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Comparison of Newton-Raphson Based Modified Laplace Adomian Decomposition Method and Newton's Interpolation and Aitken's Method for Solving Quadratic Riccati Differential Equations

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Abstract: There has been greater attempt to solving differential equations by analytic methods and numerical methods. Most of authors treated numerical approach to solve first order ordinary differential equations, specially nonlinear Riccati differential equation (RDE). Numerical Laplace transform method is applied to approximate the solution of nonlinear (quadratic) Riccati differential equations mingled with Adomian decomposition method. A new technique is given by Vinod M and Dimple R by reintroducing the unknown function in Adomian polynomial with that of well-known Newton-Raphson formula. In this paper we will study this problem by using of Newton's interpolation and Aitken's method as a hybrid technique by using these two types of interpolation to solve nonlinear Riccati differential equation, some examples in which comparisons are made among the Numerical Laplace transform method, exact solutions, ADM (Adomian decomposition method), HPM (Homotopy perturbation method), Taylor series method and the method Proposed by Vinod M and Dimple R.

Key words: Riccati differential equation · Analytic method · Numerical method · Newton's interpolation method · Aitken's method

INTRODUCTION

Most significant classes of nonlinear differential equation is Riccati differential equation (RDE) of the form [1]:

$$\frac{dy}{dx} = q(x)y + r(x)y^2 + p(x), y(0) = a \qquad (1)$$

where q(x), r(x) and p(x) are the known scalar functions and a is an arbitrary constant. This equation named after the name of Italian nobleman Count Jacopo Francesco Riccati (1676-1754) [1, 2]. The applications of this equation may be found not only in random processes, optimal control and diffusion problems, but also in stochastic realization theory, optimal control, robust stabilization, network synthesis and financial mathematics. In the field of applied science and engineering the RDEs have played an important role, A one dimensional static Schrodinger equation is closely related to RDEs. Satisfying projective Riccati equations, solitary wave solutions of nonlinear partial differential equations can be expressed as polynomials in two elementary functions [2, 3]. In some control theory problems such as dynamic games, linear system with Markovian jumps and stochastic controls RDEs act predominantly [3, 4]. Apart from these applications RDEs is also used in stochastic realization theory, robust stabilization and network synthesis and presently in financial mathematics [2]. Much attention has been given to solve these kinds of equations due to the above applications. Certain methods are there in literature to solve the RDEs. In [5], Ghorbani and Momani applied the piecewise variational iteration method (VIM) to solve the RDEs. Differential transform method [2, 6] is adopted to find the solution of RDEs. Taiwo and Osilagun [7] approximated the solution of RDEs by Iterative algorithm. Perturbation iteration algorithm (PIA) has been presented in solving RDEs [8]. Vahidi has made the comparison among HPM, ADM and LTDM in solving RDEs in [9]. For solving these kinds of equations Yang, *et al*. [3] employed the hybrid functions and Tau method. In [10] the authors developed the iterative methods ADM, MADM, VIM, MVIM, HPM, MHPM and HAM to solve the general RDE. Laplace transform is a powerful tool in

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solving linear problems but it is incapable of solving nonlinear problems. A well-known numerical algorithm Laplacetrans forms and Adomian decomposition method has conquered much importance in solving many linear and nonlinear problems which provides a series solution. Suheil A. Khuri was the first to apply Laplace decomposition algorithm to solve a class of nonlinear differential equation [11]. A combined Laplace Adomian decomposition method is used to solve nonlinear Volterra integral equation with weakly kernel [12]. In [13], Majid Khan, *et al*. solved nonlinear coupled partial differential equations with the help of Laplace Decomposition method. LDM is also implemented to obtain the series solution of nonlinear fractional differential equations [14]. Waleed Al-Hyani [15] solved nth order Integro differential equations by the usage of LT-ADM. In [16], Modified Laplace decomposition method is proposed for solving Lane-Emden type differential equation. LDM is exercised to solve the Logistic differential equations in [17]. Wazwaz [18] employed CLT-ADM for solving nonlinear volterra-integro differential equations. For handling the solutions of nonlinear system of partial differential equation Laplace decomposition method and pade approximant is used in [19]. Hence this method is utilized to solve many more problems like Singular initial value problems [20], Double singular boundary value problems [21], Higher order boundary value problems [22]. Other cited references are [23-29]. In Vinod M and Dimple R. [1] uses the Laplace transform-Adomian decomposition method to solve the Quadratic RDEs. They replace the unknown function y_i in Adomian polynomial with Newton-Raphson formula, which improves the Adomian polynomial. Faith Chelimo Kosgei [30] studied the problem of solution of first order differential equation using numerical Newton's interpolation and Lagrange method by combined the newton's interpolation and Lagrange method, In this study we will combine of Newton's interpolation and Aitken's method instead of Lagrange method to solve RDE, [31-34]. Finally we verified on a number of examples and numerical results obtained show the efficiency of the method given by present study in comparison with Vinod M, Dimple R [1].

