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Similarity Solution of Viscous Flow and Heat Transfer of Nanofluid over a Nonlinearly Stretching Sheet

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Abstract: The boundary-layer flow and heat transfer in a viscous fluid contains *metal nanoparticles* over a nonlinear stretching sheet is analyzed. The stretching velocity is assumed to vary as a power function of the distance from the origin. A similarity solution is presented which depends on the nonlinear stretching parameter *n*, Prandtl number *Pr*, Lewis number Le, Brownian motion number Nb and thermophoresis number Nt. The boundary layer velocity, temperature and nanoparticle volume fraction profiles are then determined numerically. The influence of various relevant parameters are discussed and comparison with published results is presented.

Key words: Nanofluid · Stretching Sheet · Viscous Flow · Heat Transfer

INTRODUCTION

The flow over a stretching surface is important in many engineering problem processes with application in industries such as extrusion, melt-spring, the hot rolling, wire drawing, glass-fiber production, manufacture of plastic and rubber sheets, polymer sheet and filaments are manufactured by continuous extrusion of the polymer form a die to a windup roller, which is located at a finite distance away. Experiment show that the velocity of the stretching surface is approximately proportional to the distance from the orifice, Vleggaar [1], Grane [2] and Gupta and Gupta [3] have analyzed heat and mass transfer in a stretching problem with constant surface temperature while Soundagekar [4] investigated the stokes problem for a viscelastic fluid. Magyari and Keller [5] studied the stretching problem of an incompressible fluid over a permeable wall. Vajravelu [6] studied flow and heat transfer in a viscous fluid over a nonlinear stretching sheet without viscous dissipation.

Nanotechnology has been widely used in industry since materials with size of nano meters possess unique physical and chemical properties. Nano-scale particle added fluids are called as nano fluid which is firstly utilized by Chio [7]. Chio *et al*. [8] showed that the addition of a small amount(less than 1% by volume) of nano-particles to conventional heat transfer liquids increased the thermal conductivity of the fluid up to approximately two times. Some numerical and experimental studies on nano fluids include thermal conductivity and convective heat transfer, Maiga *et al*. [9], Tiwari and Das [10], Tzou [11], Abu-Nada [12], Oztop and Nada [13], Kuznetsov and Nield [14] have examined the influence of nano particles on natural convection boundary- layer flow past a vertical plate. In our case we present study to analyze the study of nonlinearly stretching sheet of viscous flow of nano fluid and heat transfer. The dependency of the local Nusselt and Sherwood numbers is numerically investigated. Ghasemi and Aminossadati [15], Ho *et al*. [16, 17], etc. These studies have used traditional finite difference and finite volume techniques with the tremendous call on computational resources that these techniques necessitate. Hamad *et al*. [18] used the application of a one-parameter group to present similarity reductions for problems of magnetic field effects on free-convection flow of a nanofluid past a semi-infinite vertical flat plate following a nanofluid model proposed by Buongiorno [19]. The same nanofluid model has been used also by Hamad and Pop [20] for the similarity solution of the steady boundary layer flow near the stagnation-point flow on a permeable stretching sheet in a porous medium saturated with a nanofluid and in the presence of internal heat generation/absorption effect. Hamad [21] found the analytical solutions of convective flow and heat transfer of an incompressible viscous nanofluid past a semi-infinite vertical stretching sheet in

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the presence of magnetic field. We mention also the paper by Khan and Pop [22] on the steady boundary layer flow, heat and concentration over a stretching surface in its plane. Further, we mention the papers by Abu-Nada and Chamkha [23] on the natural convection heat transfer characteristics in a differentially-heated enclosure filled with a CuO-EG-water nanofluid for different variable thermal conductivity and variable viscosity models. For more information see also Das *et al*. [24], Wang and Mujumdar [25, 26].

The aim of this paper is to investigate the similarity solution for the boundary-layer flow and heat transfer in a viscous fluid contains *metal nanoparticles* over a nonlinear stretching sheet is analyzed. Therefore, discuss the effect of the influence parameters, namely, the nonlinear stretching parameter n, Prandtl number Pr, Lewis number Le, Brownian motion number Nb and thermophoresis number Nt on the flow characteristics.

