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Modeling and Forecasting Olive Oil Price Using Fuzzy Time Series and a Fractional Integrated Stochastic Process

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Abstract: Olive oil prices are a key feature for local and global market operators; despite that, the analysis and studies in this field didn't get much attention of researchers. The aim of this article is to model and forecast olive oil prices using two econometric approaches namely, autoregressive fractionally integrated moving average (ARFIMA) model and Fuzzy time series (FTS) technique over the period 1980 To 2019. The results show with strong evidence that *ARFIMA*(1, *d*, 1) and the FTS model of Singh (2008) fit better the data among the different competing others specifications. In terms of forecasting, we find that the Singh FTS model outperforms the ARFIMA model. We hope by this first study in this field could gives the farmers and operators in olive oil market a new tool of well predicted the pattern of future prices.

JEL Classification: C22, C53, Q11
Key words: Olive Oil Price・Forecast・ARFIMA・Fuzzy time series

INTRODUCTION

Olive oil is a crucial food for human dietary, [1, 2] and for therapy, [3]. Statistics shows that the world production of olive oil has continuously increased from a level of 2458 to 3314 (Million metric ton) between 2015 and 2018 with a rate of 34%. For this same period the price variable corresponds to 10% (see International Olive Council (IOC, 2019). For instance, for the only two last oil olive seasons (October of year *t* to September of year *t* + 1), the world production has decreased from 2561.5 million tons in the 2016/2017 season to 3314 million for the 2017/2018 season, which corresponds to nearly 29% decreases. In addition, the production forecast for olive oil for the current season (2018/2019) is evaluated to 3131 million tones.

The fluctuations in the price of olive oil have increased in recent years and are much larger than in the early 2000s. Thus, after a short period of stability at 3, 200 USD of tones (in 2000/2002), the (2003/2008) and (2013/2015) were the sub-periods more volatiles in terms of prices. Consequently, due to this high variability of olive oil price all market participants such farmers, consumers, governments, policy makers, speculators are highly interested by the forecast of the future prices of this food product. Under the market regulations strategies, the governments are still faced with the challenge of setting up devices for farmers when prices are low and squeezing profit margins to fight inflation of these prices to the benefit of consumers. Recently, the government in Tunisia set the profit margin for the resale of olive oil at 15%, [4]. However, from an econometric point of view, it is also important to better understand the true data generating process of the price of this agriculture product since this it allow econometricians to use the adequate model for forecasting.

Several reasons motivate our focus on only the oil olive product. *First,* most of previous agriculture studies have focused in other important products such as: wheat, [5], Rice, [6], Palm oil, [7] and Sugar: [8, 9]. *Second*, olive oil is a key agriculture product for several countries around the world such as Spain: [10] , Tunisia, [11] For instance, the share of the oil export in the total agriculture export for Spain is 36.6% and for Tunisia 21% in 2018, [12]. For Greek olive oil production accounts for approximately ten per cent of the total agricultural production and represents 9.4 per cent of the Greek agricultural GDP, [13].

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Third, the there is an expectation of demand increase of olive oil, [14]; the new market of U.S can make a big difference on dynamics of olive oil prices, they argued that this new trend is mainly supported by News about the health and culinary benefits of olive oil and the spread of Mediterranean diet contribute significantly to the rising demand. Furthermore, Canada, Brazil, Australia, Japan, China and Russia can be characterized as emerging markets, since they continuously increase the imported quantity of olive oil to fulfill the growing demand [15]. Under a marketing view, [16] tried to give answers to the functional olive oil market.

Some researchers go forward to think of a creation of new tourism segment exclusively for olive oil product [17], For [18] he was interested on cost of olive oil; he presented a new methodology to estimate cost of a unit package of extra-virgin olive oil. [19] focused on the likelihood of the relationships of olive oil price dynamic and olive yields in three Mediterranean olive cultivation areas (Spain, Italy and Tunisia). [20] taken Olive oil market in Brazil as a case study and tried to estimate the effects of extrinsic cues on olive oil price. But all studies we found don't analyzed the dynamic of Olive oil prices over time, specifically extra virgin oil, we think our study comes to fill this research gap.

In this article, we use the Autoregressive Fractional Integrated Moving Average (ARFIMA) and Fuzzy Time Series (FTS) models. The main advantage of the ARFIMA modeling is the ability to take into consideration both the short and long-run effect of shocks. The short run component correspond to the autoregressive part ($AR(p)$) and the long-run component is modeled by the fractional long memory parameter, d. For the FTS technique, the fuzzy time series (FTS), the main advantage is that there are no assumptions considered for the data set. In this later technique, the forecasting accuracy is improved by using modification of interval number of the data set. In literature these two approaches have been widely applied in agricultural field, for *AFRIMA models*, [21] have been shown the evidence of the long memory in future prices. [22], made a strong evidence of usefulness of *ARFIMA models* for predicting macroeconomic and financial time series. [23], in different markets in India have been applied an ARFIMA model for forecasting the price of agricultural commodities. In order to investigate the behavior of the Brazilian ethanol price, [24], used the ARFIMA models. For Fuzzy time series models, recently, [25] presented a new method of FTS based on weighted technique to forecast enrollment data. [26], made a featured model to forecast rice production. [27], have focused on the application of FTS to forecast the electric load. [28], to predict the stock index prices used the fuzzy sets and a multivariate fuzzy time series models. [29], have been trying to model Indonesian-Malaysian Stock Market used the fuzzy random times series approach.

Through this study, our objective is to understand first the true data generating process of this food product. For this end, two approaches have been proposed to model the oil olive prices. The first approach is the ARFIMA process which assumes that the impact of a shock on the olive price will have long-lasting effects. The second approach is the FTS technique which left to data to generate its process of estimation and forecasting. In a second step, after selecting the two final ARFIMA and TFS models, we compare their performance based on their ability to better forecast olive price. The rest of paper is organized as follows. Section 2 provides an overview about the oil olive market.Section3presents the two models ARFIMA and FTS models. Section 4 presents the data and discusses the empirical results in term of modeling and forecasting of oil olive price. Section 5 concludes and provides some economic policies.

Diagnostic of Olive Oil Market: Olive growing today is dominated by a major producing country, Spain, which produces on average more than 50% of world production, [30]. The total area occupied by the olive tree is about 11 million hectares planted by nearly 1.5 billion feet. The European Union accounts for 50% of this orchard, Africa (North Africa) 25%; the Middle East 20%, the rest is divided between America (California, Chile, Argentina ...), Australia and China. Olive oil production represents 3% of the vegetable oils produced in the world, far behind soybean oil, palm oil, rapeseed oil, sunflower oil. The global production of vegetable oils is 137.7 million tons over 2011/2012 with a growth perspective of 30% by 2020, growth driven notably by Asia (Malaysia, Indonesia, China), the Argentina, the European Union and Brazil, but also Canada, the Russian Federation and Ukraine. About 15% of vegetable oil production is used for biodiesel.

From the map and according to the IOOC statistics, the main orchards: Spain, Italy, Turkey and Tunisia, the smaller orchards: Malta, Angola, Brazil and Uruguay. With Spain as the driving force, olive oil production has experienced an upward trend during the past 25 years – reaching 2.8 million tons in 2014 from about 1.5 million tons in 1990. World production is mainly concentrated in the Mediterranean basin where the climate is favorable. The main consuming countries are also the main producing countries as shown in the diagram below.