Combined Newton's Interpolation and Lagrange Method [30]: This study combine both Newton's interpolation method and Lagrange method. It used newton's interpolation method to find the second two terms then use the three values for to form a quadratic equation using Lagrange interpolation method as follows;

Newton's interpolation method [30-34].

$$f_n(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + \dots + a_n(x - x_0)(x - x_1)\dots a_2(x - x_{-1}) \tag{2}$$

where,

$$a_0 = y_0,\ a_1 = \frac{f(x_1) - f(x_0)}{(x_1 - x_0)},\ a_2 = \frac{\frac{f(x_2) - f(x_1)}{(x_2 - x_1)} - \frac{f(x_1) - f(x_0)}{(x_1 - x_0)}}{(x_2 - x_0)} \tag{3}$$

etc,

Lagrang interpolation method [30, 34].

$$y_n = \frac{(x-x_1)-(x-x_2)}{(x_0-x_1)(x_0-x_2)}y_0 + \frac{(x-x_0)-(x-x_2)}{(x_1-x_0)(x_1-x_2)}y_1 + \frac{(x-x_0)-(x-x_1)}{(x_2-x_0)(x_2-x_1)}y_2 \tag{4}$$

Description of the Method: This method will combine both Newton's interpolation method and Aitken method. It used newton's interpolation method to find the second two terms then use the three values for y to form a linear or quadratic equations using Aitken interpolation method as follows;

$$f_n(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + \dots + a_n(x - x_0)(x - x_1)\dots a_2(x - x_{-1}) \tag{5}$$

where,

$$a_0 = y_0,\ a_1 = \frac{f(x_1) - f(x_0)}{(x_1 - x_0)},\ a_2 = \frac{\frac{f(x_2) - f(x_1)}{(x_2 - x_1)} - \frac{f(x_1) - f(x_0)}{(x_1 - x_0)}}{(x_2 - x_0)} \tag{6}$$

etc

$$y_1 = a_0 + a_1(x - x_0) \tag{7}$$

$$y_2 = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) \tag{8}$$

Note: We can use Newton's Forward Interpolation Formula instead of Newton's divided Interpolation method in (2.1).

Aitken interpolation method [33].

$$P_{o,k}(x) = \frac{1}{x_k - x_o}\begin{vmatrix} y_o & x_o - x \\ y_k & x_k - x \end{vmatrix} \tag{9}$$

$$P_{o,1,2}(x) = \frac{1}{x_2 - x_1}\begin{vmatrix} P_{o,1}(x) & x_1 - x \\ P_{o,2}(x) & x_2 - x \end{vmatrix} \quad (10)$$

$$y_n = P_{o,1,2,\ldots,n}(x) = \frac{1}{x_n - x_{n-1}}\begin{vmatrix} P_{o,1,\ldots,(n-1)}(x) & x_{n-1} - x \\ P_{o,1,\ldots,(n-2),n}(x) & x_n - x \end{vmatrix} \quad (11)$$

Examples: In this section, we will check the effectiveness of the present technique (3). First numerical comparison for the following test examples taken in [1].

Example 1:

Solve $\frac{dy}{dx} = 1 + y^2(x), \quad y(0) = 0,$

By taking the step h=0.01, which has the exact solution as: $y = \tan(x)$.

First by using Newton's interpolation, we have,

$$a_0 = 0 = y_0$$
$$a_1 = \frac{f(x_1) - f(x_0)}{(x_1 - x_0)} = [\frac{dy}{dx}]_{0,0} = 1$$
$$y_1 = 0 + 1(0.01 - 0) = 0.01$$
$$a_2 = \frac{\frac{f(x_2) - f(x_1)}{(x_2 - x_1)} - \frac{f(x_1) - f(x_0)}{(x_1 - x_0)}}{(x_2 - x_0)} = \frac{[\frac{dy}{dx}]_{0.01,0.01} - [\frac{dy}{dx}]_{0,0}}{0.02 - 0} = 0.005$$
$$y_2 = 0 + 1(0.02 - 0) + 0.005(0.02 - 0)(0.02 - 0.01) = 0.020001$$

Now, forming linear and quadratic using Aitken Method

$$P_{0,1}(x) = x$$
$$P_{0,2}9x) = 1.00005_x$$
$$P0,1,2(x) = 0.005x^2 - 0.99995_x$$

Hence, we can take the approximation solution of linear and quadratic using Aitken Method, if we take quadratic using Aitken Method, we find the same solution given by Vinod M, Dimple R [1, 8], Table 1.

