

Blast Domination Number of a Graph

¹G. Mahadevan, ²A. Ahila and ³Selvam Avadayappan

¹Department of Mathematics, Gandhigram Rural Institute-
 Deemed University, Gandhigram, Dindigul

²Assistant Professor of Mathematics, Kalasalingam University

³Department of Mathematics, VHNSN College Virudhunagar - 626 001, Tamilnadu, India

Abstract: The concept of triple connected graphs with real life applications was introduced by Paulraj Joseph, considering the existence of a path containing any three vertices of G . G. Mahadevan et.al., introduced the concept of triple connected domination number of a graph. The same authors introduced triple connected domination number of a graph. Motivated by all the above papers, in this paper, we introduce another new domination parameter called, the *Blast Domination Number of a graph* with a real life application. A subset S of V of a non-trivial graph G is said to be a *Blast Dominating Set*, if S is a connected dominating set and the induced sub graph $(V - S)$ is triple connected. The minimum cardinality taken over all Blast Dominating sets is called the Blast Domination Number and is denoted by $\gamma_c^{tc}(G)$. In this paper, we initiate the parameter and investigate many more results, values for some standard and special graphs.

AMS (2010): 05C69

Key words: Domination number • Connected domination number • Triple connected domination number and chromatic number

INTRODUCTION

By a **graph** $G(V, E)$, we mean a finite, undirected graph with neither multiple edges nor loops. The **order** and **size** of G are denoted by n and m respectively. For graph theoretic terminology, we refer Chartrand and Lesniak [1]. **Degree** of a vertex v is denoted by $d(v)$, the maximum degree of a graph, G is denoted by $\Delta(G)$. A graph G is **connected** if any two vertices of G are connected by a path. A maximal connected sub graph of a graph G is called a **component** of G . The number of components of G is denoted by $\epsilon(G)$. The **complement** \bar{G} of G is the graph with vertex set V in which two vertices are adjacent if and only if they are not adjacent in G . We denote a **cycle** on p vertices by C_p , a **path** on p vertices by P_p and a **complete graph** on p vertices by K_p . A **wheel graph** W_n of order n , sometimes simply called an n -wheel, is a graph that contains a cycle of order $n - 1$ and for which every vertex in the cycle is connected to one other vertex. A **tree** is a connected acyclic graph. A **bipartite**

graph [or bigraph] is a graph whose vertex set can be divided into two disjoint sets V_1 and V_2 , such that every vertex of V_1 is adjacent to every vertex in V_2 . A graph G is **complete** if every pair of its vertices is adjacent. A **complete graph** on p vertices is denoted by K_p . The **complete bipartite graph** with partitions of order $|V_1| = m$ and $|V_2|$, is denoted by $K_{m,n}$. A star, denoted by $K_{1,p-1}$ is a tree with one root vertex and $p - 1$ pendant vertices. The **friendship graph**, denoted by F_n can be constructed by identifying n copies of the cycle C_3 at a common vertex. A **fan graph** $F_{m,n}$ is defined as the graph join $\bar{K}_m + P_n$, where \bar{K}_m is the empty graph on m nodes and P_n is the path graph on n nodes. A **helm graph**, denoted by H_n is a graph obtained from the wheel W_n by joining a pendant vertex to each vertex in the outer cycle of W_n by means of an edge. The **ladder graph** L_n is a planar undirected graph with $2n$ vertices and $n+2(n-1)$ edges. The **m-book graph**, B_m is the graph Cartesian product $S_{m+1} \times P_2$, where S_{m+1} is a star graph and P_2 is a path on two vertices. A **cut-vertex** (cut edge) of a graph

is a vertex (edge) whose removal increases the number of components. A **vertex cut** (or) separating set of a connected graph G is a set of vertices whose removal results in a disconnected graph. A graph G is **regular** of degree 'r' if every vertex of G has degree r. Such graphs are called **r-regular** graphs. Any 3-regular graph is called a **cubic** graph. Two graphs G_1 and G_2 are **isomorphic** (symbolically denoted by $G_1 \cong G_2$) if there exists a one to one correspondence between their vertex sets, which preserves adjacency.