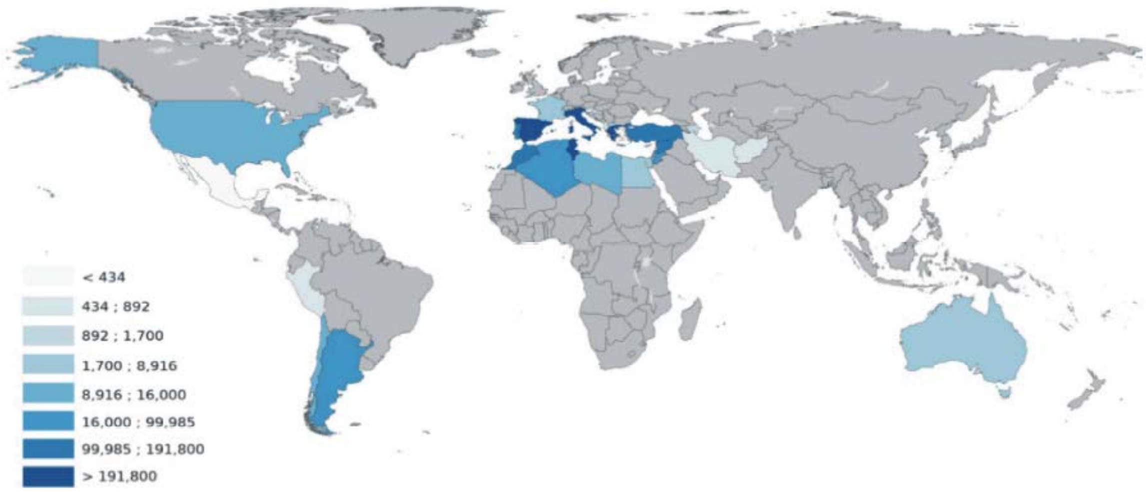

Fig. 1: World olive production by countries, Source: FAO Statistics 2016

All countries in the European Union account for 71% of world consumption. The countries around the Mediterranean account for 77% of world consumption. Other consuming countries are the United States, Canada, Australia and Japan (see the import trade flow diagram). (Source: http://www.unctad.org)The Olive oil market has changed over the period (1990-2018). Olive Oil Prices are mainly related to the levels of: production, consumption and even speculation. In term of production and according to IOC statistics it is 100 times larger than when it in 1990, with 3321(1000t) as a max production level recorded in (2011/12), also, the average yearly increase in olive oil production during the period (1990/2018) was 3%. According to the olive oil balances adopted at the end of November 2018 by the Council of IOC Members, world production for the 2018/2019 season (1 October 2018-30 September 2014) should decrease compared to the previous season which had been particularly good, from 3314 to 3131(1000 t). Climate changes could heavily affect olive oil producing areas, especially in the Mediterranean basin, [31].

Under this climatic challenge, the irrigation of olive yields was considered as best solution and its management can have a profound influence on olive oil production, [32]. This is, approximately, the same trend recorded for consumption and international trade (import and export) on these oils or on the oilseeds from which they are extracted. The major consumption regions are also relative concentrated in North Africa and UE. There is a great relationship between olive oil consumption levels, local developed level and population.

The consumption patterns and trend of olive oil have changed over the whole period of analysis (1991-2019), (Figure 2), it records a 0.165 of coefficient of variation and it's the lowest coefficient comparing Import, Export and Production ones. We note an increase of average consumption by 77% during the entire period and (2000-2019). [33] confirmed that the consumption has increased in countries where olive oil is not part of the traditional diet, as for example Germany and the UK. From the Figure 2, the four time series exhibited approximately the same pattern of trend and fluctuations, precisely we see clearly that (Production; Consumption) and (Export; Import) jointly compose a cointegrate time series, furthermore, the volatility of these time series (measured here by the coefficient of variation, see Table above) are nearly equals for the twin (Production; Consumption) and (Export; Import); furthermore, by estimating the linear correlation (Figure 3) between these four time series, the highest correlation have been estimated for: export *vs* import with a coefficient of 0.99 and 0.9 for production *vs* consumption. In 2016/17 period, world export and import of virgin olive oil was (respectively) 929 and 920 thousand tones (table), a 15% and 14% decrease under 2015/16 global export and import.

Based on the statistics delivered by the IOC on 2019, the global market potential olive oil is around 11.5 billion USD in 2018 and is expected to reach USD 13 billion by 2025. The Figure 2 reports the effects of the dry climate in the Mediterranean region on the production, over the period (2014-2016), which in turn has resulted in a price increase, where the highest price was recorded in 2015 by 5045 $U.S. Dollars per Metric Ton. This rise dynamic in prices slowed down the consumption (the demand); see the blue line on Figure 2.

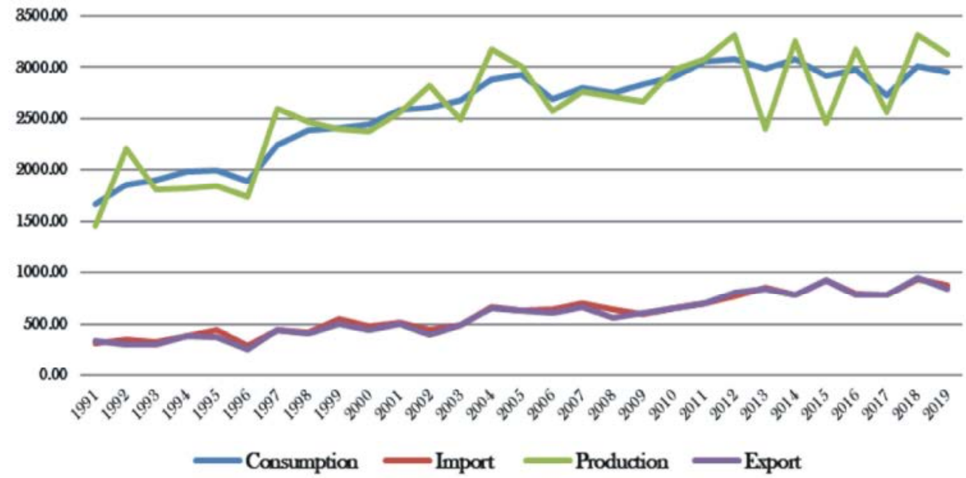

Fig. 2: Global evolutions of Consumption, Import, production and Export of olive oil. Source: Own graphs based on IOC data set, published in 2019. See the link: http://www.internationaloliveoil.org/estaticos/view/131-world-olive-oil-figures?lang=en_US.
Note: for the years (2017/18 and 2018/19) are just the predicted values.

Table 1: Coefficient of Variation of Consumption, Import, Production and Export of Olive oil

	Consumption	Import	Production	Export
CV	0, 165	0, 321	0, 195	0, 345

Source: Own estimate using IOC data set.
Note: CV: Coefficient of Variation is a ratio of standard-deviation and means value.

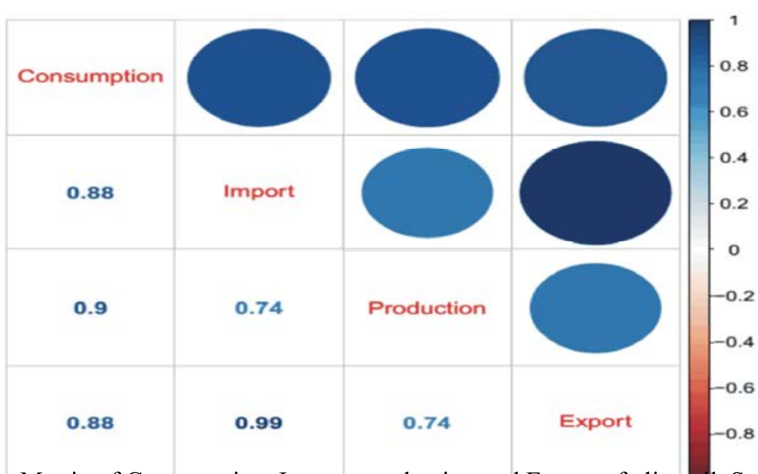

Fig. 3: Linear Correlation Matrix of Consumption, Import, production and Export of olive oil. Source: From the scale of linear correlation; we have strong positive linear correlations between the four times series have been detected.

