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Poly, Binomial and Hypergeometric Distributions Comparison Regarding Skewness and Kurtosis

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Abstract: In this paper, we treat the skewness and kurtosis of the Poly, hypergeometric and Binomial probability distributions, in case of skewness when 2p<1 and N>nc, Polya distribution will be positively skewed and the same result is obtained for Hypergeometric and Binomial when c = -1 and c = 0 respectively.

Key words:

INTRODUCTION

Polya distribution [3]: Polya distribution can be explained in better way with the help of the example. An urn contains b black and r red balls. A ball is drawn at random. It is replaced and moreover, c balls of the color drawn and d ball of the opposite color are added. A new random drawing is made from the urn (while the urn now containing r+b+c+d balls) and this procedure is repeated. Here c and d are arbitrary integers. They may be chosen negative, except that in this case the procedure may terminate after finitely many drawings for lack of balls. In particular, choosing c = -1 and d = 0 we have the model of random drawing without replacement (Hypergeometric distribution) which terminates after r+b steps. The Polya distribution will be;

$$P_{n_1,n} = \frac{\binom{-\left(\frac{b}{c}\right)}{n_1}\binom{-\left(\frac{r}{c}\right)}{n_2}}{\binom{-\left(\frac{b+r}{c}\right)}{n}}$$

where n_1 is the number of black balls, n_2, number of red balls, $n = n_1+n_2$, total number of balls and when

$$p = \frac{b}{b+r},\ q = \frac{r}{b+r},\ \gamma = \frac{c}{b+r}$$

then

$$P_{n_1,n} = \frac{\binom{-\left(\frac{p}{\gamma}\right)}{n_1}\binom{-\left(\frac{q}{\gamma}\right)}{n_2}}{\binom{-\left(\frac{1}{\gamma}\right)}{n}} = \binom{n}{n_1}\frac{b^{n_1,c}\gamma^{n_2,c}}{N^{n,c}}$$

remains meaningful for arbitrary (not necessarily rational) constants p>0, q>0, γ>0 such that p+q=1. Verify that $P_{n_1,n} > 0$ and

$$\sum P_{v.n} = 1$$

In other words, the above is the Polya probability distribution with integers 0,1,2,…,n.

Hypergeometric distributions [2]: A random variable X has the hypergeometric distribution if (for some integers n,a,N with $1 \le n \le N$ and $0 \le a \le N$)

$$P(X = x) = \frac{\binom{a}{x}\binom{N-a}{n-x}}{\binom{N}{n}}$$

$$x = \max(0, n-(N-a)), \ldots\ldots\ldots, \min(n,a))$$

Now there are $\binom{N}{n}$ possible unordered samples, of which $\binom{a}{x}\binom{N-a}{n-x}$ contain x elements of type a. Thus X is a hypergeometric with n,a,N.

Binomial distribution [1]: A discrete random variable X is said to follow the binomial distribution if,

$$P(X = x) = \binom{n}{n_1} p^x (1-p)^{1-x}$$

$$x = 0, 1, 2, \ldots\ldots\ldots\ldots, n$$

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where $0<p<1$.

The conditions that give rise to a binomial distribution are:

(i) There is a fixed number n of trails;
(ii) Only two outcomes, 'success' and 'failure', are possible at each trial;
(iii) The trails are independent;
(iv) There is a constant probability p of success at each trial;
(v) The variable is the total number of successes in n trials.

SKEWNESS AND KURTOSIS OF POLYA, BINOMIAL AND HYPERGEOMETRIC DISTRIBUTIONS.

Compare the above said distributions regarding skewness: The skewness of the Polya distribution,

$$\beta_{1(pol)} = \frac{\mu_3}{\mu_2^{\frac{3}{2}}} = \frac{1-2p}{[np(1-p)]^{\frac{1}{2}}} \frac{(N+2nc)(N+c)^{\frac{1}{2}}}{(N+2c)(N+nc)^{\frac{3}{2}}}$$

where N,n,c and p are population size, sample size, (c will be zero or -1) and probability of success respectively, while the shape of the said distribution depends on the value of c also. Now if $p<1/2$ and $N>2nc$, then the Polya distribution will be positively skewed. By putting $c = 0$ in the Polya's skewness, so we will get the binomial skewness.

$$\beta_{1(bin)} = \frac{\mu_3}{\mu_2^{\frac{3}{2}}} = \frac{1-2p}{[np(1-p)]^{\frac{1}{2}}}$$

If $p<1/2$, then the binomial distribution will be positively skewed. By putting the value of $c = -1$, we will get the skewness of the Hypergeometric distribution,

$$\beta_{1(hyp)} = \frac{\mu_3}{\mu_2^{\frac{3}{2}}} = \frac{1-2p}{[np(1-p)]^{\frac{1}{2}}} \frac{(N-2n)(N-1)^{\frac{1}{2}}}{(N-2)(N-n)^{\frac{3}{2}}}$$

Compare the above said distributions regarding Kurtosis: For

$$\beta_{2(pol)} = \frac{\mu_4}{\mu_2^2} = \frac{N(N+1)(N+c)(N+nc)+N(n+1)(N+c)}{np(1-p)(N+2c)(N+3c)(N+nc)} + \frac{3(N+c)(N+c)}{(N+2c)(N+3c)} - \frac{6N^2(N+c)}{n(N+2c)(N+3c)(N+nc)}$$

by putting the value of $c = -1$, we will get the kurtosis of Hypergeometric distribution;

$$\beta_{2(hyp)} = \frac{\mu_4}{\mu_2^2} = \frac{(N-1)}{np(1-p)(N-2)(N-3)(N-n)} \times \{N(N+1)-6N(N-n)-6N^2p(1-p) + 3np(1-p)(N+6)(N-n)\}$$

By putting the value of $c = 0$, we will get the kurtosis of Binomial distribution;

$$\beta_{1(bin)} = \frac{\mu_4}{\mu_2^2} = \frac{(N+1)-6N}{np(1-p)N} + 3 - \frac{6}{n}$$

RESULTS AND CONCLUSION

In this paper, we discussed the relationship of Polya, Binomial and hypergeometric distributions in the context of their skewness and kurtosis. We conclude that the skewness and kurtosis for all said probability distributions seems to be same.

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