In our literature survey, we are able to find many authors have introduced various new parameters by imposing conditions on the dominating sets. In that sequence, the concept of connectedness plays an important role in any network. A subset S of V of a nontrivial graph G is called a **dominating set** of G if every vertex in $V - S$ is adjacent to at least one vertex in S . The domination number $\gamma(G)$ is the minimum cardinality of a dominating set. The concept of **triple connected graphs** was introduced by Paulraj Joseph *et al* [2]. A graph is said to be triple connected if any three vertices lie on a path in G . In [3] the authors introduced triple connected domination number of a graph. A subset S of V of a nontrivial graph G is said to be **triple connected dominating set**, if S is a dominating set and $\langle S \rangle$ is triple connected. The minimum cardinality taken over all triple connected dominating sets is called the triple connected domination number of G and is denoted by $\gamma_{tc}(G)$. In [1], the authors Mahadevan *et.al* introduced the concept of **complementary triple connected domination number of a graph**. A subset S of V of a nontrivial graph G is said to be complementary triple connected dominating set, if S is a dominating set and the induced sub graph $\langle V-S \rangle$ is triple connected. The minimum cardinality taken over all complementary triple connected dominating sets is called the complementary triple connected domination number of G and is denoted by $\gamma_{ctc}(G)$. In [4], the authors have introduced **Neighborhood triple connected domination number of a graph**. A subset S of V of a nontrivial graph G is said to be a neighborhood triple connected dominating set, if S is a dominating set and the induced sub graph $\langle N(S) \rangle$ is a triple connected. The minimum cardinality taken over all neighborhood triple connected dominating sets is called the neighborhood triple connected domination number and is denoted by γ_{ntc} . In [5], the authors introduced **triple connected.com dominating set**. A subset S of V of a nontrivial connected graph G is said to be a triple connected.com dominating set, if S is a triple connected dominating set and the

induced subgraph $\langle V-S \rangle$ is connected. The minimum cardinality taken over all triple connected.com dominating sets is called the triple connected.com domination number and is denoted by $\gamma_{tc.com}(G)$.

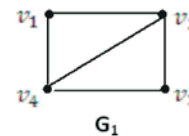
Notation 1.1:

- $K_m(P_n)$ is the graph obtained by attaching end vertices P_n in any one vertex of K_m .
- $K_m(nP_r)$ is the graph obtained by attaching n-times P_r in any one vertex of K_m .

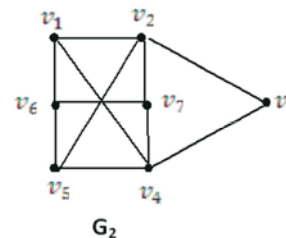
Blast Domination Number of a Graph

Definition 2.1: A subset S of V of a non-trivial connected graph G is called a **Blast dominating set (or) BD-set**, if S is a connected dominating set and the induced sub graph $\langle V-S \rangle$ is triple connected. The minimum cardinality taken over all such Blast Dominating sets is called the **Blast Domination Number (BDN) of G** and is denoted by γ_c^t . Also, any Blast Dominating set with γ_c^t vertices is called a γ_c^t -set of G .

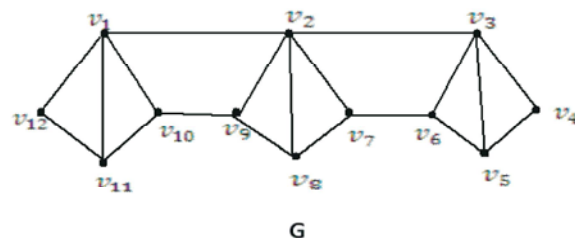
Examples 2.2: For the graph G_1 , $S = \{v_2\}$ is a Blast Dominating set and the Blast Domination Number, $\gamma_c^t(G_1) = 1$



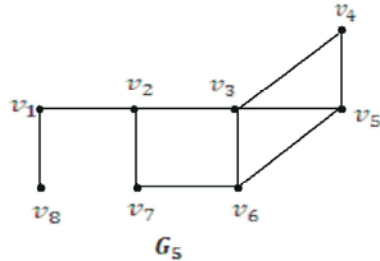
- For the graph G_2 , $\gamma_c^t(G_2) = 2$, since $\{v_1, v_4\}$ is a γ_c^t -set.