MATERIALS AND METHODS

Autoregressive Fractional Integrated Moving Average Process: The Autoregressive Fractional Integrated Moving Average Process (ARFIMA), is a case of long memory stochastic process; whose study developed in hydrology, [34, 35, 36, 37, 38]. We just present the main concepts and base principles of these models. We start by define a long memory process,

Definition 1: A stochastic process Z_t is a long memory process if the relation below holds:

$$\lim_{v\to\infty} \frac{\rho v}{c.v^{-a}} = 1$$

With, $0 < \alpha < 1$, $c > 0$ and ρ_v is the autocorrelation function (ACF) and v: the lag of ACF. The autocorrelation function ρ_v of a long memory process decays slowly at a hyperbolic rate; the opposite of ARMA models that have an exponential decay. A simple representation of an Autoregressive Fractional Integrated Moving Average model, *ARFIMA* (p, d, q) is:

$$\left(\sum_{i=1}^{p}\beta_i z_{t-i}\right)(1-L)^d\ z_t = \sum_{i=1}^{q}\theta_i\varepsilon_{t-i}$$

With: β_i (i = 1,...,p) θ_i(i = 1,...,q)$\in\mathbb{R}$ ε_t is a process with zero mean and constant variance σ_ε^2.

We can simply use binomial development to reformulate the fractional integrated term as follow:

$$(1-L)^d = 1-dL-\frac{d(1-d)}{2!}L^2-\frac{d(1-d)(2-d)}{3!}L^3-...=\sum_{i=0}^{\infty}\omega_i L^i$$

with,

$$\omega_i = \frac{\Gamma(i-d)}{\Gamma(i+1)\Gamma(-d(} = \prod_{0<h\le i}\left(\frac{h-1-d}{h}\right),\quad i=0,1,2,\ ...$$

The ARFIMA processes are long-stationary and invertible when $d\in\left[-\frac{1}{2},\frac{1}{2}\right], d\neq 0$. There is a relationship between the Hurst exponent "H", see [34] and the fractional integration parameter, which is: $d = H-\frac{1}{2}$. The estimation of Hurst exponent is based on R/S statistics, (denoted M_t here):

$$M_t = \frac{\left(\max\limits_{1\le v\le T}\sum_{j=1}^{v}(z_j-\hat{z})\right)-\max\limits_{1\le v\le T}\sum_{j=1}^{v}(z_j-\hat{z})}{\left[\frac{1}{T}\sum_{j=1}^{v}(z_j-\hat{z})^2\right]^{1/2}}$$

The Hurst exponent is given as:

$$H \sim \frac{\log M_t}{\log T}$$

If $d\in\left[0,\frac{1}{2}\right]$, the ARFIMA process is stationary with long memory. The autocorrelations are positive and decrease hyperbolically towards 0, when the delay v increases; therefore, ARFIMA is a persistent process.

Just as the ARIMA processes have been extended to the SARIMA processes; to take into account this periodic behavior of the time series, we can work with:

- A filtration technique: $(1-\varphi L+L^2)^d$
- Or the introduction of a fractional seasonal differentiation operator of the type $(1-L)^d$:

Estimation of ARFIMA and Tests for Presence of Long Memory: In literature several tests and methods used for estimating and testing long memory in stochastic process applied in time series, we found Heuristic method based on R/S statistic, [34] Regression method or GPH method, [36]. Approximate and Exact Maximum Likelihood methods [39] and [40].

We just here present briefly the *Geweke & Porter-Hudak* method; also called "Regression method", which is based on spectacle density of *ARIMA* (p, d, d) process given by:

$$f(\gamma) = |1-e^{i\gamma}|^{-2a}\ g_\pi(\gamma)$$

With: $g_\pi(\gamma)$ is the spectral density of an *ARMA*(p, q) model. After making transformations (by taking logarithm of the equation (6) and add to the two sides, ln$P(\gamma_j)$- the periodogram of the time series – we found, an equation like a linear regression model, with dependant variable z_j ln $p(\gamma_j)$,

$$Z_j = \alpha+\beta Y_j+\zeta_j$$

With:

- j = 1,2,3..,m, correspond of ordered from the periodogram;
- $\alpha = \ln g_\pi(0);\ \beta = -d;\ \ Y_j = \ln|1-e^{i\gamma_j}|^{-2}\ \ and\ \ \zeta_j = \ln\frac{p(\gamma_j)}{f(\gamma_j)}$

We used the ordinary least square method to estimate of the linear regression model, thus factionary integrated parameter is estimated by:

$$\hat{d} = -\left[\sum_{j=1}^{m}(Y_j-\bar{Y})^2\right]^{-1}\sum_{j=1}^{m}(Y_j-\bar{Y})(Z_j-\bar{Z})$$

If $\frac{1}{2}<\hat{d}<\frac{1}{2}$, [36], stated that, with a sample size too big (i.e.) $T\to\infty$, this estimation follow a normal distribution, with variance,

$$V(\hat{d}) = \pi^2/6\left[\sum_{j=1}^{m}(Y_j-\bar{Y})^2\right]^{-1}$$

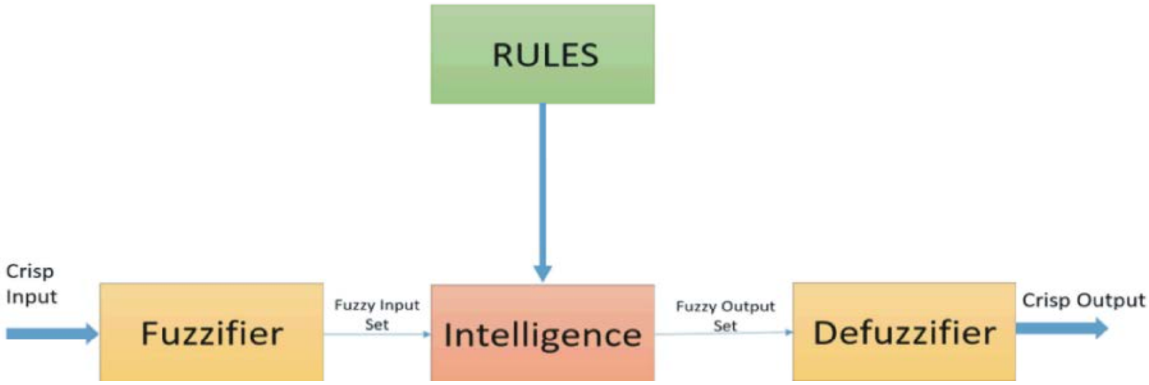

Fig. 4: The Fuzzy Logic Architecture.

Fuzzy Logic Time Series Models: Fuzzy logic (FL) is a multi-valued logic where the truth values of variables - instead of being true or false - are reels between 0 and 1. In this sense, it extends classical Boolean logic with partial truth values. The FL method imitates the way of decision making in a human which consider all the possibilities between digital values T and F. The fuzzy logic is considered as a support of decision making. This technique has been applied to different fields, from control theory to Artificial Intelligence, [41].

The diagram of a fuzzy system is shown in Figure 4. The system has as input a precise value (x_t), the latter is fuzzified (transformed into degree of membership in the input fuzzy set, see Definition (1) below; then it is transmitted to the fuzzy inference engine. Using the fuzzy IF-THEN rules stored in the rule base, the inference engine produces a fuzzy value that will be defuzzified giving the result to be usable. Partitioning the crisp dataset (Figure. 4), the identification of fuzzy logical relationships and the defuzzification play a very important role on the forecasting performance of the model, [42].