MATERIALS AND METHODS

Analysis: We consider the steady of viscous flow of nano fluid past a nonlinearly stretching sheet. The flow takes place at $y \geq 0$ where y is the coordinate measured normal to the stretching sheet, a steady uniform stress leading to equal and opposite forces is applied along the x-axis, so the sheet is stretched keeping the origin fixed. We assume that the temperature at the stretching surface is a function of x as in equation 5, while the ambient temperature has constant values T_∞. It is assumed that at the sheet, the nanoparticle fraction C take constant value C_w. It is further assumed that the base (host) fluid and the suspended nanoparticles are in thermal equilibrium and no slip occurs between them. Under the above assumptions, the boundary layer equations governing the flow and temperature in the presence of heat source or heat sink are (using the boundary layer approximations and neglecting viscous dissipation). The basic steady conservation of mass, momentum, thermal energy and nanoparticles equation of nanofluid can be written in Cartesian coordinates x and y as.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2+\tau\left\{D_B\frac{\partial T}{\partial y}\frac{\partial C}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right\} \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

Where u and v are the velocity component of the fluid in the x-and y- directions, respectively and v is the kinematic viscosity.

The appropriate boundary conditions for the problem are given by

$$u=u_w(x)=c\,x^n,\quad v=0,\quad T=T_w(x)=T_\infty+b\,x^{2n},\quad C=C_w \\ \text{at } y=0 \quad u\to 0,\quad T\to T_\infty,\quad C=C_\infty \quad \text{as} \quad y\to\infty \tag{5}$$

By introducing the following non-dimensional variables

$$\eta=y\sqrt{c(n+1)/2\nu_f}\;x^{(n-1)/2},\; u=c\,x^n F'(\eta) \\ v=-\sqrt{\frac{(n+1)c\nu_f}{2}}\,x^{(n-1)/2}\left[F(\eta)+\frac{n-1}{n+1}\eta F'(\eta)\right], \\ \theta(\eta)=(T-T_\infty)/(T_w-T_\infty),\; \phi(\eta)=(C-C_\infty)/(C_w-C_\infty) \tag{6}$$

Using (6), equations (1) to (4) become

$$F'''+FF''-\frac{2n}{n+1}F'^2=0. \tag{7}$$

$$\frac{1}{\Pr}\theta''+F\theta'-\frac{4n}{n+1}F'\theta+Nb\theta'\phi'+Nt\theta'^2+E_c F''^2=0 \tag{8}$$

$$\phi''+Le\,F\phi'+\frac{Nt}{Nb}\theta''=0 \tag{9}$$

and the corresponding boundary conditions (5) become

$$F=0,\quad F'=1,\quad \theta=1,\quad \phi=1 \quad \text{at} \quad \eta=0 \\ F'\to 0,\quad \theta\to 0,\quad \phi=0 \quad \text{as} \quad \eta\to\infty \tag{10}$$

Where $\Pr = v/k$ is the Prandtl number, $Le = v/D_B$ is the Lewis number, $Nb = \tau D_B (C_w - C_\infty)/v$ is Brownian motion parameter, $NT = \tau D_T(T_w - T_\infty)/(vT_\infty)$ is the thermophoresis parameter and $E_c=\frac{u_w^2}{C_p(T_w-T_\infty)}$ is the Eckert number.

The quantities of practical interest, in this study, are the local Nusselt number Nu_x and the local Sherwood number Sh_x which are defined as.

$$Nu_x=\frac{-x}{(T_w-T_\infty)}\left(\frac{\partial T}{\partial y}\right)_{y=0},\; Sh_x=\left|\frac{x}{(C_w-C_\infty)}\left(\frac{\partial C}{\partial y}\right)_{y=0}\right| \tag{11}$$

Using (6) and (11), we get

$$\mathrm{Re}_x^{-1/2}Nu_x=-\left(\frac{n+1}{2}\right)^{1/2}\theta'(0),\; \mathrm{Re}_x^{-1/2}Sh_x=\left(\frac{n+1}{2}\right)^{1/2}|\phi'(0)| \tag{12}$$

Where $Re_x = xu_w/v_f$ is the local Reynolds number based on the stretching velocity u_w.