- In G_3 , $S = \{v_1, v_2, v_3\}$ is a γ_c^t -set and so the Blast Domination Number, $\gamma_c^t(G_3) = 3$



- The graph G_2 has a γ_c^{tc} -set $S = \{v_8, v_1, v_2, v_3\}$ and the Blast Domination Number, $\gamma_c^{tc}(G_3) = 4$



Remark 2.3: Throughout this paper, we consider only non-trivial connected graphs for which Blast Dominating Set exists.

Observation 2.4: The complement of a Blast Dominating set need not be a Blast Dominating Set.

Example: In the graph, G_2 , we get γ_c^{tc} -set $S_1 = \{v_1, v_2\}$. But when we consider the complement set, $V - S$, then we find its complement S fails to be triple connected.

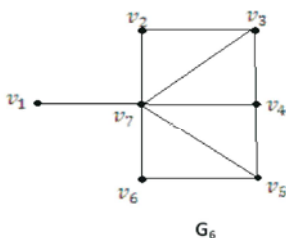
Observation 2.5: In any particular graph G , the complement of a Blast Dominating set is also Blast Dominating set, if and only if $\gamma_c^{tc}(G) \geq 3$.

Observation 2.6: Every γ_c^{tc} -set is a dominating set but converse not true.

Observation: 2.7 Every γ_c^{tc} -set is a connected dominating set but not conversely.

Since, there may be a connected dominating set excluding the pendant vertex. But which may not be γ_c^{tc} -set as $(V - S)$ has an isolated vertex, a contradiction for the triple connectedness of $(V - S)$.

Consider the graph, G_6 . Here $S = \{v_7\}$ is a connected dominating set. But $(V - S)$ is not triple connected. So S is not a γ_c^{tc} -set.



Exact Values for Some Standard Graphs:

- Every Complete graph of order $p \geq 4$, we get $\gamma_c^{tc}(K_p) = 1$.

- For Wheel Graph of order $p \geq 4$, we get $\gamma_c^{tc}(W_p) = 1$.
- For Fan Graph $p \geq 4$, we get $\gamma_c^{tc}(F_p) = 1$.
- For the Complete Bipartite graph $p \geq 4$, we get $\gamma_c^{tc}(K_{m,n}) = 2$, where both m and n are either odd or even.
- For Petersen Graph, $\gamma_c^{tc}[GP(5,2)] = 5$
- For Ladder Graph, $\gamma_c^{tc}(L_p) = \frac{p}{2}$
- For Prism Graph, $\gamma_c^{tc}[GP(n,1)] = n$

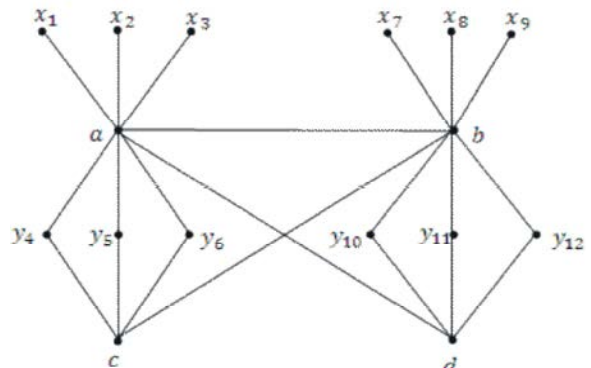
Blast Domination Number Does Not Exist for Some of the Following Standard and Special Types of Graphs:

The graph $S'(B_{k,k})$ is defined as follows:

$S'(B_{k,k})$ is a graph constructed by the vertices a, b, c, d, x, y_i where $1 \leq i \leq 2k$ and the edges

$$\{ab, bc, ad\} \cup \{ax_i / 1 \leq i \leq k\} \cup \{ay_i / 1 \leq i \leq k\} \cup \{bx_i / k+1 \leq i \leq 2k\} \cup \{by_i / k+1 \leq i \leq 2k\} \cup \{cx_i / 1 \leq i \leq k\} \cup \{dx_i / k+1 \leq i \leq 2k\}$$

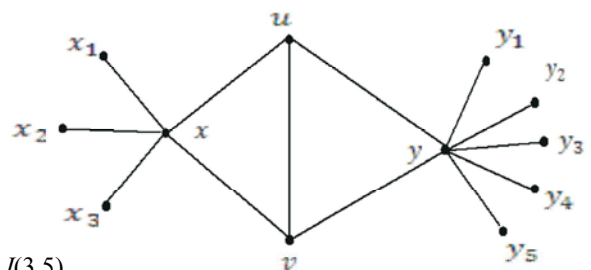
Here $\{ax_i / 1 \leq i \leq k\}$ and $\{bx_i / k+1 \leq i \leq 2k\}$ are pendant edges.



$S'(B_{3,3})$

The Jellyfish Graph Is Defined as Follows: Jellyfish Graph,

$J(m,n)$ is a collection of vertices u, v, x, y, x_i, y_j where $1 \leq i \leq m; 1 \leq j \leq n$ and the edges $uv, ux, uy, vx, vy, xx_i, yy_j$ where $1 \leq i \leq m; 1 \leq j \leq n$. Here xx_i, yy_j are the pendant edges where $1 \leq i \leq m; 1 \leq j \leq n$.



$J(3,5)$

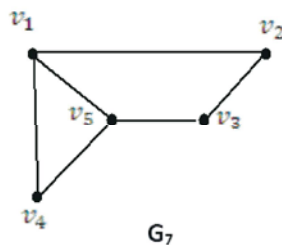
Observation: 3.2:

- For Paths, Cycles, Trees, Stars, Friendship Graphs, Helm, Book., this γ_c^{tc} - set does not exist.
- For the special graph like $S'(B_{k,k})$ and Jellyfish graphs, defined as follows, this γ_c^{tc} - set does not exist.

Theorem: 3.3: For any non-trivial connected graph G , with $p \geq 4$ vertices, we have $1 \leq \gamma_c^{tc}(G) \leq p-3$ and the bounds are sharp.

Proof: We know that, any triple connected set needs at least 3 vertices. By our definition of Blast Domination set, the induced sub graph of the complement set, $\langle V-S \rangle$ must be triple connected and so it should contain a minimum of 3 elements. Therefore, the upper bound holds. ie, as $\langle V-S \rangle \geq p-3$, $|S| \leq 3$ where $\gamma_c^{tc}(G) \leq p-3$. Also, we get $\gamma_c^{tc} = 1$ for the standard graphs like complete graph, Fan and Wheel graphs, So The lower bound holds and is sharp. Upper bound is sharp by the following example.

Example: In graph G_7 , γ_c^{tc} - set $S = \{v_1, v_2\}$ and $\gamma_c^{tc}(G_7) = 2$



Theorem: 3.4: For any non-trivial connected graph G , with $p \geq 4$, $\gamma_c^{tc}(G) = p-3$ if and only if G is isomorphic to $K_4, W_4, K_{1,3}$.

Proof: Let G be a non-trivial connected graph with $p=4$.

If Part: Let $\gamma_c^{tc}(G) = p-3 = 4-3 = 1$. Then S contains only one element and $V-S$ contains the remaining 3 elements, such that $\langle V-S \rangle$ is triple connected. This is possible only if $G \cong K_4, W_4, K_{1,3}$.

Implied by Part: Suppose $G \cong K_4, W_4, K_{1,3}$. Then obviously $\gamma_c^{tc}(G) = p-3 = 1$.