In this section, we briefly present some concepts of fuzzy time series. We mainly based on: [43, 44, 45]. The main difference between the fuzzy time series and classical time series is that the values of the former are fuzzy sets while the values of the latter are real numbers.

Definition 1: We put Ω the universe of discourse; $\Omega\{u_1, u_2,..., u_n\}$ we define a fuzzy set M of U as:

$$M = \left\{ \frac{u_M(u_1)}{u_1}, \frac{u_M(u_2)}{u_2}, ..., \frac{u_M(u_n)}{u_n} \right\}$$

With: $u_M(u_i)$ is the membership function of B, taking values in [0,1] and $1 \leq i \leq n$.

Definition 2: We have subset, H_t, (t = 1,2,...) of real numbers be the universe of discourse by which we define a fuzzy sets $m_i(t)$ are defined. If $M(t)$ is a collection of $m_1(t), m_2(t)$, then, $M(t)$ is called a Fuzzy time series (FTS) defined on H_t.

In literature according to time scale, we have two kinds of fuzzy time series (FTS) models: time variant and time invariant, [46] So, M_t is called a time-invariant fuzzy time series, If: $\forall t$, i: $R(t,t-i)$ is independent of t, where $R(t,t-i)$ is the fuzzy relationship between $F(t-i)$ and $F(t)$, if not $M(t)$ is a time-variant fuzzy time series model, for this category, we found a few number of studies, [47, 48]. Another decomposition of FTS models is when varying the values of i in $R(t, i)$; (*i.e.*): $M(t)$ depends on: $M(t-1)$, $M(t-2)$,.., $M(t-i)$.

$$\begin{cases} if \;\; i = 1, & \text{we have a first order FTS model} \\ if \;\; i > 1, & \text{we have a high order FTS model} \end{cases}$$

In this work, especially for the fuzzy time series part, we are focused mainly only on applications. For theoretical advanced, the references cited here are sufficient to give the reader a full overview of Fuzzy logic theory in general and FTS in particular.

Accuracy Comparison Criterion: Performance and forecast accuracy of the ARFIMA and FTS models is measured in terms of Root Mean Square Errors (RMSE); is defined as:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n} (\hat{y}_i - y_i)^2}{n}}$$

With: $\hat{y}_i$ are predicted values and y_i: observed values. We used also, the Mean Error (ME), the Mean Absolute

Error, the Mean Percentage Error, Mean Standard Error and Mean Absolute Percentage Error For details about forecasting accuracy measures,

$$U = \frac{\sqrt{\sum_{t=1}^{n-1}\left(\frac{\hat{y}_{i+1} - y_i}{y^i}\right)^2}}{\sqrt{\sum_{t=1}^{n-1}\left(\frac{y_{i+1} - y_i}{y_i}\right)^2}}$$

For Fuzzy Time Series models, we have based on Theil's U statistics, [49] which takes the formula below; the U2 statistic will take the value 1 under the naïve forecasting method. Values less than 1 indicate greater forecasting accuracy than the naïve forecasts, values greater than 1 indicate the opposite.

RESULTS AND DISCUSSION

Estimation and Prediction of ARFIMA Models: The Olive Oil price series is a monthly price of Extra virgin, a part of data were retrieved from FRED data base covered the period (1/1980-6/2017), for the period (7/2017-6/2019) we uploaded them form MUNDI index site, (2019). The above data set is analyzed by using, mainly, the R packages: arfima and AnalyzeTS, the choice of R program, especially for ARFIMA models, we followed the suggestion of [50].

The Olive oil market has changed over the period (1990-2018). Olive Oil Prices are mainly related to the levels of: production, consumption and even speculation. In term of production and according to IOC statistics it is 100 times larger than when it in 1990, with 3321(1000t) as a max production level recorded in (2011/12), also, the average yearly increase in olive oil production during the period (1990/2018) was 3%. This is, approximately, the same trend recorded for consumption and international trade (import and export) on these oils or on the oilseeds from which they are extracted. The major consumption regions are also relative concentrated in North Africa and UE. There is a great relationship between olive oil consumption levels, local developed level and population.

The series of OOPs as showed in graph is clearly no stationary; for making decision about the stationarity and non-stationarity, the Augmented Dickey Fuller tests (ADF), the Generalized or modified Augmented Dickey Fuller test (ADF-GLS) and Phillips-Perron unit root tests are used of the Olive Oil prices are displayed in Table above. In the test procedure, we first include the trend and constant terms; but the former was no-significant. All results for these tests indicate the non-stationarity of the olive oil prices time series.

The series exhibits long memory and no seasonality, Figure (5). We know well the inverse relation between the supply of a product and its price; we think at the beginning that during the time of the harvest of the olive the prices of the olive's oil diminish. This assumption is not verified for the olive oil price and this after the analysis of the seasons; which is well established in the Figure (A) in Appendix*; the seasonal component appears to be constant, so all the years in the sample have a very similar pattern*. Despite that and according to seasonality coefficients showed in Table(A) in appendix, we can note that the olive oil prices reaches its peak at the beginning of the year (exactly in January) and its min in June. According to the autocorrelation function (ACF) figure, $\frac{\hat{\rho}_v}{\hat{\rho}_o}$, for lags of v = 1 up to v = 40 months. We see that the decay in the function is very slow, indicating a d close to, or inside the non-stationary region [0.5, 8).

The *ARFIMA*(*p*,*d.q*) models are estimated via conditional Maximum Likelihood (ML) using the R Software.

Different *ARFIMA*(*p*,*d.q*) models specifications have been estimated with $p + q \leq 4$. There is evidence of unit roots and the selected ARFIMA model, for all samples, is ARFIMA(1, d, 1). The selected ARFIMA(1, d, 1) model corresponds to the smallest AIC and SBC information criteria. The results are reported in Table 2 above. Accordingly, all parameters are significant. $d = 0.14 \in \left[0, \frac{1}{2}\right]$, so the ARFIMA process is stationary with long memory; It so clear there is a strong long positive dependence in prices. We can see form Figures (5, 6) There are two pronounced peaks in olive oil prices, in the 1997s and in the second quarter of 2006, according to these structural change points, we tried to divide the sample into three sub-periods; the most noticed result was: estimates of *d* varied greatly across different sample periods and sample sizes and are generally not robust at all, this result have been reported by [51].

Estimation and Prediction for Fuzzy Models: The motivation for using FTS is that we find it difficult to identify a trend or cycle component in the time series observations of Olive Oil prices and can therefore be analyzed using Fuzzy Time Series methods. For the calculating fuzziness of time series, we used: [43] Heuristic method [45, 52] and Singh models [53]; the required calculations are given through the following steps:

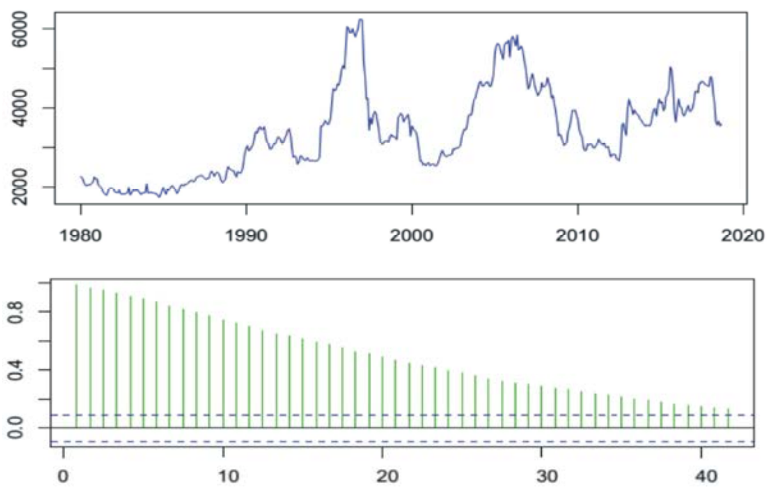

Fig. 5: Monthly Olive Oil prices, with sample autocorrelation function (ACF). Source: own plotting using data set.