RESULTS AND DISCUSSIONS

Using the boundary condition (10) and solving the e.q. (7-9) numerically for some values of the governing parameter *Pr, Nt, Le* and *Nb*. Nusselt number Nu_x and Sherwood number Sh_x are defined and the results of $F''(0)$, $-\theta'0(0)$ are compared with Cortell [27] in different value of $Nb = 0$, $Nt = 0$ and *Pr* that is shown in Table (1) and (2), Table (3) observed that $\theta'(0\_$ and $\phi'(0)$ decreasing function with *Nb values in two state of* $E_c = 0$ when $E_c = 0.1$ and *Pr*=10, *n*=10, *Le*=10 and *Nt*=0.3, Table (4) we suggest *Pr*=10, *n*=10, *Nb*=0.3 and *Nt*=0.3 and different value *Le* of we find the value of $\theta'(0)$ and $\phi'(0)$ are decrease with $E_c = 0$ sand $E_c = 0.1$. Fig. (1) As expected, the boundary layer Profiles for the temperature function $\theta(\eta)$ are essentially the same form as in the case of a regular fluid. As nonlinear stretching parameter *n* increase, the temperature decreases for the specified conditions, while it is increase with increasing the value of *Nt, Nb*. Fig. (2) However, the thickness of the boundary layer for the mass fraction function $\phi(\eta)$ is found to be smaller than the thermal boundary layer thickness when $Le = 1$, E_c and $Pr = 6.8$.

Table 1: Comparison of results for

n	Present results	Cortell [27]
0.0	0.628316	0.6276
0.2	0.767489	0.7668
0.5	0.890103	0.8895
1	1.000484	1.0000
3.0	1.148986	1.1486
10	1.235220	1.2349
20	1.257758	1.2574

It decreases with the increase in *Nb* and increase with stretching parameter *n* increasing. Fig. (3) Shows the effects of E_c and *Le* numbers on the temperature distribution which is decreases with increasing *Le* and increases with increasing E_c. In Fig. (4) It is clear that the concentration decreases as the *Le* numbers increase and also when E_c increase.

Table 2: Comparison of results for $-\theta'(0)$ when $Nb = 0$, $Nt = 0$ and $Pr = 5$

	$E_c = 0$		$E_c = 0.1$	
n	Present results	Cortell [27]	Present results	Cortell [27]
0.75	3.125152	3.1250	3.016658	3.0170
1.5	3.567797	3.5677	3.455594	3.4557
7.0	4.185335	4.1854	4.065541	4.0657
10	4.255853	4.2560	4.135057	4.1353

Table 3: Values of $-\theta'(0)$ and $\phi'(0)$ when Pr=10, n=10, Le=10 and Nt=0.3

	$-\theta'(0)$		$\|\phi'(0)\|$	
Nb	$E_c = 0$	$E_c = 0.1$	$E_c = 0$	$E_c = 0.1$
0.1	3.771628	3.658830	5.621979	5.345776
0.2	3.251502	3.149618	0.998139	0.875024
0.3	2.827929	2.735630	0.451703	0.525092

Table 4: Values of $-\theta'(0)$ and $\phi'(0)$ when Pr=10, n=10, Nb=0.3 and Nt=0.3

	$-\theta'(0)$		$-\phi'(0)$	
Le	$E_c = 0$	$E_c = 0.1$	$E_c = 0$	$E_c = 0.1$
10	2.827929	2.735630	0.451703	0.525092
20	2.311415	2.234830	2.159347	2.213024
30	2.039608	1.971317	3.303876	3.347141

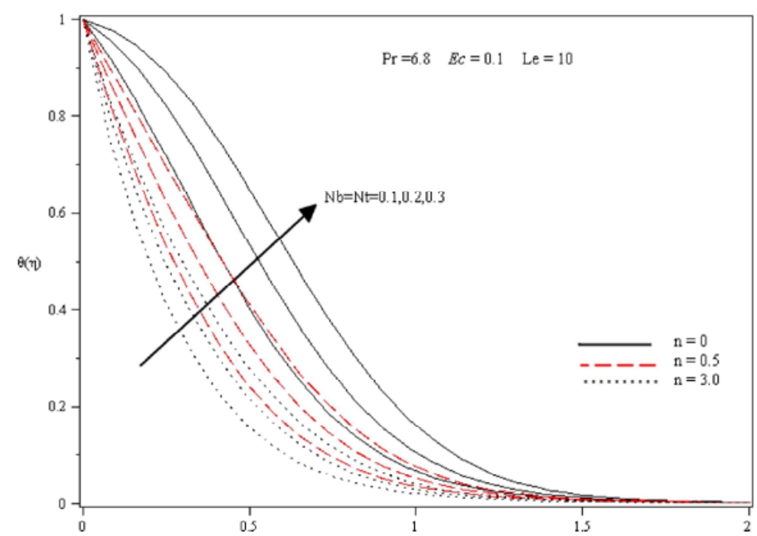

Fig. 1: Effect of Nt and Nb on temperature distribution for virous n.