Observation: 3.5 There exists no spanning sub graph of G which contains a Blast Dominating Set.

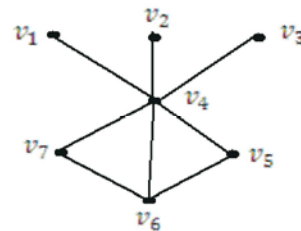
Theorem 3.6: Every γ_c^{tc} - set of G must contain the entire pendant and their supporting vertices of G .

Proof: Let S be any γ_c^{tc} dominating set of G . Let 'u' be a pendant vertex with supporting vertex 'v'. Since S is a dominating set, either u or v is in S .

Case (i): Suppose $u \notin S$ and $v \in S$ Then $u \in V-S$, which remains as an isolated vertex in $V-S$. Therefore $\langle V-S \rangle$ cannot be triple connected. This leads to a contradiction to our definition. So $u \in S$.

Case (ii): Suppose $u \in S$ and $v \notin S$ Then S containing u cannot be connected, which is a contradiction to our definition. So v should also be in S . Therefore all the pendant vertices and their supporting vertices of G must be the members of S .

Observation 3.7: The above theorem implies that $\gamma_c^{tc} \geq p(G) + 1$, where $p(G)$ is the number of pendant vertices of G . Moreover, the bound is sharp. For example, consider the graph G shown in figure below. Here $S = \{v_1, v_2, v_3, v_4\}$ is the BDN- set and $\gamma_c^{tc} = 4 = p(G) + 1$



Theorem 3.8: For any non-trivial connected graph G , $\left\lfloor \frac{p}{\Delta+1} \right\rfloor \leq \gamma_c^{tc}(G)$ and the bounds are sharp.

Proof: We know, $\left\lfloor \frac{p}{\Delta+1} \right\rfloor \leq \gamma(G) \leq p-\Delta$. Also, $\gamma(G) \leq \gamma_c^{tc}(G)$. Therefore, $\gamma_c^{tc}(G) \geq \gamma(G) \geq \left\lfloor \frac{p}{\Delta+1} \right\rfloor$ which implies $\left\lfloor \frac{p}{\Delta+1} \right\rfloor \leq \gamma_c^{tc}(G)$. For the complete graph, K_n , the bound is sharp.

Theorem 3.9: Let v be a cut-vertex of G and S be a γ_c^{tc} - set of G . If $v \in S$ then all vertices except one component of $G-v$ belongs to S .

Proof: Let $v \in S$ be a cut vertex of G and suppose that there are two vertices u and w in 2 different components of $G-v$ and not in S . Then the vertices u and w are not connected in $\langle V-S \rangle$, which is a contradiction. Hence all vertices except one component of $G-v$ belong to S .

Observation 3.10: Complement of a γ_c^{tc} - set need not be a dominating set.

Theorem 3.11: Any γ_c^{tc} - set of a graph G whose complement is a dominating set, cannot contain a cut-vertex of G .

Proof: Suppose S has a cut-vertex, say ' x '. Then by Theorem 3.9, all the vertices except one of the components of $G - \{x\}$ are in S . Hence $\langle V - S \rangle$ cannot all the vertices of S . Therefore we get a contradiction. So, S has no cut vertex.

Relation with Other Domination Parameters

Observation 4.1: For any non-trivial connected graph G , with $p \geq 5$ vertices, $\gamma_c(G) \leq \gamma_c^{tc}(G) \leq \gamma_{tc}(G) \leq \gamma_{tc.com}(G)$. Also the bounds are sharp.

Example: As per definition of both connected domination number, $\gamma_c(G)$ and Blast Domination Number, $\gamma_c^{tc}(G)$, get the same value for all graphs. Similarly, $\gamma_{tc}(G)$ and $\gamma_{tc.com}(G)$ always greater than or equal to 3, for all graph, G .