Table 2: Unite Root tests of Olive Oil Prices

Product\unit root tests	ADF	ADF-GLS	PP
Olive Oil Prices	-2.351 (0.156)	-1.558 (0.119)	-2.178 (0.214)

Source: Estimation results of Eviews program.

Table 3: ARFIMA Model estimation Results

					95% CI	
Variable	Coefficient	Std. Error	t-Statistic	Prob.	Low	High
Mean	3285.42	688.68	4.770	0.000	1932.06	4638.78
d	0.141	0.0611	2.288	0.022 (**)	0.01977	0.26027
AR(1)	0.963	0.0176	54.52	0.000	0.92866	0.99811
MA(1)	0.113	0.0567	2.004	0.045	0.00221	0.22542

Source: Estimation outputs of arfima R packages. (**) The conventional tests based on the statistic $\frac{(d*\sigma)}{\sigma_d}$ can be applied, because the maximum likelihood estimator for *d* has an asymptotic normal distribution.

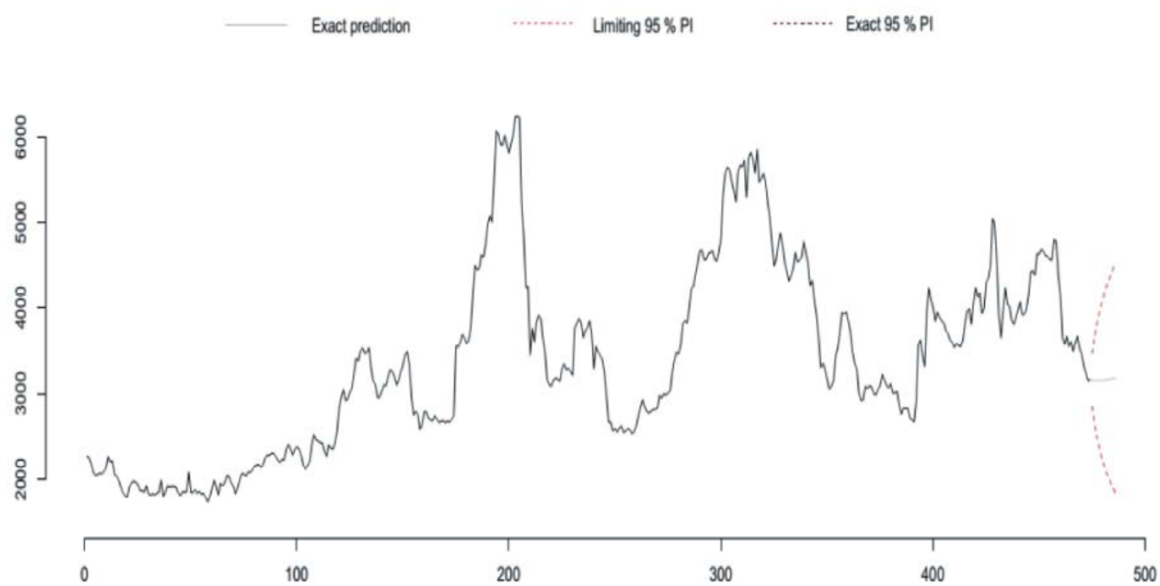

Fig. 6: Forecasts of Olive Oil Price with 95% confidence bands of ARFIMA (1, 0.14, 1) model. Source: Plot results of R program

Table 4: Fuzzy Sets determination

Set	down	up	mid	Number of observations
u_1	1700	1.919	1.810	40
u_2	1.919	2.138	2.029	34
u_3	2.138	2.357	2.248	33
u_3	2.357	2.576	2.467	17
u_5	2.576	2.795	2.686	36
u_6	2.795	3.014	2.905	32
u_7	3.014	3.233	3.124	37
u_8	3.233	3.452	3.343	29
u_9	3.452	3.671	3.562	39
u_{10}	3.671	3.890	3.781	29
u_{11}	3.890	4.110	4.000	27
u_{12}	4.110	4.329	4.219	16
u_{13}	4.329	4.548	4.438	19
u_{14}	4.548	4.767	4.657	29
u_{15}	4.767	4.986	4.876	7
u_{16}	4.986	5.205	5.095	6
u_{17}	5.205	5.424	5.314	5
u_{18}	5.424	5.643	5.533	10
u_{19}	5.643	5.862	5.752	9
u_{20}	5.862	6.081	5.971	8
u_{21}	6.081	6.300	6.190	3

Source: Own calculates using R program

Step 1: Determination of Universe of Discourse: We define the universe of discourse Ω as an interval which covers all data. $\Omega = [Min(y_t) - d_i; Max(y_t + d_2]$, where $Min(y_t) = 1736.47$ and $Max(y_t) = 6241.913$ denote the minimum and maximum values of data set respectively. d_1 and d_2 are arbitrarily selected as to cover all data of universe of discourse of Olive Oil prices data. We put, $d_1 = 36.47$ and $d_2 = 58.087$. Thus, the universe of discourse would be defined as: $\Omega = [1700; 6300]$.

Step 2: Definition of Fuzzy Sets: The universe of discourse $\Omega = [1700; 6300]$ is divided into equal sub-intervals according to the well known formula in statistics, $k = 1 + 3.32 * \log(T)$ With, k: number of sub-intervals and T: number of observations, $T = 465$ observations, so, $k = 1 + 3.32 * \log(465) \approx 21$.

According to these sub-intervals and the function of membership- see Definition (1) - , we define the fuzzy sets M_i, i: 1,2, ... ,21, as:

As indicated in definition (1), u_i are the sub-intervals, while numbers in nominator of fuzzy sets m_i denote membership degrees of u_i to m_i, taking values in the interval [0,1].

Step 3: Fuzzification: Of Oil time series time series data.

Step 4: Definition of Fuzzy Logic Relationships: In this step, fuzzy logical relationships are defined between the fuzzyfied data and then, fuzzy relationship groups are formed.

Classification process is performed based on the current status of the fuzzy logical relationships. The list of fuzzy relationship groups are illustrated in Table 4.

Step 5: Select Best Fuzzy Time Series Models Based on Accuracy Criterion: Table (7) depicts the prediction results for the sample test (1986-2018M06) estimated from the four different models. According to the table, the Singh model, [53] shows the forecast advantage, with little Accuracy criterion: (Root Mean Square Errors (RMSE), when the test period is lengthy. In contract, when the test period is relatively shorter, the fuzzy time series model has been proved to be more effective than the ARIMA model.

Overall, for FTS approach the results show the Sing model has the lowest prediction error, followed by the Abbasov - Manedova, model, [52] and ARFIM model. As it shows in the figure below, the forecasted olive oil prices describe the original prices of the corresponding time series very well.

