussed, results in Fig. 2 indicate that output quality of module assembly phase is significantly under effect of output variation quality of the board assembly phase. As a result, when the phase board assembly goes out of control, output quality of module assembly phase is affected. Hence, ordinary control diagrams (Shewhart charts etc) may mistakenly give the indication that the both phases of board assembly and module assembly are out-of-control. Therefore, this aspect lays stress on necessity of monitoring methods which take phases' quality links into account.

Execution of control method: In this section, the recommended control method using mobile phone production line data will be explained.

Step 1: Data collection: A sample size of n = 90 boards of production line has been obtained.

Step 2: Data reduction: At this stage, as has been earlier explained, factorial analysis is employed. As a result of this factorial analysis, 47 observational variables were reduced to 31 factors. Then, 90 values of each factor using the estimated weights for PLS algorithm are calculated. For instance, the observation t of the factor x_{12} is calculated as follows:

$$x_{12t} = 0.47\, v_{121t} + 0.60\, v_{122t} \quad \text{for all } (t=1, \ldots, 90) \qquad (13)$$

In which, ν_{21} and ν_{122} are observational variables of factor x_{12}.

Step 3: Model's parameters estimation: At this stage, model parameters of Fig. 2 using ordinary PLS method are estimated. Estimation of these parameters has been shown in Fig. 2.

Step 4: Residues: To obtain residue of equation y_{11}, the below term is calculated:

$$\hat{\varepsilon}_{11t} = y_{11t} - 0.59\, x_{11t} - 0.13\, x_{13t} \quad \text{for all } (t=1, \ldots, 90) \qquad (14)$$

Next, the obtained residues from equation (14) are standardized as follows:

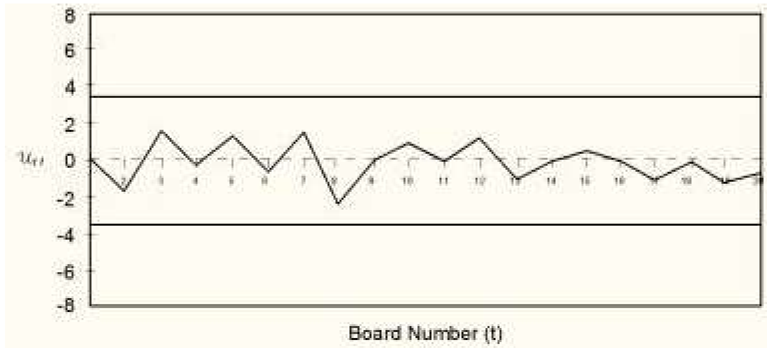

Fig. 3:

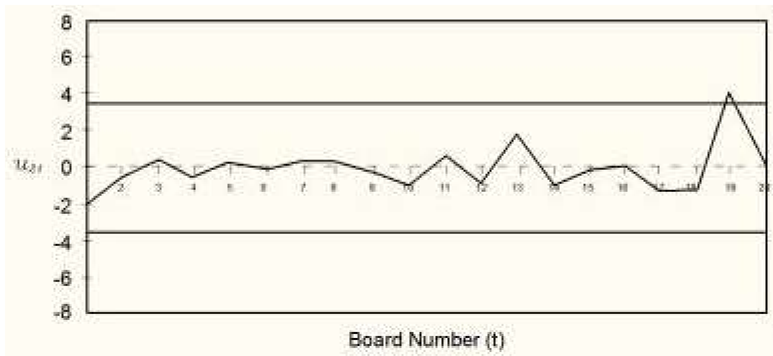

Fig. 4:

$$\hat{u}_{1t} = \frac{\hat{\varepsilon}_{1t}}{\sqrt{1-0.37}} \quad \text{for all } (t = 1, \ldots, 90) \tag{15}$$

Note that the introduced statistic in equation (15) is used to test whether the unmeasured operational variables in board assembly phase are under control or not.

Equation residues of y_{21} are calculated as follows:

$$\hat{\varepsilon}_{21t} = y_{21t} - 0.66 y_{11t} \quad \text{for all } (t=1, \ldots, 90) \tag{16}$$

Then, the obtained residues from relation (16) are standardized as follows:

$$\hat{u}_{21t} = \frac{\hat{\varepsilon}_{21t}}{\sqrt{1-0.43}} \quad \text{for all } (t=1, \ldots, 90) \tag{17}$$

Note that the introduced statistic in (17) is used to test whether the unmeasured operational variables in module assembly phase are under control or not.

Step 5: By specifying the error rate of first kind to the amount of a = 0.0027, the following critical values will be obtained for each test statistic (For some test statistics, normality assumption according to Shapiro-Wilk Test is refused. However, the normal probability chart suggests that these deviations are not serious. Therefore, it seems logical the critical values to be used. In addition, results of Durbin-Watson Test suggest that the test statistics are successively independent):

$$(3.460, -3.460) \tag{18}$$

To test under-control being of the operational variable x_{ij} (i =1, j =1, 2, 3) when gathering data, the presented test statistic is compared with critical values in (18) (t=1,…90). By performing this test, it is found that all the test statistics are within these values. Hence, it is concluded that at the time of data collection of the variables x_{11}, x_{12} and x_{13} have been under control.

Step 6: Finally, by comparing the statistics of (15) and (17) with critical values of (18) under-control being of the test's unmeasured operational variables are tested (for all t = 1,…90). Test statistic of (15) for all t values falls within range on critical values which indicate under-control being of the unmeasured operational variables in board assembly phase at the time of data gathering. Test statistic of (17) with regard to three boards fell outside the range of critical values which suggest out-of-control being of module assembly phase.

To clarify this point, the test statistic values of (3 and 4) have been shown for 20 boards of initial sample in Fig. 3 and 4. Darkly colored lines in these figures indicate critical values of (18). Results in Fig. 3 indicate under-control being of board assembly phase at the time of data collection. Figure 4 shows that test statistic of (17) with regard to number 19 is outside the control limit indicating out-of-control being of module assembly phase.

The next stage involves elimination of the three boards which have been outside the control limit and then stages 2 to 6 are iterated. Due to similarity of this stage with earlier stages, it is dispensed with its explanation.

CONCLUSION

In this study, a solution was recommended by means of which one can measure operational effect of each stage on output quality of final product. A distinct aspect of this solution is use of information related to correlation of product's quality specifications inside and between the system's phases. In general, this solution is relationship between quality specification of one stage and the measured quality specifications in earlier stages using a linear regression relation. This method intends by taking effect of quality specifications in previous phases from quality specification of a specific stage to control that stage. The recommended control method is done in two separate steps. In the first step, first, using the obtained data from production system, parameters of regression model are estimated then by backward looking it is tested whether at the time of data collection the system has been under control or not. Purpose of the first step is to obtain an under-control data set in order to found a model which is used for control of future observations of production system. In the second stage, the obtained model in the first stage is used to test under-control being of the system at the time of the next units' production.

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