The **Goldner-Harary Graph** is a undirected graph with 11 vertices and 27 edges as shown in Figure 1. It is named after A. Goldner and Frank Harary, who proved in 1975 that it was the smallest non Hamiltonian maximal planar graph. For any Goldner – Harary graph, $\gamma_c(G) = 2$, $\gamma_c^{tc}(G) = 2$, $\gamma_{tc}(G) = 3$, $\gamma_{tc.com}(G) = 3$. Here $S = \{v_1, v_2\}$ is a minimum Blast dominating set.

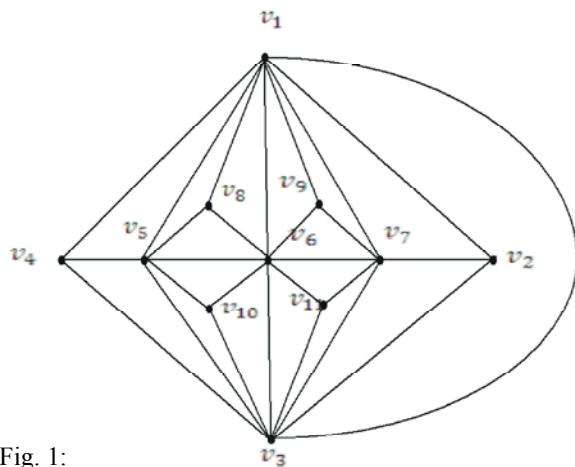


Fig. 1:

For the **Moser's Spindle graph**, we get the equality relation holds fine and so the bounds are sharp. In Figure 2, $S = \{v_1, v_2, v_3\}$ and $\gamma_c(G) = \gamma_c^{tc}(G) = \gamma_{tc}(G) = \gamma_{tc.com}(G) = 3$

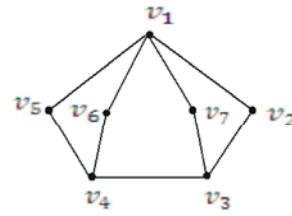


Fig. 2:

REFERENCES

1. Mahadevan, G., A. Selvam, J. Paulraj Joseph, B. Ayisha and T. Subramanian, 2013. Complementary triple connected domination number of a graph, *Advances and Applications in Discrete Mathematics*, 12(1): 39-54.
2. Paulraj Joseph, J., M.K. Angel Jebitha, P. Chithra Devi and G. Sudhana, 2012. Triple connected graphs, *Indian Journal of Mathematics and Mathematical Sciences*, 8: 61-75.
3. Mahadevan, G., A. Selvam, J. Paulraj Joseph and T. Subramanian, 2012. Triple connected domination number of a graph, *International Journal of Mathematical Combinatorics*, 3: 93-104.
4. Mahadevan, G., A. Ahila, N. Ramesh, C. Sivagnanam, Selvam Avadayappan and T. Subramanian, XXXX. Neighborhood triple connected domination number of a graph, *International Journal of Computational Engineering Research*, 04(3): 38-48.
5. Mahadevan, G., M. Lavanya, Selvam Avadayappan and T. Subramanian, 2013. Triple connected.com domination number of a graph, *International Journal of Mathematical Archive*, 4(10): 178-189.
00. Harary, F., 1972. *Graph Theory*, Addison Wesley Reading Mass.
00. Haynes, T.W., S.T. Hedetniemi and P.J. Slater, 1998. *Fundamentals of Domination in Graphs*, Marcel Dekker, Inc., New York.
00. Sampathkumar, E. and H.B. Walikar, 1979. The connected domination number of a graph, *J. Math. Phy. Sci.*, 13(6): 607-613.
00. Mahadevan, G., A. Selvam, A. Nagarajan, A. Rajeswari and T. Subramanian, 2012. Paired Triple connected domination number of a graph, *International Journal of Computational Engineering Research*, 2(5): 1333-1338.
00. Mahadevan, G., A. Selvam, A. Mydeen Bibi and T. Subramanian, 2012. Complementary perfect triple connected domination number of a graph, *International Journal of Engineering Research and Application*, 2(5): 260-265.