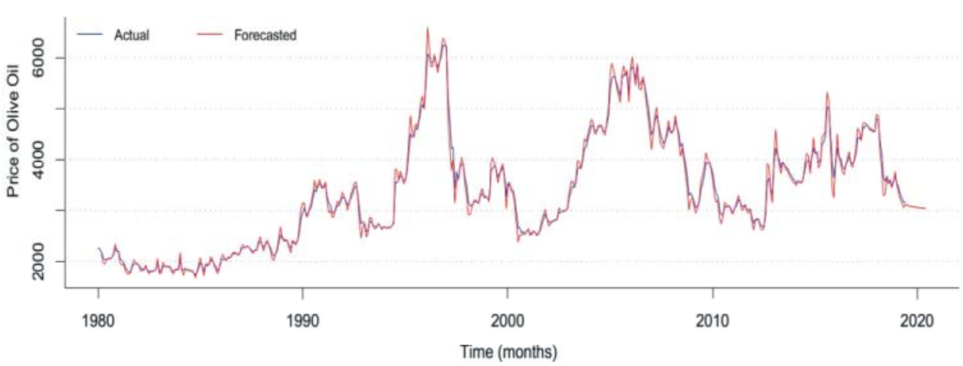

Fig. 7: Forecasts of Olive Oil Price with (Singh, 2008) FTS model. Source: Own plotting using R program. (**) The red line indicates forecasted values; we see clearly the decreased tendency of future olive oil prices according to FTS model.

Table 5: Determination of Fuzzy Sets

$m_1 =$	$^{1}/_{u_1} + {}^{0.5}/_{u_2} + {}^{0}/_{u_3} + {}^{0}/_{u_4} + {}^{0}/_{u_5} + {}^{0}/_{u_6} + {}^{0}/_{u_7} + {}^{0}/_{u_8} + {}^{0}/_{u_9} + {}^{0}/_{u_{10}} + {}^{0}/_{u_{11}} + {}^{0}/_{u_{12}} + {}^{0}/_{u_{13}} + {}^{0}/_{u_{14}} + {}^{0}/_{u_{15}} + {}^{0}/_{u_{16}} + {}^{0}/_{u_{17}} + {}^{0}/_{u_{18}} + {}^{0}/_{u_{19}} + {}^{0}/_{u_{20}} + {}^{0}/_{u_{21}}$
$m_2 =$	$^{0.5}/_{u_1} + {}^{1}/_{u_2} + {}^{0.5}/_{u_3} + {}^{0}/_{u_4} + {}^{0}/_{u_5} + {}^{0}/_{u_6} + {}^{0}/_{u_7} + {}^{0}/_{u_8} + {}^{0}/_{u_9} + {}^{0}/_{u_{10}} + {}^{0}/_{u_{11}} + {}^{0}/_{u_{12}} + {}^{0}/_{u_{13}} + {}^{0}/_{u_{14}} + {}^{0}/_{u_{15}} + {}^{0}/_{u_{16}} + {}^{0}/_{u_{17}} + {}^{0}/_{u_{18}} + {}^{0}/_{u_{19}} + {}^{0}/_{u_{20}} + {}^{0}/_{u_{21}}$
$m_3 =$	$^{0}/_{u_1} + {}^{0.5}/_{u_2} + {}^{1}/_{u_3} + {}^{0.5}/_{u_4} + {}^{0}/_{u_5} + {}^{0}/_{u_6} + {}^{0}/_{u_7} + {}^{0}/_{u_8} + {}^{0}/_{u_9} + {}^{0}/_{u_{10}} + {}^{0}/_{u_{11}} + {}^{0}/_{u_{12}} + {}^{0}/_{u_{13}} + {}^{0}/_{u_{14}} + {}^{0}/_{u_{15}} + {}^{0}/_{u_{16}} + {}^{0}/_{u_{17}} + {}^{0}/_{u_{18}} + {}^{0}/_{u_{19}} + {}^{0}/_{u_{20}} + {}^{0}/_{u_{21}}$
$m_4 =$	$^{0}/_{u_1} + {}^{0}/_{u_2} + {}^{0.5}/_{u_3} + {}^{1}/_{u_4} + {}^{0.5}/_{u_5} + {}^{0}/_{u_6} + {}^{0}/_{u_7} + {}^{0}/_{u_8} + {}^{0}/_{u_9} + {}^{0}/_{u_{10}} + {}^{0}/_{u_{11}} + {}^{0}/_{u_{12}} + {}^{0}/_{u_{13}} + {}^{0}/_{u_{14}} + {}^{0}/_{u_{15}} + {}^{0}/_{u_{16}} + {}^{0}/_{u_{17}} + {}^{0}/_{u_{18}} + {}^{0}/_{u_{19}} + {}^{0}/_{u_{20}} + {}^{0}/_{u_{21}}$
$m_5 =$	$^{0}/_{u_1} + {}^{0}/_{u_2} + {}^{0}/_{u_3} + {}^{0.5}/_{u_4} + {}^{1}/_{u_5} + {}^{0.5}/_{u_6} + {}^{0}/_{u_7} + {}^{0}/_{u_8} + {}^{0}/_{u_9} + {}^{0}/_{u_{10}} + {}^{0}/_{u_{11}} + {}^{0}/_{u_{12}} + {}^{0}/_{u_{13}} + {}^{0}/_{u_{14}} + {}^{0}/_{u_{15}} + {}^{0}/_{u_{16}} + {}^{0}/_{u_{17}} + {}^{0}/_{u_{18}} + {}^{0}/_{u_{19}} + {}^{0}/_{u_{20}} + {}^{0}/_{u_{21}}$
$m_6 =$	$^{0}/_{u_1} + {}^{0}/_{u_2} + {}^{0}/_{u_3} + {}^{0}/_{u_4} + {}^{0.5}/_{u_5} + {}^{1}/_{u_6} + {}^{0.5}/_{u_7} + {}^{0}/_{u_8} + {}^{0}/_{u_9} + {}^{0}/_{u_{10}} + {}^{0}/_{u_{11}} + {}^{0}/_{u_{12}} + {}^{0}/_{u_{13}} + {}^{0}/_{u_{14}} + {}^{0}/_{u_{15}} + {}^{0}/_{u_{16}} + {}^{0}/_{u_{17}} + {}^{0}/_{u_{18}} + {}^{0}/_{u_{19}} + {}^{0}/_{u_{20}} + {}^{0}/_{u_{21}}$
$m_7 =$	$^{0}/_{u_1} + {}^{0}/_{u_2} + {}^{0}/_{u_3} + {}^{0}/_{u_4} + {}^{0}/_{u_5} + {}^{0.5}/_{u_6} + {}^{1}/_{u_7} + {}^{0.5}/_{u_8} + {}^{0}/_{u_9} + {}^{0}/_{u_{10}} + {}^{0}/_{u_{11}} + {}^{0}/_{u_{12}} + {}^{0}/_{u_{13}} + {}^{0}/_{u_{14}} + {}^{0}/_{u_{15}} + {}^{0}/_{u_{16}} + {}^{0}/_{u_{17}} + {}^{0}/_{u_{18}} + {}^{0}/_{u_{19}} + {}^{0}/_{u_{20}} + {}^{0}/_{u_{21}}$
$m_8 =$	$^{0}/_{u_1} + {}^{0}/_{u_2} + {}^{0}/_{u_3} + {}^{0}/_{u_4} + {}^{0}/_{u_5} + {}^{0}/_{u_6} + {}^{0.5}/_{u_7} + {}^{1}/_{u_8} + {}^{0.5}/_{u_9} + {}^{0}/_{u_{10}} + {}^{0}/_{u_{11}} + {}^{0}/_{u_{12}} + {}^{0}/_{u_{13}} + {}^{0}/_{u_{14}} + {}^{0}/_{u_{15}} + {}^{0}/_{u_{16}} + {}^{0}/_{u_{17}} + {}^{0}/_{u_{18}} + {}^{0}/_{u_{19}} + {}^{0}/_{u_{20}} + {}^{0}/_{u_{21}}$
$m_9 =$	$^{0}/_{u_1} + {}^{0}/_{u_2} + {}^{0}/_{u_3} + {}^{0}/_{u_4} + {}^{0}/_{u_5} + {}^{0}/_{u_6} + {}^{0}/_{u_7} + {}^{0.5}/_{u_8} + {}^{1}/_{u_9} + {}^{0.5}/_{u_{10}} + {}^{0}/_{u_{11}} + {}^{0}/_{u_{12}} + {}^{0}/_{u_{13}} + {}^{0}/_{u_{14}} + {}^{0}/_{u_{15}} + {}^{0}/_{u_{16}} + {}^{0}/_{u_{17}} + {}^{0}/_{u_{18}} + {}^{0}/_{u_{19}} + {}^{0}/_{u_{20}} + {}^{0}/_{u_{21}}$
$m_{10} =$	$^{0}/_{u_1} + {}^{0}/_{u_2} + {}^{0}/_{u_3} + {}^{0}/_{u_4} + {}^{0}/_{u_5} + {}^{0}/_{u_6} + {}^{0}/_{u_7} + {}^{0}/_{u_8} + {}^{0.5}/_{u_9} + {}^{1}/_{u_{10}} + {}^{0.5}/_{u_{11}} + {}^{0}/_{u_{12}} + {}^{0}/_{u_{13}} + {}^{0}/_{u_{14}} + {}^{0}/_{u_{15}} + {}^{0}/_{u_{16}} + {}^{0}/_{u_{17}} + {}^{0}/_{u_{18}} + {}^{0}/_{u_{19}} + {}^{0}/_{u_{20}} + {}^{0}/_{u_{21}}$
$m_{11} =$	$^{0}/_{u_1} + {}^{0}/_{u_2} + {}^{0}/_{u_3} + {}^{0}/_{u_4} + {}^{0}/_{u_5} + {}^{0}/_{u_6} + {}^{0}/_{u_7} + {}^{0}/_{u_8} + {}^{0}/_{u_9} + {}^{0.5}/_{u_{10}} + {}^{1}/_{u_{11}} + {}^{0.5}/_{u_{12}} + {}^{0}/_{u_{13}} + {}^{0}/_{u_{14}} + {}^{0}/_{u_{15}} + {}^{0}/_{u_{16}} + {}^{0}/_{u_{17}} + {}^{0}/_{u_{18}} + {}^{0}/_{u_{19}} + {}^{0}/_{u_{20}} + {}^{0}/_{u_{21}}$
$m_{12} =$	$^{0}/_{u_1} + {}^{0}/_{u_2} + {}^{0}/_{u_3} + {}^{0}/_{u_4} + {}^{0}/_{u_5} + {}^{0}/_{u_6} + {}^{0}/_{u_7} + {}^{0}/_{u_8} + {}^{0}/_{u_9} + {}^{0}/_{u_{10}} + {}^{0.5}/_{u_{11}} + {}^{1}/_{u_{12}} + {}^{0.5}/_{u_{13}} + {}^{0}/_{u_{14}} + {}^{0}/_{u_{15}} + {}^{0}/_{u_{16}} + {}^{0}/_{u_{17}} + {}^{0}/_{u_{18}} + {}^{0}/_{u_{19}} + {}^{0}/_{u_{20}} + {}^{0}/_{u_{21}}$
$m_{13} =$	$^{0}/_{u_1} + {}^{0}/_{u_2} + {}^{0}/_{u_3} + {}^{0}/_{u_4} + {}^{0}/_{u_5} + {}^{0}/_{u_6} + {}^{0}/_{u_7} + {}^{0}/_{u_8} + {}^{0}/_{u_9} + {}^{0}/_{u_{10}} + {}^{0}/_{u_{11}} + {}^{0.5}/_{u_{12}} + {}^{1}/_{u_{13}} + {}^{0.5}/_{u_{14}} + {}^{0}/_{u_{15}} + {}^{0}/_{u_{16}} + {}^{0}/_{u_{17}} + {}^{0}/_{u_{18}} + {}^{0}/_{u_{19}} + {}^{0}/_{u_{20}} + {}^{0}/_{u_{21}}$