Example 2:

Solve $\frac{dy}{dx} = 1 + y^2(x), \quad y(0) = 0,$

By taking the step h = 0.01, which has the exact solution as: $y = \frac{e^{2x} - 1}{e^{2x} + 1}$

First by using Newton's interpolation, we have,

$$a_0 = 1 = y_0$$
$$a_1 = \frac{f(x_1) - f(x_0)}{(x_1 - x_0)} = [\frac{dy}{dx}]_{0,1} = -1$$
$$y_1 = 1 - 1(0.01 - 0) = 0.99$$
$$a_2 = \frac{\frac{f(x_2) - f(x_1)}{(x_2 - x_1)} - \frac{f(x_1) - f(x_0)}{(x_1 - x_0)}}{(x_2 - x_0)} = \frac{[\frac{dy}{dx}]_{0.01,0.99} - [\frac{dy}{dx}]_{0,1}}{0.02 - 0.01} = 0.505$$
$$y_2 = 1 - 1(0.02 - 0) + 0.505(0.02 - 0)(0.02 - 0.01) = 0.980101$$

Now, forming linear and quadratic using Aitken Method

$$P_{0,1}(x) = x$$
$$P_{0,2}(x) = 1.00005x$$
$$P_{0,1,2}(x) = 0.005x^2 + 0.99995x$$

Hence, we can take the approximation solution of linear and quadratic using Aitken Method, if we take quadratic using Aitken Method, we find the same solution given by Vinod M, Dimple R [1, 8], Table 2.

Example 3:

Solve, $\frac{dy}{dx} = 1 - y^2(x) + 2y(x), \; y(0) = 0,$

By taking the step h = 0.01

First by using Newton's interpolation, we have,

$$a_0 = 0.5 = y_0$$
$$a_1 = \frac{f(x_1) - f(x_0)}{(x_1 - x_0)} = [\frac{dy}{dx}]_{0,05} = 0.5$$
$$y_1 = 0.5 + 0.5(0.01 - 0) = 0.5$$
$$a_2 = \frac{\frac{f(x_2) - f(x_1)}{(x_2 - x_1)} - \frac{f(x_1) - f(x_0)}{(x_1 - x_0)}}{(x_2 - x_0)} = \frac{[\frac{dy}{dx}]_{0.01,0505} - [\frac{dy}{dx}]_{0,5}}{0.02 - 0} = 0.25$$
$$y_2 = 0.5 + 0.5(0.02 - 0) - 0.25(0.02 - 0)(0.02 - 0.01) = 0.50995$$

Now, forming linear and quadratic using Aitken Method

$$P_{0,1}(x) = x$$
$$P_{0,2}(x) = 1.00995x$$
$$P_{0,1,2}(x) - 0.995x^2 + 0.99005x$$

Hence, we can take the approximation solution of linear and quadratic using Aitken Method, if we take quadratic using Aitken Method, we find the same solution given by Vinod M, Dimple R [1, 8], Table 3.

Table 1: Solution of $\frac{dy}{dx} = 1 + y^2(x), \quad y(0) = 0,$

x	Combined Newton's Interpolation and Aitken	Vinod M and Dimple R method	s
0	0	0	0
0.01	0.010000000	0.010000083	0.010000333
0.02	0.020001000	0.020000667	0.020002667
0.03	0.030003000	0.03000225	0.030009003
0.04	0.040006000	0.040005334	0.040021347
0.05	0.050010000	0.050010419	0.050041708
0.06	0.060015000	0.060018006	0.060072104
0.07	0.070021000	0.070028597	0.070114558
0.08	0.080028000	0.080042694	0.080171105
0.09	0.090036000	0.090060799	0.090243790
0.1	0.100045000	0.100083417	0.100334672

Table 2: Solution of $\frac{dy}{dx} = 1 + y^2(x), \quad y(0) = 0,$

x	Combined Newton's Interpolation and Aitken	Vinod M and Dimple R method	s
0	0	0	0.0000000E+00
0.01	0.010000000	0.009999917	2.4998750E-07
0.02	0.020001000	0.019999333	1.9996001E-06
0.03	0.030003000	0.02999775	6.7469637E-06
0.04			

Table 3: Solution of $\frac{dy}{dx} = 1 - y^2(x) + 2y(x), \ y(0) = 0,$

x	Combined Newton's Interpolation and Aitken	Vinod M and Dimple R method	Taylor Series method	ADM	HPM
0	0	0	0	0	0
0.01	0.010000000	0.010100585	0.010100330	0.010100330	0.010100330
0.02	0.020199000	0.020404690	0.020402612	0.020402612	0.020402612
0.03	0.030597000	0.030915863	0.030908719	0.030908719	0.030908719
0.04	0.041194000	0.041637666	0.041620432	0.041620431	0.041620432
0.05	0.051990000	0.052573672	0.052539435	0.052539435	0.052539435
0.06	0.062985000	0.063727448	0.063667310	0.063667309	0.063667310
0.07	0.074179000	0.075102547	0.075005529	0.075005526	0.075005529
0.08	0.085572000	0.086702496	0.086555447	0.086555443	0.086555447
0.09	0.097164000	0.098530785	0.098318300	0.098318293	0.098318300
0.1	0.108955000	0.110590857	0.110295195	0.110295185	0.110295195

CONCLUSIONS

In this work, we have been solve the Riccati nonlinear first order differential equation by the combined Newton's interpolation and Aitken's method, we compare the result for some examples with Vinod M and Dimple R method, we find the same results given by Vinod M and Dimple R.

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