

Conformal Tensor and Lanczos Generator

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Abstract: For spacetimes with Petrov types O, N and III, we obtain the spinor structure of the corresponding Lanczos potential and conformal tensor.

Key words: Lanczos potential • Petrov types • Weyl and Lanczos spinors • Canonical tetrad • Weyl tensor

INTRODUCTION

The Lanczos spinor [1-7] is given by:

$$L_{ABCD} = l_A l_B l_C (\Omega_0 l_D - \Omega_4 o_D) + (l_A l_B o_C + (o_A * l_B) l_C) (-\Omega_1 l_D + \Omega_5 o_D) + \\ (o_A o_B l_C + (o_A * l_B) o_C) (\Omega_2 l_D - \Omega_6 o_D) + o_A o_B o_C (-\Omega_3 l_D + \Omega_7 o_D), \quad (1)$$

In terms of the spinors associated to the null tetrad of Newman-Penrose (NP) [8-12]:

$$l^\mu \leftrightarrow o^A o^B, \quad n^\mu \leftrightarrow l^A l^B, \quad m^\mu \leftrightarrow o^A l^B, \quad \bar{m}^\mu \leftrightarrow l^A o^B, \quad (2)$$

On the other hand, in [6, 7, 13, 14] was studied the Lanczos potential [15-18] via the expression:

$$K_{\mu\nu\alpha} = \frac{1}{3} (2F_{\mu\nu;\alpha} + F_{\alpha\nu;\mu} - F_{\alpha\mu;\nu} + F_{\nu\lambda}^{;\lambda} g_{\alpha\mu} - F_{\mu\lambda}^{;\lambda} g_{\alpha\nu}), \quad (3)$$

where $F_{\mu\nu} = -F_{\nu\mu}$ with spinor:

$$\varphi_{AB} = \phi_0 l_A l_B - \phi_1 (o_A * l_B) + \phi_2 o_A o_B, \quad (4)$$

Being ϕ_i the NP components of $F_{\mu\nu}$.

The relation [6, 19]:

$$L_{ABCD} = \frac{1}{3} (\nabla_{AD} \varphi_{BC} + \nabla_{BD} \varphi_{CA} + \nabla_{CD} \varphi_{AB}), \quad (5)$$

Is the spinor version of (3) for any $F_{\mu\nu}$ besides, the Weyl-Lanczos equations in spinor form are given by [2, 3, 20]:

$$\psi_{ACEG} = \frac{1}{2} \left(\nabla_A^{\dot{B}} L_{CEGB} + \nabla_C^{\dot{B}} L_{AEGB} + \nabla_E^{\dot{B}} L_{ACGB} + \nabla_G^{\dot{B}} L_{ACEB} \right). \quad (6)$$

We can use (5) into (6) to obtain the property:

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$$\psi_{ACEG} = -\frac{1}{3}(\square_{AC}\varphi_{EG} + \square_{AE}\varphi_{GC} + \square_{AG}\varphi_{CE} + \square_{EG}\varphi_{AC} + \square_{GC}\varphi_{AE} + \square_{CE}\varphi_{AG}), \quad (7)$$

But we know the relation [21]:

$$\square_{AC}\varphi_{EG} = \psi_{ACEQ}\varphi_G^Q + \psi_{ACGQ}\varphi_E^Q + \frac{R}{24}(\varepsilon_{EA}\varphi_{CG} + \varepsilon_{EC}\varphi_{AG} + \varepsilon_{GA}\varphi_{EC} + \varepsilon_{GC}\varphi_{EA}), \quad (8)$$

then (7) acquires the form:

$$\psi_{ACEG} = -(\psi_{EGCQ}\varphi_A^Q + \psi_{AEGQ}\varphi_C^Q + \psi_{CAEQ}\varphi_G^Q + \psi_{GCAQ}\varphi_E^Q), \quad (9)$$

which for the Petrov type O gives $0=0$, that is, (3) with arbitrary $F_{\mu\nu}$ is a Lanczos potential for any conformally flat space.

Now we consider arbitrary 4-spaces of Petrov types N and III in their respective canonical null tetrad [10, 22] and we employ the values $\phi_0 = \phi_2 = 0$, $\phi_1 = \frac{q}{2}$, being q a constant, then:

$$\varphi_{AB} = -\frac{q}{2}o_A l_B, \quad F_{\mu\nu} = q(n_\mu l_\nu - n_\nu l_\mu). \quad (10)$$

For the type N, into (9) we use (10) and the expression [3, 10]:

$$\psi_{ACEG} = \psi_4 o_A o_C o_E o_G, \quad (11)$$

To deduce that $\psi_{ACEG} = 2q \psi_{ACEG}$ that is, (3) with $F_{\mu\nu}$ given by (10) for $q = \frac{1}{2}$ is a Lanczos generator for the corresponding Weyl tensor in the canonical tetrad. Similarly, for the type III we apply (10) and the relation:

$$\psi_{EGCQ} = -\psi_3[o_E o_G(o_C * l_Q) + (o_E * l_G)o_C o_Q], \quad (12)$$

then (9) implies $\psi_{ACEG} = q\psi_{ACEG}$ hence (3) with (10) for $q = 1$ is a Lanczos potential for the conformal tensor in the canonical null tetrad.

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