$m_{14} =$	$0/u_1 + 0/u_2 + 0/u_3 + 0/u_4 + 0/u_5 + 0/u_6 + 0/u_7 + 0/u_8 + 0/u_9 + 0/u_{10} + 0/u_{11} + 0/u_{12} + 0.5/u_{13} + 1/u_{14} + 0.5/u_{15} + 0/u_{16} + 0/u_{17} + 0/u_{18} + 0/u_{19} + 0/u_{20} + 0/u_{21}$
$m_{15} =$	$0/u_1 + 0/u_2 + 0/u_3 + 0/u_4 + 0/u_5 + 0/u_6 + 0/u_7 + 0/u_8 + 0/u_9 + 0/u_{10} + 0/u_{11} + 0/u_{12} + 0/u_{13} + 0.5/u_{14} + 1/u_{15} + 0.5/u_{16} + 0/u_{17} + 0/u_{18} + 0/u_{19} + 0/u_{20} + 0/u_{21}$
$m_{16} =$	$0/u_1 + 0/u_2 + 0/u_3 + 0/u_4 + 0/u_5 + 0/u_6 + 0/u_7 + 0/u_8 + 0/u_9 + 0/u_{10} + 0/u_{11} + 0/u_{12} + 0/u_{13} + 0/u_{14} + 0.5/u_{15} + 1/u_{16} + 0.5/u_{17} + 0/u_{18} + 0/u_{19} + 0/u_{20} + 0/u_{21}$
$m_{17} =$	$0/u_1 + 0/u_2 + 0/u_3 + 0/u_4 + 0/u_5 + 0/u_6 + 0/u_7 + 0/u_8 + 0/u_9 + 0/u_{10} + 0/u_{11} + 0/u_{12} + 0/u_{13} + 0/u_{14} + 0.5/u_{15} + 0.5/u_{16} + 1/u_{17} + 0.5/u_{18} + 0/u_{19} + 0/u_{20} + 0/u_{21}$
$m_{18} =$	$0/u_1 + 0/u_2 + 0/u_3 + 0/u_4 + 0/u_5 + 0/u_6 + 0/u_7 + 0/u_8 + 0/u_9 + 0/u_{10} + 0/u_{11} + 0/u_{12} + 0/u_{13} + 0/u_{14} + 0/u_{15} + 0/u_{16} + 0.5/u_{17} + 1/u_{18} + 0.5/u_{19} + 0/u_{20} + 0/u_{21}$
$m_{19} =$	$0/u_1 + 0/u_2 + 0/u_3 + 0/u_4 + 0/u_5 + 0/u_6 + 0/u_7 + 0/u_8 + 0/u_9 + 0/u_{10} + 0/u_{11} + 0/u_{12} + 0/u_{13} + 0/u_{14} + 0/u_{15} + 0/u_{16} + 0/u_{17} + 0.5/u_{18} + 1/u_{19} + 0.5/u_{20} + 0/u_{21}$
$m_{20} =$	$0/u_1 + 0/u_2 + 0/u_3 + 0/u_4 + 0/u_5 + 0/u_6 + 0/u_7 + 0/u_8 + 0/u_9 + 0/u_{10} + 0/u_{11} + 0/u_{12} + 0/u_{13} + 0/u_{14} + 0/u_{15} + 0/u_{16} + 0/u_{17} + 0/u_{18} + 0.5/u_{19} + 1/u_{20} + 0.5/u_{21}$
$m_{21} =$	$0/u_1 + 0/u_2 + 0/u_3 + 0/u_4 + 0/u_5 + 0/u_6 + 0/u_7 + 0/u_8 + 0/u_9 + 0/u_{10} + 0/u_{11} + 0/u_{12} + 0/u_{13} + 0/u_{14} + 0/u_{15} + 0/u_{16} + 0/u_{17} + 0/u_{18} + 0/u_{19} + 0.5/u_{20} + 1/u_{21}$