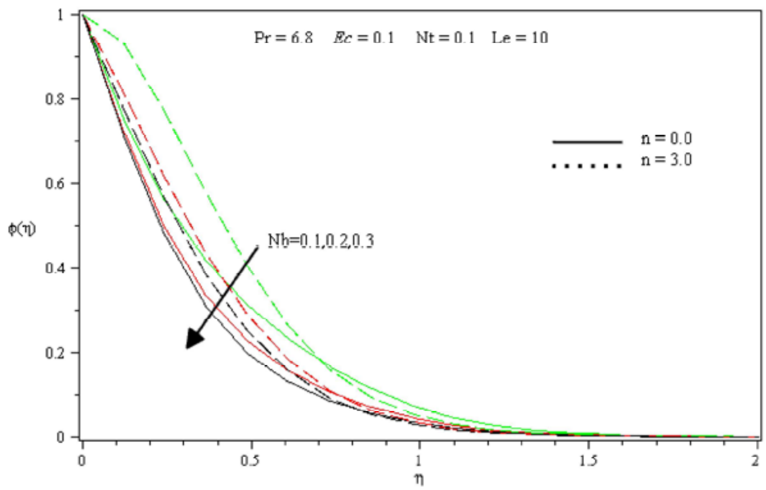

Fig. 2: Effect of Nb on concentration distribution for virous n when n =0 and n=0.3.

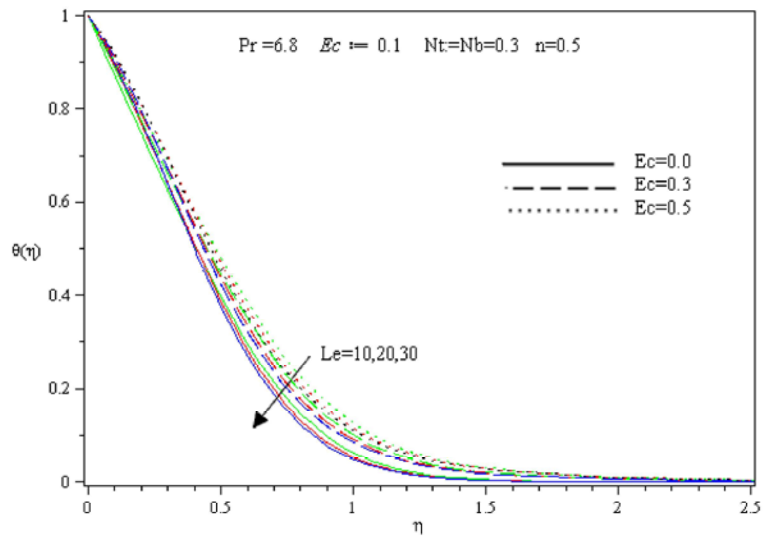

Fig. 3: Effect of Le and Ec on temperature distribution.

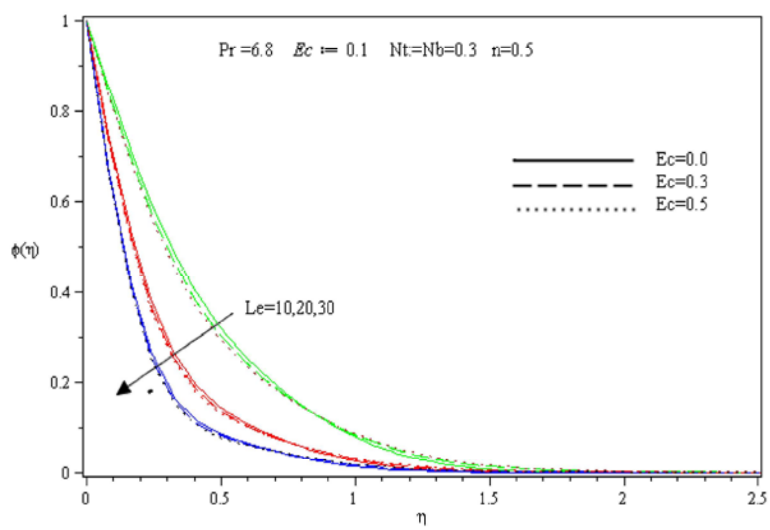

Fig. 4: Effect of Le and Ec on concentration.

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