Source: Own calculates using R program

Table 6: Fuzzy relationships 21 groups

"A1->A1, A2"	"A2->A1, A2, A3"	"A3->A2, A3, A4"
"A4->A3, A4, A5"	"A5->A4, A5, A6, A9"	"A6->A5, A6, A7, A9"
"A7->A6, A7, A8, A10"	"A8->A6, A7, A8, A9, A10, A11"	"A9->A7, A8, A9, A10, A11"
"A10->A8, A9, A10, A11, A12"	"A11->A9, A10, A11, A12"	"A12->A8, A9, A11, A12, A13"
"A13->A11, A12, A13, A14, A16"	"A14->A13, A14, A15"	"A15->A12, A13, A14, A15, A16, A17"
"A16->A13, A14, A15, A16, A18"	"A17->A16, A17, A18, A19"	"A18->A17, A18, A19, A20"
"A19->A17, A18, A19, A20"	"A20->A19, A20, A21"	"A21->A16, A21"

Source: Own calculates using R program

Table 7: Select best Fuzzy Time Series Models based on Accuracy criterion.

Models	ME	MAE	MPE	MAPE	MSE	RMSE	U
ARFIMA	8.78	105.32	0.225	2.863	26853.65	163.87	---
Chen, (1996)	-31.50	146.512	-1.522	4.475	37946.83	194.79	1.1679
Heuristic Huarng, (2001)	-15.99	128.09	-0.615	3.933	26356.09	162.345	0.9733
Abbasov – Manedova(2010)	5.349	65.377	0.246	1.859	9591.41	97.93	0.5837
Singh, (2008)	2.721	47.463	0.095	1.493	3458.67	58.81	0.3519

Source: Own calculates using R program

Table 8: Forecasting performance of ARFIMA and Fuzzy Time Series models

		Forecasted			
		Fuzzy Logic Model Singh, (2008)		ARFIMA	
Period	Observed	Forecasted	Dif (%)	Forecasted	Dif (%)
2018M10	3491.82	3457.68	0.98	3609.18	-3.36
2018M11	3591.27	3574.47	0.47	3454.54	3.81
2018M12	3679.11	3766.53	-2.38	3602.81	2.07
2019M01	3533.61	3498.85	0.98	3689.23	-4.40
2019M02	3466.16	3364.43	2.93	3496.17	-0.87
2019M03	3325.86	3226.01	3.00	3445.10	-3.59
2019M04	3243.70	3139.51	3.21	3287.97	-1.36
2019M05	3158.12	3063.66	2.99	3218.78	-1.92
2019M06	3162.49	3121.55	1.29	3132.96	0.93
2019M07	----	3108.98	----	3159.41	----
2019M08	----	3090.91	----	3156.43	----
2019M09	----	3084.80	----	3155.41	----
2019M10	----	3079.18	----	3155.87	----
2019M11	----	3074.82	----	3157.45	----
2019M12	----	3070.47	----	3159.88	----
2020M01	----	3066.13	----	3162.97	----
2020M02	----	3061.79	----	3166.57	----
2020M03	----	3057.45	----	3170.56	----
2020M04	----	3053.10	----	3174.86	----
2020M05	----	3048.76	----	3179.39	----
2020M06	----	3044.42	----	3184.09	----

Sample: 1980M01 to 2018M08. See, Table (3) and Table (4), for Models and Parameters Estimates

This quantification of the accuracy for these models is an important step to justifying the usage of the model in forecasting process. According to the estimates showed in Table (7), we have selected the Sing model form the FTS family and ARFIMA model to forecast future prices. For detailed results, see the Table (8) below.

Through the forecast results, we find that the ARFIMA model underestimates the price levels of olive oil, as opposed to the Singh FTS model, which overestimates them, as described by the sign of difference in percentage of observed and estimated values.

CONCLUSION

Several statistical time series models and techniques have been developed and implemented in the financial markets, especially to forecast prices and stocks dynamics. In this context, accurate forecast occupies an important position in this process. Therefore, in this study we focused on comparison between an ARFIMA and Fuzzy Time Series Models to select the best model for forecasting Olive Oil Prices (OOPs) from 1986M06 to 2018M06. From the application process and in terms of forecasting of OOPs the (Singh, 2008) FTS model has been shown to be more accurate and robust. Given that olive oil prices expectations (*according to our models results*) can presents a positive trend; farmers, enterprises and first-ranked producers countries in general, wanting to explore new markets, probability would be confronted to two main challenges, (1) They should preserve at the same time the existing market shares and (2) The elaboration of new production process (to maintain the production costs) and new marketing strategies which can increase the profits of these agents. Where prices increase, *have local producers become wealthy at least*? Unfortunately, it is not so simple given the supply chain (and the role of mega-companies of this agricultural product), we state that the IOC has a big challenge (under its missions) to elaborate strategies for a fair price for olive for producers and consumers.

As a perspective, we think that an econometric study on the effect of factors of production, consumption and even meteorological factors on the prices of olive oil would be a very interesting study to better understand the dynamic and patterns of prices.

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Appendixes

Table (A): Seasonality coefficients estimate

Months	1	2	3	4	5	6	7	8	9	10	11	12
Seasonal Factor	1.01	1.00	1.001	1.00	0.99	0.98	0.99	0.99	1.00	1.00	1.00	0.99

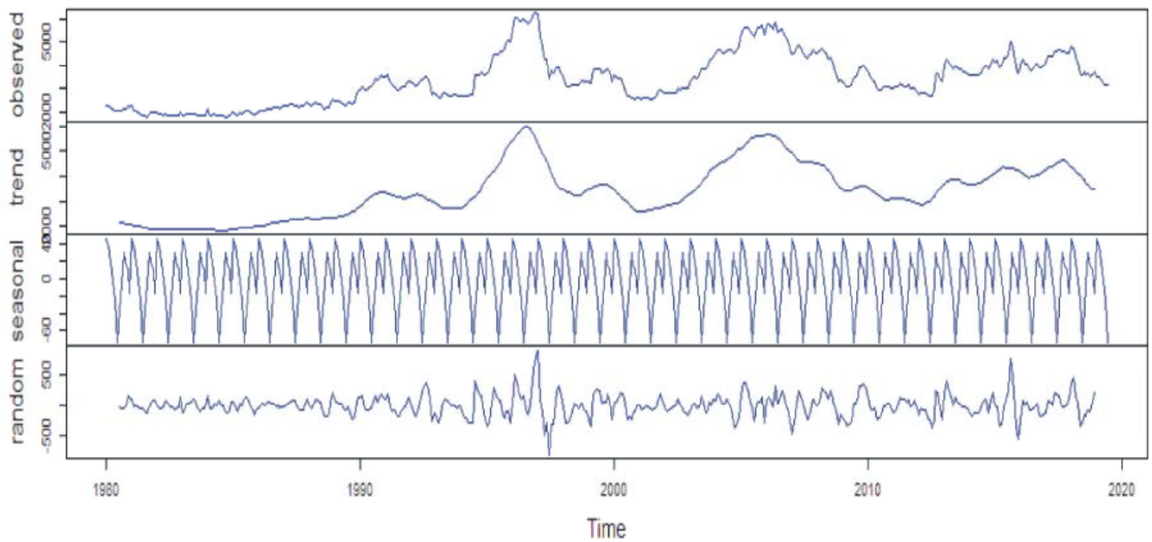

Fig. A: Seasonality components of Olive Oil Price time series. With Scaling Factors for multiplicative seasonal analysis

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